Comments on the Statistical Nature and on the Irreversibility of the Wave Function Collapse

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In a previous preprint, [1], reproduced here within the appendix in its revised version, we were confronted, to reach the validity of the second law of thermodynamics for an unique collapse of an unique quantum object, to the necessity of an ensemble of measures to be accomplished within copies of identical isolated systems. The validity of the second law of thermodynamics within the context of the wave function collapse was sustained by the large number of microstates related to a given collapsed state. Now, we will consider just one pure initial state containing just one initial state of the quantum subsystem, not an ensemble of identically prepared initial quantum subsystems, e.g., just one photon from a very low intensity beam prepared with an equiprobable eigenset containing two elements, an unique observation raising two likelihood outcomes. Again, we will show the statistical interpretation must prevail, albeit the quantum subsystem being a singular, unique, pure state element within its unitary quantum subsystem ensemble set. This feature leads to an inherent probabilistic character, even for a pure one-element quantum subsystem object.

A TOY: THE FAIR COIN EIGENSET

Let a two-state coin, fifty-fifty, with eigenset $\{\phi_1, \phi_2\}$, be our quantum subsystem. The initial state of this unique subsystem reads:

$$\Psi = \sum_{k=1}^{2} a_k \phi_k = \frac{\sqrt{2}}{2}\phi_1 + \frac{\sqrt{2}}{2}\phi_2, \tag{1}$$

with:

$$a_k = \int_V \phi_k^* \Psi dV = \frac{\sqrt{2}}{2} \ \forall \ k \in \{1, 2\}. \tag{2}$$

The eigenstates ϕ_1 and ϕ_2 are different eigenstates. The unique element [given by eqn. (1)] subsystem plus an unique ideal apparatus subsystem Φ will define an isolated system. The memory state of the subsystem apparatus is initially empty, and the initial state of the system is:

$$\Psi\Phi|_{t=0} = \left(\frac{\sqrt{2}}{2}\phi_1 + \frac{\sqrt{2}}{2}\phi_2\right)\Phi_{[\text{void}]}. \tag{3}$$

After a measure operator U acting on $\Psi \otimes \Phi|_{t=0}$, the system propagates forward in time to the $(t = \tau)$-state, the collapsed state for short:

$$\Psi\Phi|_{t=\tau} = \frac{\sqrt{2}}{2}\phi_1\Phi_{[\phi_1]} + \frac{\sqrt{2}}{2}\phi_2\Phi_{[\phi_2]}. \tag{4}$$

The observer is represented by the Φ apparatus subsystem, being in its own Hilbert state space H_Φ. Since $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$ are different states belonging to H_Φ, these apparatus states are mutually exclusive in H_Φ.

- How many final microstates of the isolated system are there?

The answer depends on which space the apparatus Φ resides. For Φ, the collapsed microstate is a member of H_Φ. The state given by the eqn. (4) cannot be observed in H_Φ, hence cannot be counted from H_Φ by the apparatus subsystem. There are two possible final states for the hypothetical one-element measure that are members of H_Φ, $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$, but both cannot be obtained at the same time. The collapse evolves but just one member of H_Φ subsists as an equilibrium apparatus subsystem state after the collapse. The entropy of a final collpased state $\Phi_{[\phi_k]}$ in $H_\Phi(\tau)$ is zero, since, under an one-element measure with an unique initial quantum coin state given by eqn. (1), there is just one manner to obtain the $\Phi_{[\phi_k]}$ collapsed state, since the other equally like manner leads to a different collapsed state and should not be considered as being another microstate of the same $\Phi_{[\phi_k]}$. But both the possible collapsed states leads to a same final null entropy. This unique object measure leads to reversible collapse, since the variation of entropy between states is null in any case. We will see this unique object quantum subsystem must be related to a global statistical context.

When one chooses an unique coin to accompish the unique measure, one is establishing there exists just an unique way to obtain the initial coin, to construct the initial coin. But, in fact, there is not. You may make the same coin with another bunch of metallic atoms. We do not take it into account, since a set of identical elements is an unitary set, being irrelevant which element we use to accomplish the measure. But two distinct but identical coins do not necessarily lead to identical outcomes. Hence, if one takes into account the identical manners, including the previous global context within the Universe, from which the system may evolve to the collapse, one does not modify the initial null entropy of the system, since identical coins are identical coins into the input ($t = 0$) but not necessarily identical coins from the output ($t = \tau$). Suppose you may construct the unique coin only from two different ways, $\mathbb{W}_1$ or $\mathbb{W}_2$. Via $\mathbb{W}_1$, there is one possible microstate for each collapsed result as observed by $\Phi_{[\phi_1]}$ *or* $\Phi_{[\phi_2]}$ in the apparatus subsystem reality. In the apparatus reality, the initial number of microstates of the system is also vanished, since the initial number of microstates is 1×1 (in the apparatus world, we do not describe the system via eqn. (3), since this is an object that is not an element of H_Φ. The initial state of the quantum subsystem, our coin, given by the eqn. (1), is unique for $\Phi_{[\mathrm{void}]}$. Initially, there is just one possibility for each subsystem state in the apparatus reality, hence $w_0 = 1 \times 1$ is the initial number of microstates of the [global] system as observed within the apparatus reality. The apparatus dialectics does not handle objects like the ones in the eqns. (3) and (4).). Analogously, via $\mathbb{W}_2$, there is one possible microstate for each collapsed state. But, when $\mathbb{W}_1$ and $\mathbb{W}_2$ are taken into consideration, two possible microstates emerge for each collapsed state, with the same initial null entropy.

When one accomplishes an one-element collapse experiment with various identical initial quantum subsystems (e.g., taking $\mathbb{W}_1$ and $\mathbb{W}_2$ into consideration), the result is one between the possible ones from identical objects (identical coins). A particular collapse result turns out to be inserted in a global probabilistic context related to the various identical manners by which the Universe may evolute from the past to their states in which there exist identical isolated experiments to be initiated at $t = 0$, in virtue of the entropic evolution of the Universe. The Universe entropically evolves under their various possibilities, and two different manners to construct a same coin are different ways under which the Universe may evolve to a same initial coin state, hence the null entropy, but not necessarily to the same collapsed state. Hence, even an isolated collapse from an unique coin has a global statistical context related to the different manners the Universe might have evolved, and an unique coin exhibits its global statistical bias. Since the Universe is large, a given initial subsystem, our two-state coin initial quantum subsystem, has a miriad of possible histories up to $t = 0$, say N_1, but with none of these manners giving a different object, all giving the same Ψ at $t = 0$. Analogously, one has, as $\Psi_{[\mathrm{void}]}$ possible initial states, a bag with N_2 identical elements, all given by $\Phi_{[\mathrm{void}]}$. When you isolate the system, you obtain an isolated bag with $N_1 \times N_2$ identical elements given by the eqn. (3). The number of microstates related to this bag is $w_0 = 1$. The number of microstates related to $\Phi_{[\phi_k]}$ is not $w_f = 1$ anymore, but [1]:

$$\lim_{N_1 N_2 \to \infty} \sum_{l=1}^{N_1 N_2} \xi_l^p = \lim_{N_1 N_2 \to \infty} \left[\frac{N_1 N_2}{2} + f(N_1 N_2) \right] > 1, \tag{5}$$

being $N_1 \times N_2$ the number of final histories of collapse, where the histories are, now, being instantaneously counted at τ within the Universe entropic evolution.

Taking into account the the different manners by which an initial subsystem may be obtained does not change the probability of a given collapsed state, conversely, defines it via a natural frequential interpretation within a global context. The probability associated to a given collapsed state when an unique experiment is accomplished with an unique one-element initial state is the one associated to the frequential interpretation taking into account the various manners to construct the initial state. Since the Universe may provide infinitely many manners to construct an isolated system, when one takes an exemplar into account, the probabilistical character is inherent to individual processes, since a particular result resides within a global statistical context related to different states of the Universe that leads to the same initial isolated system. Even a single photon within a low intensity beam may be constructed by different manners. A single photon does not know this, obviously, but it behaves under a global statistical context related to the different manners by which the Universe may evolve to that in which a beam of a single photon is within an isolated system with an apparatus.

There are not two final microstates, $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$, for the collapsed apparatus, and one should not say the entropy variation is $\Delta S = k \ln 2 - k \ln 1$, since different microstates are physically distinguishable a posteriori, carrying different measurable physical properties, encapsulated within the difference between the eigenvectors $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$. In fact, an unique one-element collapse is a reversible process for quantum initial subsystems with just an unique element. But it is very difficult to observe, since the Universe entropically evolves among a miriad of possibilities leading to identical initial quantum subsystems, inserting an individual measure within the Universe's entropy evolution statistical context, from which the number of final collapsed microstates of a given collapsed state is greater than 1, leading to an irreversible collapse even with a single photon beam as initial quantum subsystem, e.g., since this single photon within the beam turns out to be in a context of a very large number of avaiable microstates for each possible collapsed state, a context in which the final entropy of a given collapsed state is greater than the initial null entropy.

APPENDIX: COMMENTS ON THE ENTROPY OF THE WAVE FUNCTION COLLAPSE

The Boltzmann formula: a source of misconception for a reckless vision

[2] At a first glance, one may think the wave function collapse violates the second law of thermodynamics, since

a quantum system prepared as a superposition of eigenstates of a given operator suddenly undergoes to a more restrictive state. But this is not the case, in virtue of the fact that a superposition and a eigenstate are states on equal footing. The use of the Boltzmann formula:

$$S = k \ln w, \tag{6}$$

for the entropy S of a thermodinamically closed system, where $k = 1.38 \times 10^{-23} \, JK^{-1}$ is the Boltzmann constant, leads, at a first glance, to the impression that the entropy should have a greater value before the collapse, under an erroneous assumption that the initial number, w_0, of microstates, w, should be greater than the final number of microstates, w_f, in virtue of the needed quantity of eigenstates, $w_0 > 1$, used to construct the wave function before the collpse, in contrast to the apparent $w_f = 1$ after the collapse. We will see that the converse occurs. Furthermore, one should, firstly, define the thermodinamically closed system as consisting of two subsystems: the quantum object subsystem plus the classical apparatus subsystem.

A simple solution for this apparent paradox

[3] Consider a quantum subsystem Ψ: prepared as a superposition of the n eigenstates $\{\phi_k\}$, with $1 \leq k \leq n$, of a given operator Φ with finite non-degenerated spectrum:

$$\Psi = a_1\phi_1 + \cdots + a_n\phi_n = \sum_{k=1}^{n} a_k\phi_k, \tag{7}$$

where:

$$a_k = \int_V \phi_k^* \Psi dV, \tag{8}$$

is the inner product with which the Hilbert state space is equipped. The * denotes the complex conjugation and dV the elementar volume of the physical space V of a given representation.

Up to the measure, before the interaction between a classical apparatus subsystem, designed to obtain observable eingenvalues of the operator Φ, and a quantum subsystem Ψ given by eqn. (7), there exists just one microstate of the global system consisted by apparatus subsystem plus quantum subsystem, since these two subsystems are not initially correlated and the initial microstate of the quantum subsystem Ψ is just the unique state Ψ as well the initial microstate of the classical apparatus subsystem is the unique one in which it has no registered eigenvalue.

Hence, in virtue of the initial independence of the subsystems, the initial microstate of the global thermodinamically closed system has multiplicity $w_0 = 1 \times 1 = 1$, being the initial entropy of the global system given by:

$$S_0 = k \ln 1 = 0, \tag{9}$$

in virtue of the eqn. (6).

One may argue the initial state of the classical apparatus subsystem has got a multiplicity greater than 1, since this subsystem seems to have internal modes compatible with an empty memory. We emphasize this is not the case, since the state of the memory defines the apparatus state, being this state an empty one in spite of any apparatus internal modes before an accomplished measure [4]. The same comment is valid for the quantum subsystem, since the state of this subsystem is Ψ, previously defined by the superposition of a Φ operator eigenstates, $\{\phi_k\}$, being the object Ψ an unique one. These objects, by definition, are initially constrained to these defined states, and one does not need to take into account the different manners by which these subsystems should equally evolute to their respective initial states.

Once a measure is accomplished, there will exist n possible eigenvalues to be registered within the memory of the classical apparatus subsystem, viz., since there are n different final situations for the global system, where n is the number of non-degenerated eigenvectors of the Φ operator. A reckless short-term analysis would lead to the conclusion that the final number of microstates of the global system, w_f, should be $w_f = n$, since it seems to be the number of ways by which a final collapsed state is reached. But such a conclusion is wrong, since the final state is not simply a collapsed one with a label on it. Differently from a case in which a pair of unbiased dice is thrown, where a particular result of a throw of dice is not physically different from any other result, except for the labels on them, a given collapsed state encapsulates physical content. Each collapsed state is a different final state with its characteristic multiplicity, and one should not enroll the possible collapsed states within a same bag with $w_f = n$ possible collapsed elements. Comparing with the throw of dice case, if you erased the dice numbers, their labels, you could not infer the difference between the results, but the physical content within the collapsed wave function result would lead one to infer the difference between different results, between different outcomes of collapse of Ψ.

- Different physical characteristics implying different outcomes for the wave function collapse define evolutions from the initial global system to new states of the global system, instead of different configurations for a same final state.

In the throw of dice example, the different outcomes are different configurations of a same final state. If the collapsed wave function was a state with n different possible configurations for this same collapsed state, the final number of microstates would be $w_f = n$, but this is not the case.

For the collapsed states, the multiplicities of the possible final results are not necessarily the same, since they depend on the outcome probabilities of their respective eigenvalues. Let p be the label of the eigenvalue with the least reliable ($\neq 0$) [5] outcome probability. The outcome probability of a given eigenvalue is given by the Max Born's rule, from which the least probability, of the p-labeled eigenvalue, is simply given by $a_p^* a_p$, where [see eqn. (8)]:

$$a_p^* a_p = \left| \int_V \phi_p^* \Psi dV \right|^2 \neq 0. \tag{10}$$

Applying a frequential interpretation for the probability, the least multiplicity of microstates is $N a_p^* a_p$, where N is the quantity of *state-balls* within an *a posteriori interpreted quantum-subsystem-urn* (we are emphasizing that the interaction with the classical apparatus subsystem permits a classical [6], under the frequential sense, a posteriori, interpretation of probabilities, since any quantum effects of probabilistic superposition of amplitudes cease after the collapse, permitting a frequential interpretation via Born's rule). Such a frequential interpretation requires $N \to \infty$, i.e., infinitely many measures to be accomplished on identical quantum subsystems by the classical apparatus subsystem, but we will back to this point later.

The least final entropy of the global system, related to the outcome probability of the p-labeled eigenvalue, reads:

$$S_f = k \ln \left(N a_p^* a_p \right). \tag{11}$$

From the eqns. (9) and (11), the least possible entropy variation turns out to be:

$$\Delta S = S_f - S_0 = k \ln \left(N a_p^* a_p \right). \tag{12}$$

From the eqn. (12), we infer that the second law of thermodynamics holds *iff*:

$$N a_p^* a_p \geq 1 \Rightarrow a_p^* a_p \geq \frac{1}{N}, \tag{13}$$

since $N > 0$. Now, we will prove the following theorem:

Theorem: *The second law of thermodynamics holds for the wave function collapse under a frequential interpretation via Max Born's rule and, once accomplished the collapse, the collapse is an irreversible phenomenon.*

Proof: • Suppose the converse, i.e., that the second law of thermodynamics does not hold for the wave function collapse under a frequential interpretation via Max Born's rule. In virtue of eqn. (12), one has:

$$\Delta S = S_f - S_0 = k \ln \left(N a_p^* a_p \right) < 0 \Rightarrow N a_p^* a_p < 1. \tag{14}$$

Since $a_p \neq 0$ [7], $N \geq 1/(a_p^* a_p)$ violates the condition stated by the eqn. (14). But $N \to \infty$, in virtue of the frequential interpretation, hence $N > 1/(a_p^* a_p)$, and the eqn. (14) is an absurd. We conclude the second law of thermodynamics holds within the terms of this theorem. The proof the collapse is an irreversible phenomenon follows as a corollary of this theorem. In fact:

$$N > 1/(a_p^* a_p) \Rightarrow N a_p^* a_p > 1 \therefore$$

$$\Delta S = k \ln \left(N a_p^* a_p \right) > 0, \tag{15}$$

and the collapse of the wave function is an irreversible phenomenon, being $\Delta S > 0$ the entropy variation of the thermodinamically closed system: quantum subsystem plus classical apparatus subsystem. •

The law of large numbers states the probability of an event p, P_p, is given by the limit:

$$\lim_{N\to\infty} \frac{\sum_{l=1}^{N} \xi_l^p}{N} = P_p, \tag{16}$$

where ξ_l^p assumes the value 1 when the event p occurs, or zero otherwise. If $a_p^* a_p \equiv P_p \neq 0$, the limit must obey:

$$\lim_{N\to\infty} \frac{\sum_{l=1}^{N} \xi_l^p}{N} = \frac{\lim_{N\to\infty} \sum_{l=1}^{N} \xi_l^p}{\lim_{N\to\infty} N} \neq 0. \tag{17}$$

From eqn. (17), we conclude $\lim_{N\to\infty} \sum_{l=1}^{N} \xi_l^p$ cannot be finite, since N grows without limit. Hence:

$$\lim_{N\to\infty} \sum_{l=1}^{N} \xi_l^p > 1. \tag{18}$$

Particularly, the eqn. (18) gives the number of microstates of the p-labeled eigenstate, proving the above theorem. Rigorously, one should substitute:

$$N \to N + \frac{f(N)}{a_p^* a_p}, \tag{19}$$

within the above theorem proof, with:

$$\lim_{N\to\infty} \frac{f(N)}{N} = 0. \tag{20}$$

Such choice leads to:

$$\sum_{l=1}^{N} \xi_l^p = N a_p^* a_p = \left(N + \frac{f(N)}{a_p^* a_p} \right) a_p^* a_p = N \left(a_p^* a_p + \frac{f(N)}{N} \right) \quad \therefore \tag{21}$$

$$\frac{\sum_{l=1}^{N} \xi_l^p}{N} = a_p^* a_p + \frac{f(N)}{N}. \tag{22}$$

Taking the limit $N \to \infty$ in eqn. (22), we recover the law of large numbers. Taking the limit $N \to \infty$ in eqn. (21), one obtains in virtue of the eqn. (18):

$$\lim_{N \to \infty} \sum_{l=1}^{N} \xi_l^p = \lim_{N \to \infty} \left(N + \frac{f(N)}{a_p^* a_p} \right) a_p^* a_p > 1 \quad \therefore \tag{23}$$

$$\lim_{N \to \infty} \left(N + \frac{f(N)}{a_p^* a_p} \right) > \frac{1}{a_p^* a_p}. \tag{24}$$

Eqn. (24) is the argument used to prove the theorem, as one infers from the eqn. (19).

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[1] A. Assis, http://vixra.org/pdf/1106.0004v4.pdf **2011**, 1 (2011).

[2] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics* (John Wiley and Sons, 1985), second edition ed.

[3] S. Gasiorowicz, *Quantum Physics* (John Wiley and Sons, 2003), third edition ed.

[4] The irrelevance of the apparatus internal modes compatible with a given apparatus memory state asserts the hypothesis of an unbiased apparatus subsystem. Any result to be measured by the apparatus subsystem must have the same number of equally like apparatus microstates. If some result was related to a different number of apparatus compatible microstates, the results with the maximal number of apparatus compatible microstates would be biased. The collapse should not be caused by apparata biases. In virtue of this hypothesis, one may neglect the apparatus internal modes compatible with a particular apparatus memory state, since the same number of internal modes is common to all the memory states, and the variation of entropy cancels out the same common number (say w_a): $\Delta S = S_f - S_0 = k \ln (w_f \times w_a) - k \ln (1 \times 1 \times w_a) = k \ln w_f - k \ln (1 \times 1)$, where w_f is the number of microstates of a given final state of the global isolated system in which the apparatus has registered the respective collapsed state, considering the apparatus memory state as its unique degree of freedom.

[5] If $a_p = 0$, the respective eigenstate ϕ_p, within the superposition representing Ψ [see eqn. (7)], turns out to be an impossible collapsed state. Such consideration would be totally void, since the final microstate associated to it would never occur, being $\Delta S = k \ln 0 - k \ln 1 = -\infty$ [see eqns. (6) and (9)] a violation of the second law of thermodynamics, in accordance with the impossibility of a final microstate with $a_p = 0$.

[6] Here, the classical designation resides within the counting process after the collapse. We are not saying the final collapsed state leads to a classical interpretation of the quantum object, we are enphasizing that the dialectics after the collapse to interpret frequency of a given collapsed state is the classical one via Born's rule. One does not count quantum waves, but the discret signals of a collapsed object. Surelly, alluding, e.g., to the double-slit canonical example, the diffraction pattern on the screen has not a discrete counterpart, but the points on the screen, when the intensity of the source is reduced, have and may be counted.

[7] Remember the reliability defining the p-labeled eigenstate, see eqn. (10) again and its inherent paragraph.