A lagrangian formulation of the Lambda CDM model with predictions relating to particle astrophysics.

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Abstract. The Lambda CDM model or "concordance model" is the standard model of modern cosmology. This model contains a number of separate theories with different mathematical formulations. The subject of this paper is a proposed Lagrangian which would provide a unified mathematical framework for the concordance model of cosmology. This unification is achieved by a combination of the f(R) approach, with the standard LCDM approach. It is postulated that Dark matter-energy fields depend on the Ricci curvature R, and dark energy fields weaken as the Ricci scalar (R) increases or strengthen as R decreases. The utility of this is a great simplification compared to the currently accepted formulation. One Lagrangian plus one constraint can model the same physics as the three Lagrangian's found in the standard formulations. The unexpected degree of difficulties in observing the fermion like WIMPS of dark matter in Earth based observatories are also explained by this theory.

1. Introduction

The ΛCDM model or "concordance model" is the standard model of modern cosmology. This model contains a number of separate theories with different mathematical formulations. $f(R)$ gravity is an actively researched alternative in which gravity is modeled with functions of the Ricci curvature R in the action. The f(R) program, and the inflation with dark matter plus dark energy program both have desirable traits. Suppose they were both combined, by parameterizing the scalar and vector fields of inflation using the Ricci curvature. This unification would in effect make the scalar and vector inflationary models into f(R) models. What would be the consequences of such a unification? Can a unified model explain the negative results of searches for dark matter particles on earth[1, 2], or the halos of dark matter around galaxy's[3], or the apparent lack of dark matter within 13,000 light years of the sun [4]?

The subject of this paper is a proposed Lagrangian which would provide a unified mathematical framework for the concordance model of cosmology. In the process new insight will be gained into the nature of dark matter and dark energy which the separate formulations do not provide. The motivation for writing this paper is to provide a unified mathematical basis for Lambda CDM.

There are certain mysteries to the standard model of cosmology. It contains vast amounts of matter and energy of a mysterious type described as "dark". Dark matter which we cannot detect in spite of massive efforts such as the cryogenic dark matter search II (CDMS II) and XENON100[1, 2]. Energy which we can only detect by it's effect on the acceleration of the expansion of the universe. Energy which is then modeled with a simple constant Λ. This simple model makes very good predictions and matches observations.

There has to be a mathematically more elegant, informative, and dynamic formulation than the current collection of no less than three very different parts (depending on how one counts). The following outlines an attempt at a unified and ultimately simpler model.

2. The Lagrangian

We have not observed any dark matter particles on Earth to date. The best results available are signals indistinguishable from noise[1, 2]. It has also been observed that dark matter halo's form at a characteristic distance from galaxies[3]. One way to explain these observations would be to have dark matter decay as the Ricci curvature increases. Based on those observations I postulate the following:

Dark matter-energy fields depend on the Ricci curvature R, dark energy fields weaken as (R) increases or strengthen as R decreases.

The fields precise behavior will depend on which metric and hence which R is in effect. In the case of a galaxy the Ricci curvature corresponding to Schwarzchild's metric would be used, in the case of the universe the Friedman-Lemaitre-Robertson-Walker R would be used.

To realize the postulate mathematically first write the fields with R as a parameter.

$$A^{\mu} = A^{\mu}(R), \phi = \phi(R) \tag{1}$$

Upon review of the published literature one finds Lagrangian's for inflation, dark matter, dark energy, etc[5, 6, 7, 8, 9, 10, 11]. The standard formulation of Lambda CDM would consist of Einsteins field equation, a Lagrangian for inflation, another one for dark matter, and another one for dark energy. These all model the universe very well. So, it makes sense to use these theories as a starting point. To realize this postulate mathematically let us write the Lagrangian for a scalar and vector field, parameterized with and dependent upon Ricci curvature R, in curved space time. Each field has a mass which is at least an effective mass that has no assumed dependence on any dynamical variables. The resulting action is....

$$\begin{array}{c} s = \int \sqrt{-g}\left(-\frac{R}{16\pi} - k(\phi)\nabla^{\mu}\phi\nabla_{\mu}\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left(D_{\mu}\gamma^{\mu} - m_{\psi}\right)\psi \right. \\ \left. -\frac{1}{2}\left(m_{\phi}^2\phi^2 + m_A^2 A^{\mu}A_{\mu}\right) - \frac{R}{6}\left(\phi^2 + A^{\mu}A_{\mu}\right) - \beta\bar{\psi}\gamma^{\mu}\psi A_{\mu}\right) d^4x \end{array} \tag{2}$$

Using functions of the Ricci curvature has been done before in a program known as f of R gravity. Here the functions f(R) are identified with the scalar and vector fields of inflation. It is assumed that said fields have at least an effective mass. This mass is not assumed to depend on any variables at the outset, however it will be shown that a value for this effective mass is derivable and can depend on both R and Λ. Mass dependence on Ricci curvature is a feature of many published models of f(R) gravity[12, 13].

These fields are similar, yet not identical, to those found in theories of inflation in which they drive the rapid expansion [5, 6, 7, 8, 9, 10, 11]. To see how inflation arises in this theory the equations of motion need to be derived and solved.

3. Equations of Motion.

Following all the elementary steps of classical field theory the Euler-Lagrange equations for this theory can be derived. One of those equations is for R itself. That is none other than the Einstein field equation. Then there are two more equations one for the scalar and one for the vector fields. One more constraint is desirable. The Stress energy tensor of this field must be proportional to the cosmological constant. This ensures agreement with known observations. The result is a set of three equations, derived from the above action.

$$\left\{ \begin{array}{c} R^{\mu\nu} - Rg^{\mu\nu} = \frac{8\pi G}{c^2}T^{\mu\nu} \\ \nabla_{\alpha}F^{\alpha\mu} - \left(\frac{m_A^2}{2} + \frac{R}{6}\right)A^{\mu} = 0 \\ \nabla_{\mu}\nabla^{\mu}\phi - \left(\frac{m_{\phi}^2}{2} + \frac{R}{6}\right)\phi = 0 \\ \left(\nabla_{\mu}\gamma^{\mu} - m_{\psi}\right)\psi = 0 \end{array} \right\} \tag{3}$$

In which the stress energy tensor has the following form.

$$\begin{aligned} T^{\mu\nu} = -2k(\phi)\nabla^{\mu}\phi\nabla^{\nu}\phi - F^{\mu\nu}g_{\lambda\delta}F^{\lambda\delta} - \left(m_A^2 + \tfrac{R}{3}\right)A^{\mu}A^{\nu} + \tfrac{i}{2}\bar{\psi}\gamma^{\mu}\nabla^{\nu}\psi \\ -g^{\mu\nu}\left(k(\phi)\nabla_{\mu}\phi\nabla^{\mu}\phi + F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}\left(m_{\phi}^2\phi^2 + m_A A^{\mu}A_{\mu}\right)\right. \\ \left. -\tfrac{R}{6}\left(\phi^2 + A^{\mu}A_{\mu}\right) + \bar{\psi}\left(D_{\mu}\gamma^{\mu} - m_{\psi}\right)\psi\right) \end{aligned} \tag{4}$$

The stress energy needs to be at least proportional to the cosmological constant times the metric. This results in the following equation of constraint, which is not derivable from the Lagrangian. In the following $\lambda$is simply a constant of proportionality. This constraint is introduced in the same spirit as the cosmological constant. Λ is an important part of most any viable cosmological model. This equation of constraint ensures that the proposed model can match observations which have already been made.

$$T^{\mu\nu} = \lambda g^{\mu\nu}\Lambda \tag{5}$$

3.1. Solutions

The next task is to solve these equations for the scalar and vector fields. First the scalar fields solution.

$$\phi\left(R\right) = \phi_0 Exp\left(\int_1^R \frac{\left(\frac{m_\phi^2}{2} + \frac{R'}{6}\right)}{\Box R'} dR'\right) \tag{6}$$

Next we will solve for the vector field. The A^0 component must be zero in order to satisfy the equation of motion. The derivatives which make up F^{00} work out that way just as one would expect for an electromagnetism like field. In the process of solving for A^0 the effective mass of the A field can be calculated.

$$\frac{m_A^2}{2} + \frac{R}{6} = 0 \rightarrow m_A = \sqrt{\frac{-R}{3}} \tag{7}$$

On the cosmic scale space time is very nearly flat. In fact the curvature of space time observed to date it slightly negative. Therefore this effective mass would be small but at a characteristic distance from concentrations of luminous matter such as galaxies. This is in accord with the observations reported in[3]. In a positively curved space time the mass of this field is imaginary. This would appear to be a problem, but for the fact that so far no dark matter particles have been detected on Earth in spite of very concerted efforts [1, 2]. This theory predicts that no WIMP corresponding to the type of vector field described here will ever be detected near a concentration of luminous matter such as the Earth.

For the space like components the solution is almost identical to that for the scalar field.

$$A^{\mu} = \left(0,\ A_0^i Exp\left(\int_1^R \frac{\left(\frac{m_A^2}{2} + \frac{R'}{6}\right)}{\Box R'} dR'\right)\right) \tag{8}$$

Where $i \in \{1, 2, 3\}$.

The effective masses of these fields are fixed theoretically by the constraint that the stress energy tensor $T^{\mu\nu}$ needs to be proportional to the cosmological constant. It is possible to determine the effective mass m_ϕ from that constraint. To find an expression for this mass note that the T^{00} component of the stress energy tensor will be of a simple form. Terms which depend on the vector field drop out as it's zero in that component. Terms which depend on the velocity $\nabla^0\phi$ can be set to zero to ensure the resulting effective mass acts as a rest mass of the particle. The resulting equation is

$$T^{00} = g^{00}\left(\frac{1}{2}m_\phi^2 + \frac{R}{6}\right)\phi^2 = \lambda g^{00}\Lambda \tag{9}$$

Which simplifies to...

$$m_\phi = \sqrt{\frac{6\lambda\Lambda - R\phi}{3\phi}} \tag{10}$$

The effective mass of this scalar field cannot be zero unless the following equation holds true.

$$R\phi(R) = 6\lambda\Lambda \tag{11}$$

This equation determines a characteristic radius at which a dark matter halo would be observed from a galaxy. This is a point at which the Schwarzchild curvature due to the galaxy gives way to the large scale FLRW space time. This is in accordance with the observations in [3]. Within this radius the space time curvature would be large enough to make the mass of the scalar field imaginary, meaning no particles. Only outside of this radius can particles associated with this field exist.

This effective mass was not a priori assumed to depend on explicitly on the Ricci curvature R. However in the f of R gravity regime implicit dependence of effective mass m on R is a standard feature found in many publications [12, 13]. The bare rest masses of these particles would be found by setting R equal to zero. When R equals zero m_A is zero. The vector field is then fundamentally massless much like an EM field. The scalar fields effective massm_ϕ wold be not be zero at that point. The scalar field has a bare rest mass the vector field only has an effective mass. The fields would still contribute stress energy to the stress-energy tensor regardless of their effective mass.

3.2. Probability of fermion fermion annihilation to curvature.

In terrestrial experiments which search for dark matter we have assumed that the dark matter will be fermionic. The way that fermion like dark matter particles behave in this theory, in terms of their effective masses, will be the same as for the above particles. However there is an even more interesting interaction in this theory. Let us consider the amplitude and cross section for the annihilation of four of these fermions into R.

$$< R|\bar{\psi}\psi\bar{\psi}\psi >=< R|A^\mu A_\mu >< A^\mu A_\mu|\bar{\psi}\psi\bar{\psi}\psi > \tag{12}$$

After some computation the answer works out to the following.

$$< R|\bar{\psi}\psi\bar{\psi}\psi >= \frac{(A_0^{\mu} A_{0\mu}) \left(\bar{\psi}_{dir}\psi_{dir}\right) e^{G[R]}}{s'[R]e^{S[R]}} \left(\frac{R}{G'[R]} - 1\right) \tag{13}$$

In equation 13 the term $G[R]$ is a functional of the Ricci curvature scalar R which results from multiplying these fields together,$S[R]$ is the action as a functional of the Ricci curvature scalar R . The terms A_0^{μ} is constant., and $\bar{\psi}_{dir}$ is the standard solution for the Dirac fields. G and S will oscillate about. The interesting part of the squared probability will look like.

$$\left|< R|\bar{\psi}\psi\bar{\psi}\psi >\right|^2 \approx (R-1)^2 = R^2 - 2R + 1 \tag{14}$$

Equation 14 shows us that the cross section for these particles simply annihilating increases in area as the curvature of space time increases, and decreases as the curvature of space time decreases. Therefore as gravity becomes stronger, the particles lifetime becomes shorter. This behavior would explain why we have had so much trouble observing dark matter fermions in experiments on earth, while their astronomical existence is beyond question.

3.3. Inflation

Inflation is in this model. To see it consider the effective masses shown in equations seven and ten. The physics of standard big bang theory is modeled using the FLRW metric. In this metric at time equals zero the curvature of space time is infinite. At that point the effective masses of these fields would be imaginary infinity. At the same time the strength of the fields would be zero. When the universe begins to expand the curvature begins to decrease, this in turn causes the mass of the field to roll towards zero. As the mass rolls it drives the inflationary expansion of the universe. All the while the dark mass of the particles is converted into dark energy of the associated fields.

Thus the story of the universe is the story of two massive fields transforming one form of energy into another, along with some other stuff we call ordinary matter.

4. Conclusions

The proposed Lagrangian contains all the physics needed to represent the Lambda CDM model. There is a source of dark matter, dark energy, and inflation. The behavior of the fields is in agreement with our overall observations . This Lagrangian also provides a minimal explanation for why dark matter has been so hard to observe in experiments such as CDMS II and XENON100. The dark matter simply decays into dark energy when the curvature R is too high. Thus there are not "particles" to detect in a region of high space time curvature, like on Earth. This would provide an explanation for why it would be harder than expected to detect these particles in a ground based experiment.

This model also explains observations of a dark matter halo around galaxies at a characteristic distance in a simple and natural way. The dark matter's effective mass is imaginary when the curvature is positive. Which means it physically and classically cannot exist.

The dark matter mass in this theory is simply the effective mass of the fields and their associated bosonic particles. There may well be other fermionic and super symmetric types of dark matter. Certainly numerous particles which will be discovered at accelerator laboratories in the future which may or may not be dark matter candidates exist. I have no hypothesis about such dark matter, or how the hypothesized particles could be produced via accelerator based experiments in this model at this time. Their is a disputed observation by Moni Bidin et. al. which may support this theory Conclusions

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