Mass of an Electromagnetic Wave

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In this paper we investigate whether an electromagnetic wave can have mass, whilst also still having a maximum velocity equal to the speed of light. We find that their mass is inversely proportional to their velocity, such that they have no mass when travelling at the speed of light. This proportionality may also help explain the duality of light.

1 Introduction

Currently it is argued that a particle which has mass cannot travel at the speed of light, since it would require an infinite amount of energy to accelerate to this speed [2]. Thus it has been argued that electromagnetic waves or photons must be massless, as these by definition travel at the speed of light when in a vacuum. In this paper we will use some relativity equations to show that a photon does have mass, but that the amount of mass varies with its speed.

2 Calculating the Mass of a Photon

We know from relativity [2] that the total energy of a particle, (e.g. an electron, a photon, etc) is given by

$$E^2 = m^2c^4 + p^2c^2, \qquad (1)$$

where E is the energy, m is the mass, p is the momentum and c is the speed of light (in a vacuum). Additionally we know that the total energy of a photon in a vacuum is given by Planck's relation [10], i.e.

$$E = hf, \tag{2}$$

where h is Planck's constant and f is the frequency of the photon. We also have a formula for the momentum of a photon [2], being

$$p = \frac{hf}{c}, \tag{3}$$

assuming that it is travelling at the speed of light. However the speed of an electromagnetic wave is dependent upon the medium that it is travelling through, its maximum being in a vacuum. For that reason we propose that the correct formulation for the momentum of a photon is

$$p = \frac{hfv}{c^2}, \tag{4}$$

which degenerates into equation (3), when $v = c$. Additionally if we take de Broglie's mass-frequency relationship, given by

$$f = \frac{mc^2}{h}, \tag{5}$$

and substitute it into equation 4, then we obtain the standard Newtonian equation for momentum, i.e. $p = mv$. Furthermore, we know that energy is conserved, so let us consider how the energy of the same photon is manifested in two different locations. In the first location we will assume that the photon is travelling through a vacuum. In the second location we will assume that it is passing through some medium such that it is slowed down. We will also assume that these two locations are far away from any gravitational fields (as they change the frequency of electromagnetic waves), and so it is only the medium in which the photon is travelling through that is changing. Ergo

$$E_1^2 = E_2^2, \tag{6}$$

$$\Longrightarrow \quad h^2 f^2 = m^2 c^4 + p^2 c^2, \tag{7}$$

$$= m^2 c^4 + \frac{h^2 f^2 v^2}{c^2}, \tag{8}$$

$$\Longrightarrow \quad m^2 = \frac{h^2 f^2}{c^4}\left(1 - \frac{v^2}{c^2}\right), \tag{9}$$

$$\Longrightarrow \quad m = \frac{hf}{c^2}\sqrt{1 - \frac{v^2}{c^2}}. \tag{10}$$

Therefore equation (10) shows us that the mass of a photon is dependent upon its speed, such that

- at speed c, the photon has no mass,
- if the photon is stopped (i.e. it "collides" with a "piece of matter") then its impact "mass" is given by hf/c^2.

These situations also demonstrate the two extremes commonly mentioned when dealing with photons, i.e. as waves and particles, respectively [8]. This is intuitive from the descriptions given above, as in the first situation the photon has no mass, and so behaves completely like a wave. Whilst in the second situation, the photon has its maximum amount of mass, and so acts in the same manner as a particle.

3 Conclusion

In this paper we investigated how an electromagnetic wave could have mass, and how the amount of mass would change with speed. We found that the wave would have zero mass when travelling at the speed of light, which correlates with current arguments [7, 3]. Conversely the wave has its maximum mass when its velocity is zero, i.e. when it "collides" with matter. Hence this explains the duality of light [8], in that it can behave like a wave (i.e. when it has no mass) and like a particle (i.e. when it has mass).

4 Further Work

Here we have considered that an electromagnetic wave has mass. However since there is a link between mass and gravity [4, 7], we could consider that an electromagnetic wave has a gravitational component. This though, would imply that it is gravity that "generates" mass and not mass that "generates" gravity, as currently thought [4, 7]. Given this situation, it would imply that an electromagnetic wave has a third component, gravity. The energy of the wave would then transfer between the three components, similar to the transfer already given between the electric and magnetic components [9], given by $E/B = v$ (where E is the electric field strength, B is the magnetic field strength and v is the velocity of the wave). It is then just a question of how this gravitational component would manifest itself. We could consider that as the wave slows down its energy density becomes sufficient to bend spacetime [2], i.e. a relativity type idea. Conversely we could consider the gravitational component to be a third wave, which generates a gravitational field around it, i.e. a Newtonian idea [1]. Some work on this has already been done in "Gravity & Electromagnetic Waves" [6].

Additionally, the concept of electromagnetic waves having mass or gravity would give credence to the idea that matter consists of these waves. Again some work on this has already been done in "The Atom Uncovered" [5].

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