The constancy of the rotational velocity curves of the spiral galaxies from large
distances from their galactic centers could be due to their geometries in form of arms.

Key words: dark matter, spiral galaxies.

To explain why the rotational velocity curves of the spiral galaxies at large radii are
constants; it was assumed the existence of dark matter [1, 2]. However, it could be due
to their geometries in form of arms [3].

In effect, as in the gravitational rotation the centrifugal (repulsive) force is compensated
by the gravitational (attractive) force, we would have that

\[ m \frac{v^2}{r} = \frac{GMm}{r^2} \]  

where \( G \) is the Newton’s gravitational constant, \( M \) and \( r \) the mass and the radius of the
spiral galaxy, respectively; \( m \) the mass of a star, and \( v = \omega r \) and \( \omega \) the linear and
angular speeds of the star, respectively; \( v \) is the velocity of the orbit corresponding to
the radius (or distance) \( r \). We have put \( M \) instead of \( M - m \) because \( M - m = M \). From
(1), it results:

\[ v = \sqrt{\frac{GM}{r}} \]  

Until a relatively small distance \( d_0 \) from the galactic center compared with \( r \), the spiral
galaxy would be a disc. For distances \( s \) between the galactic center and \( d_0 \) (0 < \( s \) ≤ \( d_0 \)),
the disc would have a thickness \( h_s \), base area \( \pi s^2 \) and mass \( M_s = \rho_s V_s = \rho_s \pi s^2 h_s \),
where \( \rho_s \) and \( V_s = \pi s^2 h_s \) are, respectively, the corresponding density and volume of
the spiral galaxy until \( s \). Then, from (2), it would be

\[ v_s = \sqrt{\frac{GM_s}{s}} = \sqrt{\frac{G \rho_s \pi s^2 h_s}{s}} = \sqrt{G \rho_s \pi s h_s} = \text{const.} \times \sqrt{s} \]
and $v_s$ varies proportionally to $\sqrt{s}$. And, the corresponding mass of the spiral galaxy until $d_0$ would be

$$M_{d_0} = \rho_s \pi d_0^2 h_s = \text{const.} \quad (4)$$

and, from (2), the velocity of the orbit at $d_0$ would be

$$v_{d_0} = \frac{GM_{d_0}}{d_0} = \frac{G\rho_s \pi d_0^2 h_s}{d_0} = \sqrt{G\rho_s \pi d_0 h_s} = \text{const.} \quad (5)$$

For distances $d$ between $d_0$ and $r$ ($d_0 < d < r$), the spiral galaxy has a structure in form of arms. If we suppose that the arms are similar, then

$$M_q = nM_{qa} = n\rho_{qa} V_{qa} \quad (6)$$

where $M_q$ is the corresponding mass of the spiral galaxy from $d_0$ until $d$, $n$ the number of arms, and $M_{qa}$, $\rho_{qa}$ and $V_{qa}$ are, respectively, the corresponding mass, density and volume of a generic arm from $d_0$ until $d$. Although the arms are curved, any cross section of them can be considered circular; hence, the volume $V_{qa}$ can be calculated as a sum of volumes of right cylinders:

$$V_{qa} = \sum_j \pi \frac{h_{qa}^2}{4} \ell_j = \pi \frac{h_{qa}^2}{4} \sum_j \ell_j = \pi \frac{h_{qa}^2 q}{4 \alpha} \quad (7)$$

where $h_{qa}$ is the thickness of the generic arm, $\ell_j$ the length of the right cylinder $j$, $q = d - d_0 = \alpha \sum_j \ell_j$ and $\alpha$ a real number, $0 < \alpha \leq 1$. Substituting (7) into (6), it results

$$M_q = n\rho_{qa} \pi \frac{h_{qa}^2}{4 \alpha} \quad (8)$$

If we substitute $\rho_s$ and $\rho_{qa}$ by $\rho$ and $h_s$ and $h_{qa}$ by $h$, where $\rho$ and $h$ are, respectively, the density and the thickness of the spiral galaxy, then

$$M_d = M_{d_0} + M_q = \rho \pi d_0^2 h + n\rho \pi \frac{h^2}{4 \alpha} \frac{d - d_0}{\alpha} \quad (9)$$

where $M_d$ is the corresponding mass of the spiral galaxy until $d$. For large values of $d$ compared with $d_0$ (which implies large radii) and very small values of $\alpha$ (which implies very large arms), together with certain values of $h$ and $n$; it results
\[ M_d = n \rho \pi \frac{h^2 d}{4 \alpha} \]  

(10)

And, from (2), it would be

\[ v_d = \sqrt{\frac{GM_d}{d}} = \sqrt{\frac{Gn\rho \pi (h^2/4)(d/\alpha)}} = \sqrt{\frac{Gn\rho \pi h^2}{4\alpha}} = \text{const.} \]  

(11)

Therefore, we conclude that the constancy of the rotational velocity curves of the spiral galaxies from large distances from their galactic centers could be due to their geometries in form of arms.


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