A Note on the Dark Matter

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The constancy of the rotational velocity curves of the spiral galaxies from large distances from their galactic centers could be due to their geometries in form of arms.

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To explain why the rotational velocity curves of the spiral galaxies at large radii are constants; it was assumed the existence of dark matter [1, 2]. However, it could be due to their geometries in form of arms.

In effect, as in the gravitational rotation the centrifugal (repulsive) force is compensated by the gravitational (attractive) force, we would have that

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \tag{1}$$

where G is the Newton's gravitational constant, M and r the mass and the radius of the spiral galaxy, respectively; m the mass of a star, and $v = \omega r$ and ω the linear and angular speeds of the star, respectively; v is the velocity of the orbit corresponding to the radius (or distance) r. We have put M instead of $M - m$ because $M - m \approx M$. From (1), it results:

$$v = \sqrt{\frac{GM}{r}} \tag{2}$$

Until a relatively small distance s from the galactic center compared with r, the spiral galaxy would be a disc of thickness h_s, base area πs^2 and mass $M_s = \rho_s V_s = \rho_s \pi s^2 h_s$, where ρ_s and $V_s = \pi s^2 h_s$ are, respectively, the corresponding density and volume of the spiral galaxy until s. Then, from (2), it would be

$$v_s = \sqrt{\frac{GM_s}{s}} = \sqrt{\frac{G\rho_s \pi s^2 h_s}{s}} = \sqrt{G\rho_s \pi s h_s} = const. \times \sqrt{s} \tag{3}$$

and v_s varies proportionally to $\sqrt{s}$.

But, for distances d between s and r ($s < d \le r$), the spiral galaxy has a structure in form of arms. If we suppose that the arms are similar, then

$$M_q = nM_{qa} = n\rho_{qa}V_{qa} \tag{4}$$

where M_q is the corresponding mass of the spiral galaxy from s until d, n the number of arms, and M_{qa}, ρ_{qa} and V_{qa} are, respectively, the corresponding mass, density and volume of a generic arm from s until d. Although the arms are curved, any cross section of them can be considered circular; hence, the volume V_{qa} can be calculated as a sum of volumes of right cylinders:

$$V_{qa} = \sum_j \pi \frac{h_{qa}{}^2}{4} \ell_j = \pi \frac{h_{qa}{}^2}{4} \sum_j \ell_j = \pi \frac{h_{qa}{}^2}{4} \frac{q}{\alpha} \tag{5}$$

where h_{qa} is the thickness of the generic arm, ℓ_j the length of the right cylinder j, $q = d - s = \alpha \sum_j \ell_j$ and α a real number, $0 < \alpha \le 1$. Substituting (5) into (4), it results

$$M_q = n\rho_{qa}\pi \frac{h_{qa}{}^2}{4} \frac{q}{\alpha} \tag{6}$$

If we substitute ρ_s and ρ_{qa} by ρ and h_s and h_{qa} by h, where ρ and h are, respectively, the density and the thickness of the spiral galaxy, then

$$M_d = M_s + M_q = \rho\pi s^2 h + n\rho\pi \frac{h^2}{4} \frac{d - s}{\alpha} \tag{7}$$

where M_d is the corresponding mass of the spiral galaxy until d. For large values of d compared with s (which implies large radii) and very small values of α (which implies very large arms), together with certain values of h and n; it results

$$M_d \approx n\rho\pi \frac{h^2}{4} \frac{d}{\alpha} \tag{8}$$

And, from (2), it would be

$$v_d = \sqrt{\frac{GM_d}{d}} \approx \sqrt{\frac{Gn\rho\pi\left(h^2/4\right)\left(d/\alpha\right)}{d}} = \sqrt{\frac{Gn\rho\pi h^2}{4\alpha}} = const. \tag{9}$$

Therefore, we conclude that the constancy of the rotational velocity curves of the spiral galaxies from large distances from their galactic centers could be due to their geometries in form of arms.

[1] F. Zwicky, "Die Rotverschiebung von extragalaktischen Nebeln", Helvetica Physica Acta 6: 110–127, 1933.

[2] F. Zwicky, Ap. J., **86**, 217, 1937.
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