First Draft

Higgs-Free Symmetry Breaking from Critical Behavior near Dimension Four

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Abstract

Starting from the infrared limit of Yang-Mills theory, we introduce here a Higgs-free model in which dynamical symmetry breaking arises from critical behavior near dimension four. Gauge bosons develop mass as a result of condensation at the Wilson-Fisher point of Renormalization Group flow. We recover the family structure of Standard Model using the technique of "epsilon expansion". Our approach also suggests a straightforward solution to the cosmological constant problem.

1. Introduction

The Standard Model of particle physics (SM) is a highly successful theory that has been in place for more than 35 years. It incorporates the $SU(3)\otimes SU(2)\otimes U(1)$ gauge model of strong and electroweak interactions along with the Higgs mechanism that spontaneously breaks the electroweak $SU(2)\otimes U(1)$ group down to the $U(1)$ group of electromagnetism. Despite its outstanding reliability, SM is viewed as a low-energy framework that is likely to be amended by new phenomena in the Terascale region. The elementary Higgs boson picture of electroweak (EW) and flavor symmetry breaking suffers from several drawbacks. In particular [ ]:

- Elementary Higgs models provide no dynamical explanation for electroweak symmetry breaking (EWSB).
- Elementary Higgs models appear highly contrived, requiring fine tuning of parameters to enormous precision.
- Elementary Higgs models with grand unification have a hierarchy problem of widely different energy scales.
- Elementary Higgs models are trivial.

- Elementary Higgs models provide no insight to flavor physics.

Similar or different drawbacks exist in supersymmetric extensions of Higgs theories (MSSM) and alternate models of EWSB such as Technicolor [ ].

2. Challenges of Yang-Mills theory

In our view, there are a couple of key roadblocks that have slowed down progress on the theoretical side of high-energy physics for the past 35 years:

- Because Yang-Mills field is self-interacting, it is inherently nonlinear and prone to undergo complex behavior [ ].
- Dynamics of Yang-Mills field is strongly coupled in infrared (IR) where perturbation theory breaks down and traditional methods of QFT fail to apply.

3. New tools: nonlinear dynamics and critical behavior

To deal with these challenges, we start from a far less explored vantage point. Specifically, we exploit the fact that both mapping theorem [ ] and the Landau-Ginzburg-Wilson (LGW) model of critical behavior [ ] enable understanding of the IR regime of gauge field theory using the principles of Renormalization Group program (RG).

- **The mapping theorem**

The electroweak group $SU(2)\otimes U(1)$ is broken at a scale approximately given by $M_{EW} = G_F^{-1/2} = 293$ GeV, in which G_F is the Fermi constant. Yang-Mills fields associated with $SU(2)$ are vectors denoted as $A_\mu^a(x)$, in which $\mu = 0,1,2,3$ is the Lorentz index and $a = 1,2,3$ is the group index. To manage the large number of equations derived from the Yang-Mills theory, it is desirable to devise a method whereby $A_\mu^a(x)$ are reduced to analog fields having less complex structure. The *mapping theorem* allows for such a reduction [ ]. Consider the action functional of classical scalar field theory defined in four-dimensional space-time:

$$S[\Phi]=\int d^4x[\frac{1}{2}(\partial\Phi)^2-\frac{1}{4!}g^2\Phi^4] \tag{1}$$

An extremum of (1) is also an extremum of the $SU(2)$ Yang-Mills action provided that:

a) g represents the coupling constant of the Yang-Mills field,

b) some components of $A_\mu^a(x)$ are chosen to vanish and others to equal each other.

In the most general case, the following approximate mapping between Yang-Mills fields and scalar $\Phi(x)$ holds:

$$A_\mu^a(x)=\eta_\mu^a\Phi(x)+O(\frac{1}{\sqrt{2}g}) \tag{2}$$

where η_μ^a are properly chosen constants. The mapping becomes exact in the Lorenz gauge $\partial^\mu A_\mu^a(x)=0$ and in the IR regime of strong coupling ($g\to\infty$).

- **<u>LGW theory near dimension four: a brief overview</u>**

Consider the Euclidean space LGW action in $D-$dimensional space-time [ ]

$$S[\Phi]=\int d^Dx[\frac{1}{2}(\partial\Phi)^2+V(\Phi)] \tag{3}$$

In particular,

$$V(\Phi)=\frac{r}{2}\Phi^2+\frac{g^2}{4!}\Phi^4-j\Phi \tag{4}$$

in which j denotes the external current coupled to Φ. According to the RG program, rescaling the cutoff $\Lambda\to\Lambda'=\frac{\Lambda}{b}$, $b>1$ and integrating out fast modes within

$\Lambda' < |k| < \Lambda$, turns the original action into an *effective* action. The effective theory represents a lower-energy image of the original theory as in

$$S[\Phi], \Lambda \to S_{eff}[\Phi_<], \Lambda' \tag{5}$$

Here, $\Phi_<(x)$ are the slow modes of the field ($|k| < \Lambda'$),

$$\Phi_<(x) = \int_{|k|<\Lambda'} \frac{d^D k}{(2\pi)^D} \Phi(k) \exp(ikx) \tag{6}$$

and

$$\int D_\Lambda[\Phi] \exp(-S[\Phi]) = \exp(-S_{eff}[\Phi_<]) \tag{7}$$

with

$$S_{eff}[\Phi_<] = \int d^D x [\frac{1}{2}(\partial\Phi_<)^2 + V_{eff}(\Phi_<)] \tag{8}$$

Invoking the limit of infinitesimal scaling $b = 1 + ds$, $ds << 1$ along with the local potential approximation leads to [ ],

$$S_{eff}[\Phi_<] = S[\Phi_<] + \frac{Q}{2} \int d^D x \log[\Lambda^2 + \frac{\partial^2 V[\Phi_<]}{\partial \Phi_<^2}] \tag{9}$$

where

$$Q = \frac{\Lambda^D ds}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)} \tag{10}$$

When applied to (4), the logarithmic correction on the right hand side of (9) may be expanded as

$$\log[1 + \frac{\partial^2 V}{\partial \Phi_<^2}] = \log[1+r] + \frac{g^2}{2(1+r)} \Phi_<^2 - \frac{g^4}{8(1+r)} \Phi_<^4 + \ldots \tag{11}$$

where we set $\Lambda = 1$. Thus, for sufficiently small deviations from criticality ($r << 1$) and sufficiently small couplings obtained after a large number of scaling iterations ($g^2 << 1$), the following approximations hold

$$S_{eff}[\Phi_<] \sim S[\Phi], \qquad V_{eff}[\Phi_<] \sim V[\Phi] \tag{12}$$

A side-by-side comparison between parameters of LGW and Yang-Mills theories is shown below:

Landau –Ginzburg -Wilson theory	**Yang-Mills theory**
Dimensional parameter ($\varepsilon = 4 - D$)	Dimensional regulator ($\varepsilon = 4 - D$)
Normalized momentum cutoff ($\Lambda = 1$)	Normalized Planck scale ($\Lambda_{Pl} = 1$)
Temperature (T)	Energy scale ($\mu_{EW} < \mu << \Lambda_{Pl}$)
Critical temperature (T_c)	EW scale (μ_{EW})
Temperature parameter (r)	Deviation from the EW scale ($\delta\mu = \mu - \mu_{EW}$)
Coupling parameter (u)	Coupling constant (g^2)
External field (h)	Fermion current (j)
Renormalization Group scale (l)	Renormalization Group scale (s)

Tab. 1: Comparison between LGW and Yang-Mills theories

Under these circumstances, RG flow equations for $r = \delta\mu$, $u = g^2$ and fermion current $j = j_f$ read, respectively [ ]

$$\frac{\partial(\delta\mu)}{\partial s} = (\delta\mu)(2 + bg^2) + ag^2$$

$$\frac{\partial g^2}{\partial s} = \varepsilon g^2 - 3b(g^2)^2 \tag{13}$$

$$\frac{\partial j_f}{\partial s} = (3 - \frac{\varepsilon}{2}) j_f$$

Here,

$$a = 3K_4\Lambda_{Pl}^2 , \qquad b = 3K_4 , \qquad K_4 = (8\pi^2)^{-1} \tag{14}$$

and the cutoff Λ_{Pl} has been explicitly shown to highlight its contribution as a variable parameter under the sliding scale s, where

$$s = \log(\Lambda_{Pl} / \mu_{EW})^2 \tag{15}$$

The Wilson-Fisher (WF) fixed point of (13) is defined by the pair

$$(\delta\mu)^* = -\frac{a}{6b}\varepsilon, \qquad (g^2)^* = \frac{\varepsilon}{3b} \tag{16}$$

(16) acts as a non-trivial attractor of the RG flow. Because it resides on the critical line $\mu = \mu_{EW}$, it describes by definition a *massless* field theory ($r = \delta\mu = 0$) [ ]. The vacuum state of Φ corresponding to the WF point results from minimization of (4), that is,

$$\mathrm{v}^* = \pm\sqrt{\frac{6(-\delta\mu)^*}{(g^2)^*}} = \pm 3(K_4)^{1/2}\Lambda_{Pl} \tag{17}$$

The above formula shows how Φ and its gauge boson counterpart (2) gains mass at the WF point without any additional breaking mechanism or external fields. Let $\mathrm{v}^*=M$ denote the mass acquired by the gauge boson. Combining (14), (16) and (17) yields

$$\begin{gathered} \boxed{(g^2)^* M^2 = \mu_{EW}^2 = const.} \\ \boxed{(g^*)^2 \sim m_f^* \sim \varepsilon} \end{gathered} \tag{18}$$

in which $m_f^* = O(j_f)$ stands for the normalized fermion mass [ ]. It is apparent now that, as Λ_{Pl} flows towards μ_{EW} under RG transformation ($\Lambda_{Pl} \to \mu_{EW}$), so does the dimensional parameter $\varepsilon = 4 - D$. As a result, the WF attractor (16) turns from a *single isolated* point to a *distribution* of points. Our next step is to explore the link between the structure of the WF attractor and the parameters of SM.

4. Assumptions

4.1) As previously indicated, the mapping theorem applies when comparing Yang-Mills fields with *classical* scalar fields. We extend this ansatz and assume that the theorem holds sufficiently well for *quantum* scalar field theory. This assumption may be motivated by considering the close analogy between quantum field theory (QFT) and statistical systems *near criticality* [ ]. On this basis, the Yang-Mills model may be approximated reasonably well by the LGW theory of equilibrium critical behavior.

4.2) From (4.1) it follows that the Wilson-Fisher parameter and the dimensional regulator of Yang-Mills theory are identical entities. This identity is made explicit in the first row of Tab. 1.

4.3) We analyze on the IR regime of Yang-Mills theory in which $\mu_{EW} = G_F^{-1/2}$ stands for the electroweak scale and $\Lambda = \Lambda_{Pl} >> \mu_{EW}$ the cutoff scale. Thus [ ],

$$\varepsilon = \frac{1}{s} = \frac{1}{\log(\frac{\Lambda_{Pl}^2}{\mu_{EW}^2})} > 0 \qquad (19)$$

Moreover, to simplify the derivation, it is convenient to take advantage of the large numerical difference between the two scales entering the logarithm and substitute (19) with

$$\varepsilon \sim \frac{\mu_{EW}^2}{\Lambda_{Pl}^2} \tag{20a}$$

Therefore,

$$d\varepsilon = -\frac{2\mu_{EW}^2}{\Lambda_{Pl}} d\Lambda_{Pl} \tag{20b}$$

On the other hand, an incremental change of scale $0 < ds << 1$ yields a correspondingly small drop in the cutoff, that is, [ ]

$$\Lambda' = \Lambda_{Pl}(1 - ds) \rightarrow d\Lambda_{Pl} = -\Lambda_{Pl} ds \tag{21}$$

From (20b) and (21) we obtain

$$\frac{d\varepsilon}{ds} = 2\mu_{EW}^2 \rightarrow \varepsilon \sim O(d\varepsilon) = 2\mu_{EW}^2 ds \tag{22}$$

As expected at leading order, dimensional parameter ε grows linearly with ds.

5. Wilson-Fisher point as source of particle masses and gauge charges

We are now ready to analyze the dynamics of (13) using the standard methods employed in the study of nonlinear systems [ ]. To this end, we first note that the last equation in (13) is uncoupled to the first two. This enables us to reduce the dimensionality of (13) to a planar system of differential equations. We next cast (13) in the form of a two-dimensional map, namely

$$(g^2)_{n+1} = (1 + \varepsilon\Delta s)(g^2)_n - 3b\Delta s(g^2)_n \tag{23a}$$

$$(\delta\mu)_{n+1} = (\delta\mu)_n[1 + 2\Delta s + b\Delta s(g^2)_n] + a\Delta s(g^2)_n \tag{23b}$$

where Δs denotes the increment of (15). Linearizing (23) and computing its Jacobian J, leads to

$$J = 1 + (2 + \varepsilon)\Delta s = 1 + \mu_{EW}^{-2}\varepsilon > 1 \tag{24}$$

on account of (22). It follows that map (23) is dissipative for $\varepsilon \neq 0$ and becomes asymptotically conservative in the physical limit $\varepsilon = 0$. Invoking universality arguments [ ] we conclude that, near criticality, (23) shares the same universality class with the quadratic map. Furthermore, in the neighborhood of Feigenbaum's attractor, ε approaches $\varepsilon_\infty = 0$ according to:

$$\varepsilon_n - \varepsilon_\infty \approx a_n \cdot \overline{\delta}^{-n} \tag{25}$$

Here, $n >> 1$ is the index counting the number of cycles generated through the period doubling cascade, $\overline{\delta}$ is the rate of convergence (in general different from Feigenbaum's constant for the quadratic map) and a_n is a coefficient which becomes asymptotically independent of n, that is, $a_\infty = a$ [ ]. Substitution of (25) in (18) yields

$$P_j(n) = \left[M_n^{-2} \quad (g^*)_n^2 \quad (m_f^*)_n \right] \propto \overline{\delta}^{-n} \quad \text{if} \quad n >> 1 \tag{26}$$

in which $j = 1,2,3$ indexes the three entries of (26). Period-doubling cycles are characterized by $n = 2^p$, with $p >> 1$. The ratio of two consecutive terms in (26) is then given by

$$\boxed{\frac{P_j(p+1)}{P_j(p)} = O[\overline{\delta}^{-(2^p)}]} \tag{27}$$

Numerical results derived from (27) are displayed in Tab. 3. This table contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks and a similar comparison of coupling ratios. Tab. 2 contains the set of known quark and gauge boson masses as well as the SM coupling strengths. All quark masses are reported at the energy scale given by the top quark mass and are averaged using reports issued by the Particle Data Group [ ]. Gauge boson masses are evaluated at the EW scale and the coupling strengths at the scale set by the mass of the Z boson. The best-fit rate of convergence is $\bar{\delta} = 3.9$ which falls close to the numerical value of the Feigenbaum constant corresponding to hydrodynamic flows [ ].

Parameter	**Value**	**Units**
m_u	2.12	MeV
m_d	4.22	MeV
m_s	80.90	MeV
m_c	630	MeV
m_b	2847	MeV
m_t	170,800	MeV
M_W	80.46	GeV
M_Z	91.19	GeV
α_{EM}	1/128	-
α_W	0.0338	-
α_{QCD}	0.123	-

Tab. 2: Actual values of selected SM parameters

Parameter ratio	Behavior	Actual	Predicted
m_u / m_c	$\bar{\delta}^{-4}$	3.365×10^{-3}	4.323×10^{-3}
m_c / m_t	$\bar{\delta}^{-4}$	3.689×10^{-3}	4.323×10^{-3}
m_d / m_s	$\bar{\delta}^{-2}$	0.052	0.066
m_s / m_b	$\bar{\delta}^{-2}$	0.028	0.066
m_e / m_μ	$\bar{\delta}^{-4}$	4.745×10^{-3}	4.323×10^{-3}
m_μ / m_τ	$\bar{\delta}^{-2}$	0.061	0.066
M_W / M_Z	$(1-\frac{1}{\bar{\bar{\delta}}})^{1/2}$	0.8823	0.8623
$(\alpha_{EM} / \alpha_W)^2$	$\bar{\delta}^{-2}$	0.053	0.066
$(\alpha_{EM} / \alpha_s)^2$	$\bar{\delta}^{-4}$	4.034×10^{-3}	4.323×10^{-3}

Tab 3: Actual versus predicted ratios of SM parameters

6. A natural solution for the hierarchy problem

It is known that the technique of renormalization in perturbative QFT is conceived as a two-step program: regularization and subtraction. One first controls the divergence present in momentum integrals by inserting a suitable "regulator", and then brings in a set of "counter-terms" to cancel out the divergence. Momentum integrals in QFT have the generic form

$$I = \int_0^\infty d^4q\, F(q) \tag{28}$$

Two regularization techniques are frequently employed to manage (28), namely "momentum cutoff" and "dimensional regularization". When the momentum cutoff scheme is applied for regularization in the UV region, the upper limit of (28) is replaced by a finite cutoff Λ,

$$I \to I_\Lambda = \int_0^\Lambda d^4q\, F(q) \tag{29}$$

Explicit calculation of the convergent integral (29) amounts to a sum of three polynomial terms

$$I_\Lambda = A(\Lambda) + B + C(1/\Lambda) \tag{30}$$

Dimensional regularization proceeds instead by shifting the momentum integral (28) from a four-dimensional space to a continuous D - dimensional space

$$I \to I_D = \int_0^\infty d^Dq\, F(q) \tag{31}$$

Introducing the parameter $\varepsilon = 4 - D$ leads to

$$I_D \to I_\varepsilon = A'(\varepsilon) + B' + C'(1/\varepsilon) \tag{32}$$

In general, Λ and ε are not independent regulators and relate to each other via the approximate connection (19)

$$\varepsilon = 4 - D = \frac{1}{\log(\Lambda^2/\mu_0^2)} \tag{33}$$

where $\mu_0 < \Lambda$ stands for an arbitrary but non-vanishing reference scale.

A similar technique can be used to regularize field theory in the IR limit whereby Γ is taken to represent the lowest bound scale. A strictly positive ε on less than four dimensions ($D < 4$) requires taking the reciprocal of the logarithm in (33) to comply with $\mu_0 > \Gamma$. The infrared version of (33) accordingly reads:

$$\varepsilon' = 4 - D = \frac{1}{\log(\mu_0^2 / \Gamma^2)} \tag{34}$$

We next proceed with the following assumptions

6.1) The deep IR cutoff of field theory is set by the cosmological constant scale

$$\Gamma = (\Lambda_{cc})^{1/4} \tag{35}$$

where Λ_{cc} represents the cosmological constant.

6.2) The deep UV cutoff of field theory is set by the Planck scale:

$$\Lambda = \Lambda_{Pl} \tag{36}$$

Combining 6.1) and 6.2) implies that, as the electroweak scale (μ_{EW}) is approached from above or below, (33) and (34) naturally converge to each other. Taking $\mu_0 = \mu_{EW}$ and substituting in (33) and (34) yields

$$\boxed{\frac{\mu_{EW}}{\Gamma} = \frac{\Lambda_{Pl}}{\mu_{EW}} \to (\Lambda_{cc})^{1/4} = \frac{\mu_{EW}^2}{\Lambda_{Pl}}} \tag{37}$$

Several conclusions may be drawn from (37),

a) Asymptotic approach to four-dimensional space-time explains the existence of the deep IR cutoff (Λ_{cc}) and deep UV cutoff (Λ_{Pl}). Stated differently, fractal space-time description supplied by the condition $\varepsilon > 0$ and $\varepsilon' > 0$ appears to be linked to these natural bounds [ ].

b) Fixing two out of the three scales involved in (37) automatically determines the third one.

c) The gauge hierarchy problem, cosmological constant problem and the existence of the electroweak phase transition appear to be deeply interconnected.

d) The derivation presented here stands in sharp contrast with sophisticated approaches to the hierarchy problem based on SUSY, Technicolor, extra-dimensions, anthropic arguments, fine-tuning or gauge unification near the Planck scale.

8. Summary and conclusions

(*to follow*)

References

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