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A Note on QCD Corrections to A_{FB}^b using Thrust to determine the b-Quark Direction

Bodo Lampe

Max Planck Institut für Physik
Föhringer Ring 6, D-80805 München

Abstract

I discuss one-loop QCD corrections to the forward backward asymmetry of the b-quark in a way appropriate to the present experimental procedure. I try to give insight into the structure of the corrections and elucidate some questions which have been raised by experimental experts. Furthermore, I complete and comment on results given in the literature.

The forward backward asymmetry of the b quark is one of the most interesting quantities which has been measured at LEP. It is defined as the ratio

$$A_{FB} = \frac{\sigma_{F-B}}{\sigma_{F+B}}$$

and in lowest order is given by $A_{FB}^{\text{Born}} = 3\frac{v_b a_b}{v_b^2+a_b^2}\frac{v_e a_e}{v_e^2+a_e^2}$ and therefore sensitive to the couplings of electron and b quark to the Z. Even more interesting might be the measurement of the combined left right forward backward asymmetry $A_{FB}^{LR} = \frac{(\sigma_{F-B})_L-(\sigma_{F-B})_R}{\sigma_{\text{total}}}$ projected by SLC because in lowest order it involves the b quark couplings v_b and a_b only [1].

For a precision measurement of these quantities the understanding of higher order corrections is very important. Oneloop (electroweak and QCD) corrections have been reviewed in [1] and [2] and twoloop QCD corrections have been calculated in [3]. Usually, these results are presented under the assumption that the b-quark direction can be experimentally precisely determined. However, with the existing detectors this is not the case. Instead, the LEP experiments apply the following procedure [4]:

- Events in which the b (or the $\bar{b}$) decays semileptonically, $b \to c\mu^-\bar{\nu}(\bar{b} \to \bar{c}\mu^+\nu)$ are selected.
- For these events the thrust axis T is determined as the $max_{\vec{n}} \sum_i |\vec{p}_i \vec{n}_i|$ where the sum is over all charged momenta $\vec{p}_i$ in the event.
- The orientation of the thrust axis is chosen in such a way that $\vec{T}\vec{\mu}^-$ is positive (resp. $\vec{T}\vec{\mu}^+$ is negative). Then the event is counted forward if $\vec{T}$ points in the forward direction ($\vec{T}\vec{e}^- > 0$) and backward if $\vec{T}$ points in the backward direction ($\vec{T}\vec{e}^- < 0$).

This procedure will be called the "$\underline{T\text{ procedure}}$" in the following (in contrast to the "$\underline{b\text{ procedure}}$" where the b quark is used to determine the asymmetry). The T procedure has several deficiencies:

First, due to the missing momentum of the neutrino, $\vec{T}$ is not the "true" thrust axis.

Secondly, due to the nature and kinematics of the b decay, there are events where the μ^- goes forward while the b goes backward (and vice versa).

Thirdly, gluon emission may spoil the connection between thrust axis and b-quark direction.

These deficiencies must all be corrected for. They can be corrected for separately. In this note we concentrate on point 3. The corrections to items no. 1 and 2 can be made using existing Monte Carlo programs. Note that in this procedure the muon is only used to determine the hemisphere in which its parent quark is to be expected. One may ask why not determine the asymmetry of the muon ("μ procedure"). The answer is that the muon asymmetry will be notably smaller than the b asymmetry, because there is a loss of the original information through the missing neutrino. The μ procedure is therefore worse than the T procedure.

Before I address item no. 3 I will consider the structure of oneloop QCD corrections to A_{FB} in general. For simplicity, I will neglect mass terms $O(m_b/m_Z)$ in the following. In addition to $e^+e^- \to b\bar{b}$ one has processes $e^+e^- \to b\bar{b}g$. When they are included the QCD effect can be written as an overall correction factor

$$A_{FB} = A_{FB}^{\text{Born}} \cdot (1 + c\frac{\alpha_s}{\pi})$$

which we decompose as

$$1 + c\frac{\alpha_s}{\pi} = \frac{1 + \frac{\alpha_s}{\pi}(p_2 + p_3)}{1 + \frac{\alpha_s}{\pi}(q_2 + q_3)}$$

i.e. we write it as a correction factor p to σ_{F-B} defined by a correction factor q to $\sigma_{F+B} = \sigma_{\text{total}}$. Both p and q can be split into a 2 jet and a 3 jet piece in the sense that one can split $\sigma_{F\pm B}$ in a 2 jet and a 3 jet piece,

$$\sigma_{F\pm B} = \sigma_{F\pm B}^2(y) + \sigma_{F\pm B}^3(y),$$

with an invariant mass cut y to define the jets. $p_{2,3}(y)$ are defined by

$$\sigma_{F-B}^2(y) = \sigma_{F-B}^{\text{Born}}(1 + \frac{\alpha_s}{\pi}p_2)$$

$$\sigma_{F-B}^3(y) = \frac{\alpha_s}{\pi}p_3\sigma_{F-B}^{\text{Born}}$$

$$\sigma_{F+B}^2(y) = \sigma_{F+B}^{\text{Born}}(1 + \frac{\alpha_s}{\pi}q_2)$$

$$\sigma_{F+B}^3(y) = \frac{\alpha_s}{\pi}q_3\sigma_{F+B}^{\text{Born}}$$

Note that the sums $p_2 + p_3$ and $q_2 + q_3$ are independent of y. One could define a forward backward asymmetry based on 2 jet resp. 3 jet events only

$$A_{FB}^2(y) = \frac{\sigma_{F-B}^2}{\sigma_{F+B}^2} = A_{FB}^{\text{Born}}(1 + \frac{\alpha_s}{\pi}(p_2 - q_2))$$

$$A_{FB}^3(y) = \frac{\sigma_{F-B}^3}{\sigma_{F+B}^3} = \frac{p_3}{q_3}A_{FB}^{\text{Born}}$$

but we shall not consider these quantities in the following. No simple relation holds between A_{FB}, A^2_{FB} and A^3_{FB}.

The functions $p_{2,3}(y)$ and $q_{2,3}(y)$ have been given in the literature [3] and I do not want to repeat them here because I am only interested in the inclusive correction factor c. Within the b procedure one has $c = -1$ (for $m_b = 0$) and $c \approx -0.8$ (for $m_b = 4.7$ GeV). It is a question of some interest to know the value of c for the T procedure, too. To determine this value we shall work on the parton level and mimic the T procedure on the parton level. On the parton level the role of the muon direction is played by the b quark direction and the thrust dirction $\vec{t}$ is given by the parton with the maximum energy. In lowest order and in the exact 2 jet limit ($y \to 0$) the thrust direction and the b quark direction are identical so that no correction needs to be applied (as compared to the b procedure). A difference arises, however, in the 3 jet region, where $\vec{t}$ can be either $\vec{b}$, $\bar{b}$ or $\vec{g}$. In $O(\alpha_s)$ the T and b procedure are equivalent only in the strict 2 jet limit $y \to 0$. An event is forward if either $\vec{t}\vec{b} > 0$ and $\vec{t}\vec{e^-} > 0$ or $\vec{t}\vec{b} < 0$ and $\vec{t}\vec{e^-} < 0$, and backward otherwise. We have used this procedure and applied it to the QCD matrix element for the process $e^+e^- \to b\bar{b}g$. One obtains the following results:

$$c(T \text{ procedure}, m_b = 0) = -0.893$$

This number can be decomposed into a 2 jet and 3 jet contribution.

$$c = (k_2 + k_3)\frac{C_F}{2} \qquad \text{with} \qquad \frac{C_F}{2}k_{2,3} = (p - q)_{2,3}$$

The colour factor $C_F = 4/3$ has been introduced for convenience. The results for k_3 are given in the table, both for the b and the T procedure for several values of y, assuming $m_b = 0$. $k_2 = \frac{3}{2}c - k_3$ vanishes for $y \to 0$ because in the $m_b = 0$ theory there is no contribution from the virtual gluon exchange diagram $e^+e^- \to b\bar{b}$. Of course it is desirable to have the $0(m_b)$ dependence of c in the T procedure. This is done in a forthcoming publication.

In summary one may state: when going from the b procedure ($c = -1$) to the T procedure ($c = -0.893$) one gets a correction of about 10% to the correction, i.e. the effect is small and irrelevant on the basis of the present experimental accuracy and only important for some future precision experiment. This statement remains true if $0(m_b)$ corrections are included.

y	k_3 (b procedure)	k_3(T procedure)
0	-1.500	-1.340
0.001	-1.474	-1.328
0.005	-1.393	-1.283
0.01	-1.313	-1.225
0.02	-1.183	-1.148
0.04	-0.974	-0.996
0.06	-0.811	-0.875
0.08	-0.671	-0.753
0.10	-0.553	-0.653
0.12	-0.451	-0.557
0.14	-0.363	-0.472
0.16	-0.288	-0.388
0.20	-0.165	-0.244
1	0	0

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Appendix:Some Details of the Calculation

We start with 3 different representations of the cross section

$$\frac{d\sigma}{d\cos\theta_b} = \sigma_V^b \cos\theta_b + \frac{3}{4}\sigma_L^b \sin^2\theta_b + \frac{3}{8}\sigma_U^b(1+\cos^2\theta_b)$$

+ aximuthal terms (ϕ_b);

$$\frac{d\sigma}{d\cos\theta_{\bar{b}}} = \sigma_V^{\bar{b}} \cos\theta_{\bar{b}} + \frac{3}{4}\sigma_L^{\bar{b}} \sin^2\theta_{\bar{b}} + \frac{3}{8}\sigma_U^{\bar{b}}(1+\cos^2\theta_{\bar{b}})$$

+ aximuthal terms $(\phi_{\bar{b}})$;

$$\frac{d\sigma}{d\cos\theta_g} = \sigma_V^g \cos\theta_g + \frac{3}{4}\sigma_L^g \sin^2\theta_g + \frac{3}{8}\sigma_U^g(1+\cos^2\theta_g)$$

+ aximuthal terms (ϕ_g); where

$$\sigma_V^i = \frac{3}{8} J\sigma_o \frac{\alpha_s}{2\pi} C_F B_V^i$$

$$\sigma_{U+L}^i = R\sigma_o \frac{\alpha_s}{2\pi} C_F B_{UL}$$

and

$$B_{UL} = \frac{x_1^2 + x_2^2}{y_{13}y_{23}}$$

$$B_V^i = \frac{x_1^2\cos\theta_{1i} - x_2^2\cos\theta_{2i}}{y_{13}y_{23}}$$

$\cos\theta_{ij} = 1$ for i=j and

$$\cos\theta_{ij} = 1 + \frac{2}{x_i x_j} - \frac{2}{x_i} - \frac{2}{x_j}$$

for $i \neq j$. The x_i are the normalized energies of partons i, $x_1 + x_2 + x_3 = 2$, $y_{13} = 1 - x_2$ etc. as usual. σ_o is the tree level cross section for $e^+e^- \to \mu^+\mu^-$ (photon exchange only). Note that at leading order $e^+e^- \to b\bar{b}$

$$A_{FB}^0 = \frac{3J}{8R}.$$

Expressions for J and R can be found, for instance, in Nachtmann's book. In [3] it was shown that in the b procedure the QCD correction factor can be written as

$$1 + \frac{\alpha_s}{2\pi} C_F (B_V^b - B_{UL})_{\text{all}} = 1 - \frac{3}{2}\frac{\alpha_s}{2\pi} C_F$$

where $(\ )_{\text{all}}$ denotes $\int_0^1 dx_1 \int_0^1 dx_2 \theta(x_1 + x_2 - 1)$ and the singularities of B_V^b and B_{UL} for $y_{13} \to 0, y_{23} \to 0$ drop out in the difference $B_V^b - B_{UL}$.

In the T procedure the QCD correction factor is given by

$$1 + \frac{\alpha_s}{2\pi} C_F \Big\{ (B_V^b - B_{UL})_{x_1>} + (-B_V^{\bar{b}} - B_{UL})_{x_2>} + (-B_V^g - B_{UL})_{x_3>} \Big\}$$

where $(\ )_{x_1>}$ denotes $\int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \theta(x_1+x_2-1)\theta(x_1-x_2)\theta(x_1-x_3)$ etc.

Numerically one finds

$$(B_V^b - B_{UL})_{x_1>} = (-B_V^{\bar{b}} - B_{UL})_{x_2>} = -0.21$$

$$(-B_V^g - B_{UL})_{x_3>} = -B_{UL}\Big|_{x_3>} = -0.92$$

For comparison we also give here

$$(B_V^b - B_{UL})_{x_2>} = -0.74$$

$$(B_V^b - B_{UL})_{x_3>} = -0.55$$

The difference between b and T procedure is given by

$$\frac{\alpha_s}{2\pi} C_F [(B_V^b + B_V^{\bar{b}})_{x_2>} + B_V^b\Big|_{x_3>}] = \frac{\alpha_s}{2\pi} C_F(-0.5273 + 0.3675)$$

$$= (-0.160 \pm 0.003)\frac{\alpha_s}{2\pi} C_F$$

References

[1] See, for example, F.M. Renard, A. Blondel, C. Vezegnassi, in Proc of the Workshop Polarization at LEP, CERN 88-06 (1988)

[2] A. Djouadi, J.H. Kühn, P.M. Zerwas, *Z. Phys.* **C46**

[3] G. Altarelli, B. Lampe, *Nucl. Phys.* **B391** (1993) 3

[4] R. Tenchini, D. Abbaneo, private communication