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QCD Corrections to W Pair Production at LEP200

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Abstract

One loop QCD corrections to hadronic W decay are calculated for arbitrary W polarizations . The results are applied to W pair production and decay at LEP200. We focus on the corrections to angular distributions with particular emphasis on azimuthal distributions and correlations. The relevance of our results to the experimental determination of possible nonstandard triple gauge bosons interactions is discussed.

1. Introduction

The LEP200 experiment at CERN will start data taking above the WW threshold in 1997. While much of the emphasis of the LEP200 program has been on the possible discovery of the Higgs and the precise determination of m_W, there remains the important aspect of probing the triple gauge boson couplings which are an essential feature of the non-abelian gauge symmetry of the standard model. Of course, there is already indirect evidence for the existence of triple gauge boson couplings from the high precision of the data at LEP1 [1], but extracting bounds on these couplings requires unavoidable model dependent assumptions. Even though the statistical accuracy of the LEP200 data will not be as impressive as at LEP1, it will be sufficient to remove any doubts concerning the structure of the pure gauge sector in the standard model.

It is expected that at threshold, radiative corrections may play a significant role in W pair production. Electroweak corrections have been known for some time [2] and there has been some progress towards including finite W width effects, which arise, for example, from the interference between the processes $e^+e^- \to f_1\bar{f}_2f_3\bar{f}_4$ with and without intermediate W's [3]. In this paper we present the one loop perturbative QCD corrections in the case where at least one of the W's decays hadronically, $W \to q\bar{q}'(g)$, which to the best of our knowledge have never been calculated along the lines we follow [4].

In principle it is possible that higher order corrections modify the standard model predictions in such a way that they mimic the existence of nonstandard triple gauge boson couplings, if not taken into account. Therefore, our results should be compared to possible effects from nonstandard couplings as parameterised in [5] [6]. It is conceivable that nonstandard effects enter at the few percent level and are thus of the same order of magnitude as the QCD effects discussed in this paper.

It would be possible to completely neglect higher order QCD corrections by studying only final states in which both the W's decay leptonically, however much information concerning the triple gauge boson vertices will be inevitably washed out due to incomplete kinematical reconstruction, quite apart from large statistical errors due to low event rates in this channel. Therefore hadronic decays are important. For definiteness, we shall mostly consider in this article the process $e^+e^- \to W^+W^- \to l^+\nu q\bar{q}'(g)$ where only the W^- decays hadronically, although our result can easily be extended to $e^+e^- \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4$. Our treatment is along the lines of [6] modifying the discussion of differential distributions sensitive to non standard triple gauge boson vertices to include the effects of higher order QCD corrections. Let us stress once again that the W's are assumed on–shell throughout the paper (as in [6]) so that our result does not take care of finite width effects whatsoever.

In section 2 we shall discuss "inclusive jet angular distributions" (i.e. the matrix element for $W \to q\bar{q}'g$ is integrated in such a way that it adds to the lowest order cross section $e^+e^- \to W^+W^- \to l^+\nu q\bar{q}'$) , but for completeness, we include the fully differential results in the appendix. In eq. 25 we present a nice compact formula, which summarises the most important piece of our result. In section 3 we shall discuss in detail some phenomenological consequences of our result on various differential distributions.

2. A Detailed Description of the Calculation

Non expert readers might be tempted to think that QCD corrections to $W \to q\bar{q}'$ and consequently also for $e^+e^- \to W^+W^- \to l^+\nu q\bar{q}'$, are given by a correction factor $(1 + \alpha_s/\pi)$. However, this is true only as far as the total rate is concerned, but is not the case if one considers some of the angular disributions relevant for probing triple gauge boson vertices. The corrections to the angular distributions are related to QCD corrections to hadronic Z decay [7], but are more complicated due to the larger number of particles in the final state. Correspondingly, more angles are necessary to describe the cross section.

Following the treatment of [6] the cross-section can be written as

$$\frac{d\sigma(e^+e^- \to W^+W^- \to l^+\nu_l q\bar{q}')}{d\cos\vartheta d\cos\theta_l d\phi_l d\cos\theta d\phi} = \sum_{A,B,A',B',\lambda} F^{\lambda}_{ABA'B'}(s,\cos\vartheta) D^0_{AB}(\theta,\phi) D^0_{A'B'}(\pi-\theta_l,\phi_l-\pi) \tag{1}$$

where the sum runs over the polarizations A,B=L,+,– of the W^- and A',B'=L,+,– of the W^+ and the electron helicity λ. D^0_{AB} are the "decay functions" of the decay $W^- \to q\bar{q}'$, normalized such that

$$\int_{-1}^{+1} d\cos\theta \int_0^{2\pi} \frac{d\phi}{2\pi} D^0_{AB}(\theta,\phi) = \frac{4}{3}\delta_{AB} \tag{2}$$

They depend on two angles (polar angle θ and azimuthal angle ϕ) which specify the direction of the outgoing quark q in the W^- rest frame with respect to the direction defined by the W^- motion in the lab frame. The definition of θ_l and ϕ_l is similar. The functions $F^{\lambda}_{ABA'B'}$ can be constructed from the helicity amplitudes for $e^+e^- \to W^+W^-$. They depend on the total e^+e^- energy $\sqrt{s}$ and the production angle ϑ of the W's. Since we are interested in QCD corrections to W–decay we shall not discuss the functions $F^{\lambda}_{ABA'B'}$ any further. Explicit expressions can be found in ref. [6].

For the sake of convenience we reproduce from [6] the lowest order decay functions,

$$\begin{aligned} D^0_{LL}(\theta,\phi) &= \sin^2\theta \\ D^0_{\pm\pm}(\theta,\phi) &= \frac{1}{2}(1\pm\cos\theta)^2 \\ D^0_{+-}(\theta,\phi) &= \frac{1}{2}\sin^2\theta e^{2i\phi} \\ D^0_{\pm L}(\theta,\phi) &= (\pm\cos\theta\sin\theta - \sin\theta)\frac{e^{\pm i\phi}}{\sqrt{2}} \end{aligned} \tag{3}$$

These results can be derived by contracting the "hadron tensor" $H_{\mu\nu}(W^- \to q\bar{q}')$ with the corresponding W polarization vectors ϵ^μ_A, $D_{AB} = H_{\mu\nu}\epsilon^\mu_A\epsilon^{*\nu}_B$. All the rest of the decay functions can be obtained by using the relation

$$D_{AB}(\theta,\phi) = D^*_{BA}(\theta,\phi) \tag{4}$$

which is true beyond the leading order. By summing up all diagonal decay functions one gets

$$D^0_{total} = 2 \tag{5}$$

Alternatively, this can be obtained from $D_{total} = \sum_{A,B} H_{\mu\nu}(-g_{\mu\nu} + \frac{W_\mu W_\nu}{m^2_W})$, where W denotes the 4–momentum of the W^-. It should be stressed that throughout this work, even when discussing the phenomenology in section 3, θ and ϕ are defined in the rest-frame of the decaying W and not in the lab-frame.

The D functions can be decomposed into the sum of a symmetric and an antisymmetric part under the simultaneous exchange $\theta \leftrightarrow \pi - \theta$ and $\phi \leftrightarrow \phi + \pi$ [6]. When we discuss the phenomenology of hadronic W decays in section 3 we will consider only the symmetric pieces of the D functions, because in hadronic W–decays, quark, antiquark and gluon jets cannot be distinguished and therefore the antisymmetric parts of the D functions drop out. In this section, however, we shall present complete results both for the symmetric and the antisymmetric terms of the W decay functions.

Our aim is to calculate the one loop QCD corrections to the decay functions D_{AB}. If gluon emission is taken into account, these functions depend on three more variables χ, x_1 and x_2 in addition to θ and ϕ, all defined in the W^- rest frame, as follows: x_1 and x_2 are the rescaled energies of the quark q (with 4–momentum p_1) and the antiquark q' (with 4–momentum p_2). They can be given in terms of invariant dot products as

$$2p_2p_3 = m_W^2(1-x_1) \qquad 2p_1p_3 = m_W^2(1-x_2) \qquad 2p_1p_2 = m_W^2(1-x_3) \tag{6}$$

x_3 is the rescaled energy of the gluon, with momentum p_3 and with $x_3 = 2 - x_2 - x_1$ from energy conservation. θ, ϕ and χ are the angles needed to fix the spatial orientation of the triangle which is formed by the vectors $\vec{p_1}$, $\vec{p_2}$ and $\vec{p_3}$ in the W^- rest frame (in lowest order there are only two angles θ and ϕ necessary to fix the quark(=–antiquark) direction in the W^- rest frame). After integration over the additional variables χ, x_1 and x_2, the form of the cross section eq. 1 will be left unchanged, with

$$D_{AB}(\theta,\phi) = D^0_{AB}(\theta,\phi) + C_F\frac{\alpha_s}{2\pi}D^1_{AB}(\theta,\phi) \tag{7}$$

replacing D^0_{AB} in eq. 1. D^1_{AB} is the sum of all one loop QCD corrections (virtual, soft and hard gluons). We shall actually integrate in such a way that the triangle is 'reduced' to the thrust direction, where the thrust is defined in the ordinary way except that it is determined in the W rest frame. For example, for three partonic jets, q ,$\bar{q}'$ and g, the thrust direction is just the direction of the most energetic parton. This definition is useful, because experimentally quark, antiquark and gluon jets cannot be distinguished. The transformation to the W rest frame should be no problem, too, because the hadronic part of the event in general can be completely reconstructed.

The calculation of the $D^1_{AB}(\theta,\phi)$ will be described in the following. More precisely, we shall calculate the correction to the ratio $\frac{D_{AB}}{D_{total}}$, because in this ratio all ultraviolet, infrared and collinear singularities present in the higher order matrix element drop out. This is a consequence of the universality of those singularities, which has been known to be true in QCD for many years. Otherwise such singularities would appear in intermediate steps of the calculation. For example, one has

$$D_{total}(q\bar{q}'g) = C_F\frac{\alpha_s}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}D^0_{total} \tag{8}$$

with singularities for $x_{1,2} \to 1$. Those ratios have the further advantage that the virtual gluon exchange corrections drop out (for the case of massless quarks which we assume throughout). Writing

$$D_{total} = D^0_{total} + C_F \frac{\alpha_s}{2\pi} D^1_{total} = 2(1 + \frac{\alpha_s}{\pi}) \tag{9}$$

(defined including virtual gluon exchange) we have

$$\frac{D_{AB}}{D_{total}} = \frac{D^0_{AB}}{2}(1 + C_F \frac{\alpha_s}{2\pi} \frac{X_{AB}}{2D^0_{AB}}) \tag{10}$$

with

$$\begin{aligned} X_{AB} &:= 2D^1_{AB} - D^0_{AB} D^1_{total} \\ &= \int_0^{2\pi} \frac{d\chi}{2\pi} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 H_{\mu\nu}(q\bar{q}'g)(2\epsilon^\mu_A \epsilon^{*\nu}_B - D^0_{AB}(-g_{\mu\nu} + \frac{W_\mu W_\nu}{m^2_W})) \end{aligned} \tag{11}$$

What is the form of the W^- polarization vectors ϵ^μ_A? In the actual calculation we worked in the W^- rest system and we choose the quantization axis to be given by the direction of the quark momentum, $q_1 = \frac{m_W}{2} x_1(1, 0, 0, 1)$ (or, more generally, the thrust direction). Furthermore, we defined the coordinate system such that $q_2 = \frac{m_W}{2} x_1(1, 0, \sin\theta_{12}, \cos\theta_{12})$ is the antiquark momentum, where $\cos\theta_{12}$ is given by $1 + \frac{2}{x_1 x_2} - \frac{2}{x_1} - \frac{2}{x_2}$ simply from energy–momentum conservation. Therefore we had to produce the polarization vectors from the ordinary vectors $(0, 0, 0, -1)$ and $\frac{1}{\sqrt{2}}(0, \pm 1, i, 0)$ by an arbitrary Euler rotation given in terms of ϕ, θ and χ,

$$\epsilon_L = (0, -\sin\theta\cos\chi, -\sin\theta\sin\chi, -\cos\theta) \tag{12}$$

$$\epsilon_\pm = \frac{e^{\pm i\phi}}{\sqrt{2}}(0, -i\cos\chi \mp \cos\theta\sin\chi, -i\sin\chi \pm \cos\theta\cos\chi, \pm\sin\theta) \tag{13}$$

Note that the angles χ and θ enter the angular correlation structure of the cross section for $Z \to q\bar{q}g$ as well. The generic form of the lowest order LEP1 cross section is an expansion in powers of $\cos\theta$

$$\frac{d\sigma(e^+e^- \to Z \to q\bar{q})}{d\cos\theta} = \frac{3}{8}\sigma^0_U(1 + \cos^2\theta) + \frac{3}{4}\sigma^0_L \sin^2\theta + \frac{3}{4}\sigma^0_P \cos\theta \tag{14}$$

where the last term is the parity violating vector–axialvector interference term, which cannot be measured, because quarks and antiquarks cannot be distinguished. For a three jet process one has an expansion

$$\begin{aligned} \frac{2\pi d\sigma(e^+e^- \to Z \to q\bar{q}g)}{d\cos\theta d\chi dx_1 dx_2} &= \frac{3}{8}\frac{d\sigma_U(x_1, x_2)}{dx_1 dx_2}(1 + \cos^2\theta) + \frac{3}{4}\frac{d\sigma_L(x_1, x_2)}{dx_1 dx_2}\sin^2\theta \\ &+ \frac{3}{4}\frac{d\sigma_P(x_1, x_2)}{dx_1 dx_2}\cos\theta - \frac{3}{2\sqrt{2}}\frac{d\sigma_N(x_1, x_2)}{dx_1 dx_2}\sin 2\theta\cos\chi \\ &+ \frac{3}{4}\frac{d\sigma_T(x_1, x_2)}{dx_1 dx_2}\cos 2\chi\sin^2\theta - \frac{3}{2\sqrt{2}}\frac{d\sigma_M(x_1, x_2)}{dx_1 dx_2}\sin\theta\cos\chi \end{aligned} \tag{15}$$

with $\frac{d\sigma_L(x_1,x_2)}{dx_1 dx_2} = 2\frac{d\sigma_T(x_1,x_2)}{dx_1 dx_2}$ (as long as one neglects higher orders, i.e. two gluon emission). If one integrates over χ, one recovers the structure of the lowest order eq. 14. In that case, for normalized distributions, there are only two independent quantities which fix the cross section completely, namely $\frac{\sigma_L}{\sigma_{total}} = -\frac{\sigma_U}{\sigma_{total}}$ and $\frac{\sigma_P}{\sigma_{total}}$. These quantities, after integration, correspond to L and P eqs. 18 and 21, and have been introduced in a different context in the last two references of [10]. In hadronic Z decays the parity violating piece $\frac{\sigma_P}{\sigma_{total}}$ cannot be determined, because quark and antiquark jets cannot be distinguished.

In our case we shall find matrix elements which depend on all three angles ϕ, χ and θ and have to integrate over χ to get the D^1_{AB}, eq. 7. The appearance of an additional angle is due to the presence of an additional plane, the plane spanned by the momenta of $e^\pm$ and $W^\pm$.

In the difference X_{AB} all singularities and contributions from virtual gluon exchange drop out. For example, for A=B=L one has

$$\frac{D_{LL}}{D_{total}} = \frac{\sin^2\theta}{2} + C_F\frac{\alpha_s}{2\pi}\int_0^{2\pi}\frac{d\chi}{2\pi}\int dx_1\int dx_2\Big\{2\frac{x_1+x_2-1}{x_1^2}(1-2\sin^2\theta + \cos^2\chi\sin^2\theta) - (2\frac{x_2}{x_1} - \frac{x_2^2}{(1-x_1)(1-x_2)})\sin\theta_{12}\sin\chi\sin\theta\cos\theta\Big\} \quad (16)$$

in case that the thrust (=quantization) axis is given by $\vec{p}_1$, i.e. $x_1 > x_{2,3}$. Note that $\frac{\sin\theta_{12}}{(1-x_1)(1-x_2)}$ is integrable. Analogous results are obtained in the other two cases, $x_2 > x_{1,3}$ and $x_3 > x_{1,2}$. Integrating these results over the appropriate phase space regions [8] one obtains

$$\frac{D_{LL}}{D_{total}} = \frac{\sin^2\theta}{2}(1-3LC_F\frac{\alpha_s}{2\pi}) + LC_F\frac{\alpha_s}{2\pi} \quad (17)$$

where

$$L = 0.4875 \quad (18)$$

is the numerical result of the integration over x_1 and x_2. Similarly, for the other decay functions:

$$\frac{D_{++}+D_{--}}{D_{total}} = \frac{1+\cos^2\theta}{2}(1-3LC_F\frac{\alpha_s}{2\pi}) + 2LC_F\frac{\alpha_s}{2\pi} \quad (19)$$

$$\frac{D_{++}-D_{--}}{D_{total}} = -\cos\theta(1+PC_F\frac{\alpha_s}{2\pi}) \quad (20)$$

where

$$P = -1.340 \tag{21}$$

is the characteristic correction for a parity violating contribution as can be derived from eq. 36 of the appendix. This and various details concerning the matrix elements will be discussed in the appendix. For the phenomenological applications we have in mind here, it is not relevant, because quark and antiquark jets cannot be distinguished and therefore the parity violating contribution cannot be determined experimentally. Note that, in addition, there are other contributions from the matrix elements, which disappear when the integration over χ is performed (see the appendix).

Next, we find

$$\frac{D_{+-}}{D_{total}} = e^{2i\phi}\frac{\sin^2\theta}{4}(1-3LC_F\frac{\alpha_s}{2\pi}) \tag{22}$$

$$\frac{D_{\pm L}}{D_{total}} = \frac{e^{\pm i\phi}}{2\sqrt{2}}\Big\{\pm\sin\theta\cos\theta(1-3LC_F\frac{\alpha_s}{2\pi}) - \sin\theta(1+PC_F\frac{\alpha_s}{2\pi})\Big\} \tag{23}$$

Note that we know D_{total} from eq. 9. In any case, the ratios $\frac{D_{AB}}{D_{total}}$ are most interesting, because they enter in normalized distributions, which are more sensitive to anomalous triple gauge boson couplings than the total cross section.

Forgetting about the parity violating contributions, our results eqs. 17 – 23 can be brought to a very compact form by defining

$$\hat{D}_{AB} = \frac{1}{1+\frac{\alpha_s}{\pi}}\frac{D_{AB}}{1-3LC_F\frac{\alpha_s}{2\pi}} \tag{24}$$

Using this, eqs. 17 – 23, can be rewritten as

$$\hat{D}_{AB} = D^0_{AB} + 2\delta_{AB}LC_F\frac{\alpha_s}{2\pi} + O(\alpha_s^2) \tag{25}$$

This is the central result of this article. It means that the QCD corrections can be organized such that apart from an overall normalization factor only the diagonal decay functions D_{LL}, D_{++} and D_{--} are modified. They are modified by a constant term $2LC_F\frac{\alpha_s}{2\pi}$. The absolute numerical value of $2LC_F\frac{\alpha_s(m_W^2)}{2\pi} = 0.024$ is relatively small, but may be relevant in regions, in which the lowest order terms vanish, like D^0_{LL} at $\theta = 0$. To see the significance of the term $2LC_F\frac{\alpha_s}{2\pi}$ one may also consider $\hat{D}_{++}$. The constant correction may be written as $\sim \cos^2\theta + \sin^2\theta$. Comparing with the lowest order decay function D^0_{++}, eq. 3, we see that a purely transverse state appears to accquire a longitudinal component, thus changing the relative proportion of longitudinal and transverse polarisations in the final state. Such effects are among the hallmarks of non-standard triple gauge boson couplings. Although the absolute numerical value of $2LC_F\frac{\alpha_s(m_W^2)}{2\pi}$ is relatively small, it is certainly comparable in size with possible non-standard effects, which may be even smaller. Concrete phenomenological applications will be presented in the next section.

3. Phenomenological Applications

Although the QCD corrections are remarkably simple, they modify the shapes of all the distributions relevant for the analysis of the triple gauge boson vertex, like azimuthal or polar angle distributions of one or both W's. We will present both analytical and numerical results for these distributions. Our basic input parameters will be $\sqrt{s} = 190$ GeV for the total beam energy and $\alpha_s = 0.11$ for the magnitude of the strong coupling constant. The latter value is reasonable in view of the LEP1 result for α_s and the uncertainty in the renormalization scale. (The argument of α_s should be somewhere between m_W and $\sqrt{s}$.) We shall present all results in the form of a ratio of higher order to lowest order prediction, because this emphasizes the QCD effects and reduces side effects like the uncertainty in m_W etc.

Let us first discuss the case, in which only one of the two W decays is considered, i.e. $e^+e^- \to W^+W^- \to W^+q\bar{q}'$. The azimuthal differential distribution may be written as [6]

$$\frac{d\sigma(e^+e^- \to W^+j_-X)}{d\cos\vartheta d\cos\theta_- d\phi_-} = \frac{3}{8\pi}\frac{d\sigma(e^+e^- \to W^+W^-)}{d\cos\vartheta}\sum_{A,B}\rho_{AB}D_{AB} \tag{26}$$

where $\rho_{A,B}$ is the spin density matrix of the W^- as given, for example, in [6]. j_- denotes the highest energetic jet from the W_- decay used to define the thrust direction. The angles θ and ϕ defining the thrust direction have been renamed to θ_- and ϕ_- to stress that they refer to the W^- decay. Note that for hadronic W–decays it is usually possible to transform to the W rest system, because the W–momentum can be fully reconstructed from the momenta of the decay products. Using our result eq. 25 one can rewrite eq. 26 as

$$\frac{d\sigma(e^+e^-\to W^+q\bar{q})}{d\cos\vartheta d\cos\theta_- d\phi_-} = \frac{3}{8\pi}\frac{d\sigma(e^+e^-\to W^+W^-)}{d\cos\vartheta}(1+\frac{\alpha_s}{\pi})(1-3LC_F\frac{\alpha_s}{2\pi}) \times \left\{\sum_{A,B}\rho_{AB}(D^0_{AB}+2LC_F\frac{\alpha_s}{2\pi}\delta_{AB})\right\} \quad (27)$$

Carrying out the polar angle integrations one obtains

$$\frac{d\sigma(e^+e^-\to W^+j_-X)}{d\phi_-} \sim \frac{4}{3}+4LC_F\frac{\alpha_s}{2\pi}+\frac{4}{3}(Re\ r_{+-}\cos 2\phi_- - Im\ r_{+-}\sin 2\phi_-) \quad (28)$$

for the shape of the azimuthal distribution. Here r_{+-} is the weighted average of ρ_{+-},

$$r_{+-} = \frac{\int d\cos\vartheta \frac{d\sigma}{d\cos\vartheta}\rho_{+-}}{\int d\cos\vartheta \frac{d\sigma}{d\cos\vartheta}} \quad (29)$$

In fig. 1 the ratio $\frac{\frac{d\sigma}{d\phi_-}|_{ho}}{\frac{d\sigma}{d\phi_-}|_{lo}}$ is shown using the standard model values for r_{+-} (Im $r_{+-}=0$ and Re $r_{+-}=-0.53$). Nonstandard triple gauge boson vertex interactions modify the ϕ_- (as well as other) distributions, by modifying r and ρ, respectively. For example, if there are CP violating gauge boson couplings, r and ρ may acquire non-vanishing imaginary pieces, apart from modifications of the standard model predictions for the real pieces induced by all nonstandard effects whether CP violating or not.

In fig. 1, ϑ and θ_- have been averaged out completely. Interesting information may also be gained by retaining some of the dependence in the polar angles. For example, one may consider the ϕ dependence of the forward-backward asymmetry in ϑ, in order to keep track of the parity violating terms ($\sim\cos\vartheta$) in the triple gauge boson interaction. The ratio of this distribution for ho and lo is also included in fig. 1. We conclude that the QCD effects must be taken into account, if one wants to check the standard model triple gauge boson vertex to an accuracy of 1%.

Furthermore, one may examine distributions, in which ϕ_- instead of θ_- is integrated out,

$$\frac{d\sigma(e^+e^-\to W^+j_-X)}{d\cos\vartheta d\cos\theta_-} = \frac{3}{4}\frac{d\sigma(e^+e^-\to W^+W^-)}{d\cos\vartheta}\Big\{(\rho_{++}+\rho_{--}) \times \Big\{\frac{1}{2}(1+\cos^2\theta_-)+2LC_F\frac{\alpha_s}{2\pi}\Big\}+\rho_{LL}\Big\{\sin^2\theta_-+2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\}(1+\frac{\alpha_s}{\pi})(1-3LC_F\frac{\alpha_s}{2\pi}) \quad (30)$$

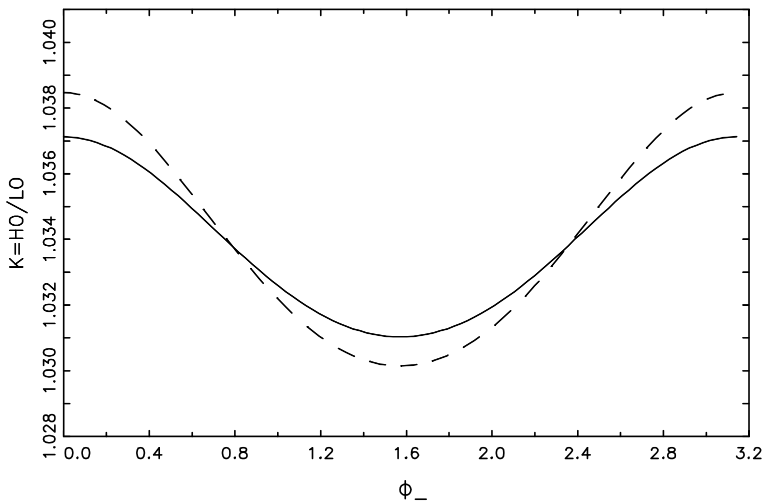

Figure 1: The ratio $\frac{\frac{d\sigma}{d\phi_-}|_{ho}}{\frac{d\sigma}{d\phi_-}|_{lo}}$ as a function of ϕ_-. The dependence on θ_- and ϑ have been averaged out (full curve). To obtain the dashed curved we have integrated over $\cos\vartheta$ in an antisymmetric way, i.e. the integrand is counted negative, if $\cos\vartheta$ is negative.

In this equation the parity violating piece $\sim \cos\theta_-$ has been left out, because it cannot be seen in the hadronic W^- decay. Since the relative size of QCD corrections is not the same for the transverse and longitudinal part of eq. 30 the shape of the θ_- distribution is modified by the QCD term. In fig. 2 the ratio of the ho to lo distribution is shown as a function of θ_-, both for symmetric and antisymmetric integration over ϑ. Again, the quantitative results contained in this figure will allow to discriminate QCD from nonstandard effects. Clearly, the variation of the curves in fig. 2 must be taken into account, if one wants to check the standard model triple gauge boson vertex to an accuracy of 1%. Note that in all the figs. 1, 2 and 3 below, the average of the curves correspond to the overall QCD correction factor $1 + \frac{\alpha_s}{\pi}$.

If the decay of the W^+ is taken into account and both W's decay hadronically, the situation becomes more complicated. For example, the shape of the double differential azimuthal distribution is of the generic form

$$\frac{d\sigma(e^+e^- \to W^+W^- \to j_+j_-X)}{d\phi_+ d\phi_-} \sim \sum_{ABA'B'} r_{ABA'B'} D^0_{AB} D^0_{A'B'} \\ + 2LC_F\frac{\alpha_s}{2\pi}\sum_{A'B'} r_{A'B'} D^0_{A'B'} + 2LC_F\frac{\alpha_s}{2\pi}\sum_{AB} r_{AB} D^0_{AB} \quad (31)$$

where $r_{ABA'B'}$, r_{AB} and $r_{A'B'}$ are defined in analogy with r_{+-}, eq. 29. The angles ϕ_- and ϕ_+ refer to the decay of the W^- and W^+, respectively. If only one of the

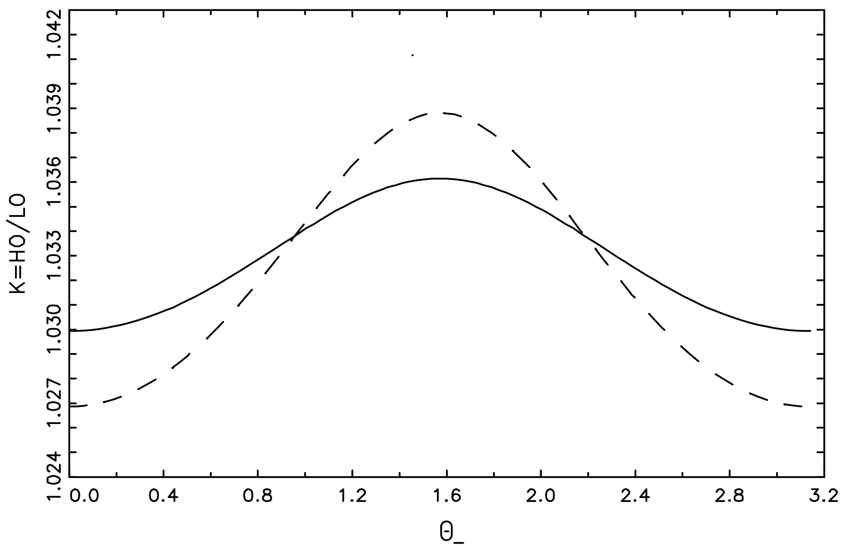

Figure 2: The ratio of the ho to lo as a function of θ_-, both for symmetric and antisymmetric integration over ϑ.

W's decays hadronically one obtains a similar result for $\frac{d\sigma}{d\phi_+ d\phi_-}$, with only one term $\sim 2LC_F...$ instead of two such terms. Note that in eq. 1 ϕ_- and ϕ_+ were denoted by ϕ and ϕ_l, respectively. We present the numerical results on the effect of the higher order corrections on the double differential azimuthal distribution $\frac{d\sigma}{d\phi_- d\phi_+}$ in table 2 (for the case that one W decays leptonically) and table 3 (for the case that both W's decay hadronically). The entries in the table are the ratio of higher order to leading order cross-sections in the respective ϕ_+–ϕ_- bins. All angles are in units of π. We see that the variation of the numbers in table 2 is en gross about the same as the variations of the curves in figs. 1 and 2, and conclude that in all the cases it is necessary to take these QCD corrections into account, if one wants to pin down the standard model triple gauge boson vertex to an accuracy of 1-2%. Of course, the overall QCD K factor $1 + \frac{\alpha_s}{\pi}$ was known before, but in figs. 1, 2 and 3 and tables 2 and 3 we see that it is also important to know the deviations from this average.

Conversely, if one leaves the polar angle dependence and integrates over the azimuthal angles, one obtains

$$\frac{d\sigma(e^+e^-\to j_+j_-X)}{d\cos\vartheta d\cos\theta_- d\cos\theta_+}=(\frac{3}{4})^2\frac{d\sigma(e^+e^-\to W^+W^-)}{d\cos\vartheta}\Big\{(\rho_{++++}+\rho_{++--}\rho_{--++}+\rho_{----})$$
$$\times\Big\{\frac{1}{2}(1+\cos^2\theta_-)+2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\{\frac{1}{2}(1+\cos^2\theta_+)+2LC_F\frac{\alpha_s}{2\pi}\Big\}$$
$$+\rho_{LLLL}\Big\{\sin^2\theta_-+2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\{\sin^2\theta_++2LC_F\frac{\alpha_s}{2\pi}\Big\}$$
$$+(\rho_{LL++}+\rho_{LL--})\Big\{\sin^2\theta_-+2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\{\frac{1}{2}(1+\cos^2\theta_+)+2LC_F\frac{\alpha_s}{2\pi}\Big\}$$
$$+(\rho_{++LL}+\rho_{--LL})\Big\{\sin^2\theta_++2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\{\frac{1}{2}(1+\cos^2\theta_-)+2LC_F\frac{\alpha_s}{2\pi}\Big\}\Big\}$$
$$\times(1+\frac{\alpha_s}{\pi})^2(1-3LC_F\frac{\alpha_s}{2\pi})^2 \quad (32)$$

where $\rho_{ABA'B'}(s,\cos\vartheta)$ denotes the two-particle joint density matrix for $W^\pm$ production as defined in ref. [6]. The complicated looking formulae eqs. 31 and 32 can be verified easily by combining the lowest order formulae of reference [6] with the compact form of our higher order result eq. 25. Terms $\sim\cos\theta_+$ and $\sim\cos\theta_-$ have been left out, because it has been assumed that both W's decay hadronically and those terms cannot be detected in that case.

This case of both W's decaying hadronically poses additional QCD problems, which we will not discuss, but which have recently become the focus of some interest [9] [3]. These have to do with finite width effects, which are partially nonperturbative and thus cannot be calculated from first principles, with important consequences for the precision to which the Standard Model can be tested at LEP200.

The distributions eq. 32 may also be considered in case that only one W (e.g. the W^-) decays hadronically. In that case in eq. 32 one has to add the parity violating

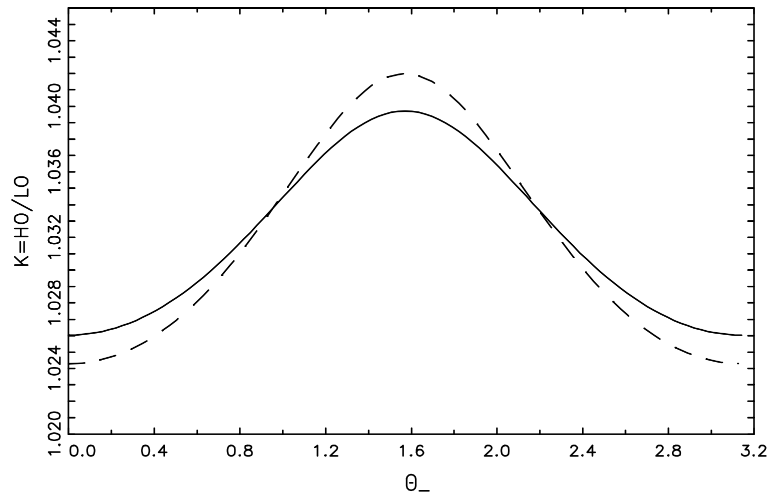

Figure 3: The ratio of the ho to the lo of the antisymmetric piece in $\cos\theta_+$ (coefficient of $\cos\theta_+$ in eq. 33) as a function of θ_-, both for symmetric (solid curve) and antisymmetric (dashed curve) integration in ϑ.

pieces $\sim \cos\theta_+$, because they can be determined from the direction of the l^+.

$$
\begin{aligned}
\frac{d\sigma(e^+e^- \to l^+\nu j_- X)}{d\cos\vartheta\, d\cos\theta_- d\cos\theta_+} = (\frac{3}{4})^2 \frac{d\sigma(e^+e^- \to W^+W^-)}{d\cos\vartheta} \Big\{ & (\rho_{++++} + \rho_{++--} + \rho_{--++} + \rho_{----}) \\
& \times \Big\{\frac{1}{2}(1+\cos^2\theta_-) + 2LC_F\frac{\alpha_s}{2\pi}\Big\}\frac{1}{2}(1+\cos^2\theta_+) \\
& + \rho_{LLLL}\Big\{\sin^2\theta_- + 2LC_F\frac{\alpha_s}{2\pi}\Big\}\sin^2\theta_+ \\
& + (\rho_{LL++} + \rho_{LL--})\Big\{\sin^2\theta_- + 2LC_F\frac{\alpha_s}{2\pi}\Big\}\frac{1}{2}(1+\cos^2\theta_+) \\
& + (\rho_{++LL} + \rho_{--LL})\sin^2\theta_+\Big\{\frac{1}{2}(1+\cos^2\theta_-) + 2LC_F\frac{\alpha_s}{2\pi}\Big\} \\
& - (\rho_{++++} - \rho_{++--} + \rho_{--++} - \rho_{----})\Big\{\frac{1}{2}(1+\cos^2\theta_-) + 2LC_F\frac{\alpha_s}{2\pi}\Big\}\cos\theta_+ \\
& - (\rho_{LL++} - \rho_{LL--})\Big\{\sin^2\theta_- + 2LC_F\frac{\alpha_s}{2\pi}\Big\}\cos\theta_+\Big\} \\
& \times (1+\frac{\alpha_s}{\pi})(1-3LC_F\frac{\alpha_s}{2\pi}) \quad (33)
\end{aligned}
$$

Note that now the factors depending on θ_+ do not get the QCD term. In fig. 3 the ratio of the ho to the lo of the antisymmetric piece in $\cos\theta_+$ (coefficient of $\cos\theta_+$ in the last equation) is shown as a function of θ_-, both for symmetric and antisymmetric integration in ϑ. The variation of the curves is somewhat larger than in figs. 1 and 2, so that essentially the same conclusions can be drawn.

Conclusions

In this article we have presented a complete calculation of one loop QCD corrections to W–pair production and decay at LEP200, including W–polarization effects. We have put special emphasis on the effects on those angular distributions sensitive to the structure of triple gauge boson vertices. Our results are of the order of a few percent and thus are certainly as important as finite W–width effects and must be taken into account for a precision study of the pure gauge sector of the standard model.

This work is part of a larger project, which one of the authors has pursued over the last years, namely to calculate QCD corrections to differential distributions using the known QCD effects on total rates plus the real gluon matrix elements [10]. This method has been applied successfully even to two loop problems [10] at LEP1. It could certainly be used to extend eq. 25 to two loops. We have not attempted this, because it is unlikey to be relevant given the magnitude of statistical and systematic errors expected at LEP200.

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Appendix: The complete W decay functions to O(α_s)

Before integration over χ, x_1 and x_2 one has a cross section

$$\frac{d\sigma(e^+e^- \to W^+W^- \to l^+\nu_l q\bar{q}'g)}{d\cos\vartheta d\cos\theta_l d\phi_l d\cos\theta d\phi dx_1 dx_2 d\chi}$$

containing decay functions $D_{AB}(\theta,\phi,\chi,x_1,x_2)$ which depend on χ, x_1 and x_2 in addition to θ and ϕ. These will be given in the following. First, for A=B=L, one has

$$\frac{D_{LL}(\theta,\phi,\chi,x_1,x_2)}{D_{total}} = \frac{\sin^2\theta}{2}\delta(1-x_1)\delta(1-x_2)\delta(\chi) + C_F\frac{\alpha_s}{2\pi}\Big\{l(x_1,x_2)(1-2\sin^2\theta + \cos^2\chi\sin^2\theta) + m(x_1,x_2)\sin\chi\sin\theta\cos\theta\Big\} \quad (34)$$

where $D_{total} = 2(1+\frac{\alpha_s}{\pi})$. The functions $l(x_1,x_2)$ and $m(x_1,x_2)$ are shown in table 1. The integral of l over dx_1dx_2 is the number L=0.4875 as given in the main text eq. 18. The integral M of m over dx_1dx_2 is given in table 1. m does not contribute to $\frac{d\sigma}{d\cos\vartheta d\cos\theta_l d\phi_l d\cos\theta d\phi}$ because the coefficient of m vanishes when the integral over χ is performed. It contributes only to the azimuthal (=χ) dependence in 3-jet decays. Secondly, one finds

$$\frac{(D_{++}+D_{--})(\theta,\phi,\chi,x_1,x_2)}{D_{total}} = \frac{1+\cos^2\theta}{2}\delta(1-x_1)\delta(1-x_2)\delta(\chi) - C_F\frac{\alpha_s}{2\pi}\Big\{m(x_1,x_2)\sin\chi\sin\theta\cos\theta + l(x_1,x_2)(1-2\sin^2\theta+\cos^2\chi\sin^2\theta)\Big\} \quad (35)$$

$$\frac{(D_{++}-D_{--})(\theta,\phi,\chi,x_1,x_2)}{D_{total}} = -\cos\theta\delta(1-x_1)\delta(1-x_2)\delta(\chi) - C_F\frac{\alpha_s}{2\pi}\Big\{p(x_1,x_2)\cos\theta - n(x_1,x_2)\sin\chi\sin\theta\Big\} \quad (36)$$

The function $p(x_1,x_2)$ together with its integral P over x_1 and x_2 is given in table 1. It gives a parity violating contribution which is not measurable in hadronic W decays. The function $n(x_1,x_2)$ and its integral N are given in table 1, too. They do not contribute to the inclusive distributions discussed in the main text because their coefficient vanishes when the integral over χ is performed. Furthermore they arise from the parity violating part of the W decay and are not measurable in hadronic W decays. Finally, one finds

$$\frac{D_{+-}(\theta,\phi,\chi,x_1,x_2)}{D_{total}} = \frac{e^{2i\phi}}{2}\frac{\sin^2\theta}{2}\delta(1-x_1)\delta(1-x_2)\delta(\chi) + \frac{e^{2i\phi}}{2}C_F\frac{\alpha_s}{2\pi}\Big\{l(x_1,x_2)(1-2\cos^2\chi+\sin^2\theta\cos^2\chi - 2\sin^2\theta + 2i\sin\chi\cos\chi\cos\theta) - m(x_1,x_2)\sin\theta(i\cos\chi+\sin\chi\cos\theta)\Big\} \quad (37)$$

$$
\begin{aligned}
\frac{D_{\pm L}(\theta,\phi,\chi,x_1,x_2)}{D_{total}} &= \frac{e^{\pm i\phi}}{2\sqrt{2}}(\pm\cos\theta\sin\theta-\sin\theta)\delta(1-x_1)\delta(1-x_2)\delta(\chi) \\
&+ \frac{e^{\pm i\phi}}{\sqrt{2}}C_F\frac{\alpha_s}{2\pi}\Big\{l(x_1,x_2)\sin\theta(i\cos\chi\sin\chi\pm\cos\theta(\cos^2\chi-2)) \\
&- \frac{1}{2}p(x_1,x_2)\sin\theta+m(x_1,x_2)(\pm\sin\chi(\cos^2\theta-\frac{1}{2})-\frac{i}{2}\cos\chi\cos\theta) \\
&+ n(x_1,x_2)(\pm\frac{i}{2}\cos\chi-\frac{1}{2}\sin\chi\cos\theta)\Big\} \qquad (38)
\end{aligned}
$$

Note that $D_{-+}(\theta,\phi,\chi,x_1,x_2)$ and $D_{L,\pm}(\theta,\phi,\chi,x_1,x_2)$ can be obtained via the relation $D_{AB}(\theta,\phi,\chi,x_1,x_2) = D^*_{AB}(\theta,\phi,,\chi,x_1,x_2)$.

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	q T	$\bar{q}$ T	g T	integral
$l(x_1,x_2)$	$2\frac{x_1+x_2-1}{x_1^2}$	$2\frac{x_1+x_2-1}{x_2^2}$	$4\frac{1-x_3}{x_3^2}$	L=0.4875
$n(x_1,x_2)$	$\sin\theta_{12}\left\{\frac{1}{(1-x_1)(1-x_2)}-\frac{1+x_2}{1-x_1}\right\}$	$\sin\theta_{12}\left\{\frac{1}{(1-x_1)(1-x_2)}-\frac{1+x_1}{1-x_2}\right\}$	$\sin\theta_{23}\left\{\frac{2}{(1-x_1)(1-x_2)}-\frac{2+x_2}{1-x_1}-\frac{1}{1-x_2}+1\right\}$	N=3.236
$p(x_1,x_2)$	$-2\frac{x_2}{x_1}$	$-2\frac{x_1}{x_2}$	$2\frac{x_2+1-2/x_3}{1-x_1}$	P=-1.340
$m(x_1,x_2)$	$\sin\theta_{12}\left\{\frac{1}{(1-x_1)(1-x_2)}-\frac{1+x_2}{1-x_1}-2\frac{x_2}{x_1}\right\}$	$\sin\theta_{12}\left\{\frac{1}{(1-x_1)(1-x_2)}-\frac{1+x_1}{1-x_2}-2\frac{x_1}{x_2}\right\}$	$\sin\theta_{23}\left\{-\frac{4}{x_3}+\frac{2}{1-x_1}-\frac{4}{x_3(1-x_1)}+1 +\frac{2}{(1-x_1)(1-x_2)}+\frac{x_2}{1-x_1}-\frac{1}{1-x_2}\right\}$	M=2.650

Table 1: Coefficients of the QCD helicity cross sections. The results depend on whether quark, antiquark or gluon are used to define the quantization (=thrust) axis.

$\downarrow \phi_+ \| \phi_- \rightarrow$	0 -0.4	0.4 -0.8	0.8 -1.2	1.2 -1.6	1.6 -2.0
0 -0.4	1.039	1.029	1.038	1.033	1.031
0.4 -0.8	1.037	1.031	1.034	1.035	1.030
0.8 -1.2	1.034	1.033	1.034	1.033	1.034
1.2 -1.6	1.030	1.035	1.034	1.031	1.037
1.6 -2.0	1.031	1.032	1.038	1.029	1.039

Table 2: Ratio of ho to lo for the double differential azimuthal distribution $\frac{d\sigma}{d\phi_+ d\phi_-}$, for various ϕ_+ and ϕ_- bins. ϕ_+ and ϕ_- are measured in units of π. It is assumed here that the W^- decay is hadronic and the W^+ decay is leptonic. $\phi_+ = \phi_l$ refers to the direction of the charged lepton in the W^+ decay.

$\downarrow \phi_+ \| \phi_- \rightarrow$	0 -0.2	0.2 -0.4	0.4 -0.6	0.6 -0.8	0.8 -1.0
0 -0.2	1.076	1.069	1.063	1.064	1.072
0.2 -0.4	1.069	1.068	1.063	1.061	1.064
0.4 -0.6	1.063	1.063	1.063	1.063	1.063
0.6 -0.8	1.064	1.061	1.063	1.068	1.069
0.8 -1.0	1.072	1.064	1.063	1.069	1.076

Table 3: Ratio of ho to lo for the double differential azimuthal distribution, eq. 31, for various ϕ_+ and ϕ_- bins. $\phi_\pm$ are the azimuthal angles defined in the decay of the $W^\pm$ and are measured in units of π. It is assumed here that both W's decay hadronically. Therefore the average here is $(1 + \frac{\alpha_s}{\pi})^2 \approx 1 + 2\frac{\alpha_s}{\pi}$ in contrast to table 2, where the average is $1 + \frac{\alpha_s}{\pi}$.