The General Theory of Relativity, Metric Theory of Relativity and Covariant Theory of Gravitation. Axiomatization and Critical Analysis

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The axiomatization of general theory of relativity (GR) is done. Axioms of GR are compared with the axioms of the metric theory of relativity and the covariant theory of gravitation. The need to use the covariant form of total derivative with respect to proper time of the invariant quantities, the 4-vectors and tensors is indicated. The definition of the 4-vector force density in a Riemannian spacetime is deduced.

Keywords: *general relativity; metric theory of relativity; covariant theory of gravitation; axiomatization.*

Establishing of axiomatic foundations is considered as an important step in the development of any modern physical theory. This is due to the fact that from a given complete set of mutually independent axioms is possible uniquely and unambiguously to deduce the whole theory. In addition, on the basis of the axioms it is easy to define the scope of applicability of the theory and its difference to alternative approaches.

Presented in 1915 by Albert Einstein [1] and David Hilbert [2] the equations of general theory of relativity (GR) are based on several principles and heuristic analogies, but has not been axiomatized. Available in GR mathematical apparatus made it possible to solve various problems, that allowed the theory to become generally accepted model of gravity. The problem with the axiomatization of GR has become mature in mid-twentieth century, when it became clear that GR can not be quantized in the same way, as electromagnetic theory.

In general theory of relativity is also not defined the tensor of gravitational field, which prevents to recognize GR as the full theory of the gravitational field. The equations of GR predict singularities with infinite energy density, and black holes with a magnitude of gravity, that it must hold within itself not only substance, but even the rays of light. However, in the framework of GR to give answer about the real existence of such exotic objects apparently not possible.

For the theoretical foundation of GR usually applies the following principles:

1) The principle of equivalence in different forms, including:

1.1) The equality of inertial and gravitational masses.

1.2) The equivalence of inertial and gravitational accelerations in the description of phenomena in the frame of the infinitely small test particle.

1.3) The equivalence of the state of free falling in any gravitational field and the inertial motion in the absence of a gravitational field, under the assumption that the instantaneous velocity of falling is equal to velocity of inertial motion.

1.4) The equivalence of forms of movement with the same initial conditions for any uncharged and non-rotating test particles in a gravitational field regardless of the structure and composition of their substance.

1.5) The equivalence of natural phenomena for the free falling in the gravitational field of an observer in his reference system, understood as a form of independence of events from the fall velocity and location in the gravitational field.

1.6) The equivalence of the effects of gravitation and deformation of spacetime; description of gravitation through the metric tensor and its derivatives over the coordinates and time.

2) The principle of motion along geodesics arising from 1.1), 1.3) and 1.4).

3) The principle of the distortion of spacetime by substance, electromagnetic field and other non-gravitational fields.

4) The principle of linear relationship between the curvature of spacetime and energy-momentum of substance and nongravitational fields (tensor equation of the Einstein-Hilbert for metric).

5) The principle of determining of the force and the equations of motion through the covariant derivative of the energy-momentum tensor.

6) Correspondence principle: in the weak field equations of GR become the classical equation of Newton's gravitation and the metric of spacetime becomes the metric of flat Minkowski spacetime.

7) The principle of covariance: physical quantities and the equations of GR must be written in covariant form, does not depend on the choice of the reference system.

It is most convenient to measure the metric in GR by means of electromagnetic waves by determining the deflection of light rays and the effect of time dilation of electromagnetic hours, depending on the coordinates and time. Hence there is a metric tensor that defines the gravitational field. Therefore, in GR is suggested that the rate of change and propagation of gravitation equals the speed of light, which has the electromagnetic wave at a given point of space-time. The speed of light in a gravitational field depends on the coordinates and time and is considered as a maximum transfer speed of interactions. Metric tensor in GR represents a gravitational field so that the covariance of the metric tensor under the transformations of any reference system defines the covariance of the gravitational field.

After the appearance in 2009 of the metric theory of relativity (MTR) and the covariant theory of gravitation (CTG), which were originally axiomatized [3], the need to conduct an axiomatization of GR appeared in order to compare the physical basis of these theories with a joint point of view. Axiomatization of GR can be useful for comparison with other alternative theories of gravitation.

Analysis of GR shows that it contains two closely related components. The first of these is the general relativity of phenomena in different reference systems. This part of the theory can link the results of spacetime measurements of different observers and recalculate the physical quantities from one frame to another. The second part of GR is the theory of gravitational field and its interaction with matter. Both parts of GR could be completely withdrawn from the respective systems of axioms [4]. By the merger of general relativity and the theory of gravitation in these systems of axioms, there is one common axiom that describes the connection of the metric and matter in the equation for calculating of the metric.

Axioms of general relativity in GR

1. Properties of spacetime defined by uncharged and noninteracting test particles and waves do not depend on the type of particles and waves.

2. Characteristic of the spacetime is the symmetric metric tensor $g_{\mu\nu}$, which depends in general on the coordinates and time. With the help of the tensor $g_{\mu\nu}$ are computed various invariants associated with 4-vectors and tensors.

3. Square of the interval Ds gives the square of the length of the 4-vector differential of coordinates and time, which does not depend on the choice of the reference system:

$$(Ds)^2 = g_{\mu\nu} Dx^{\mu} Dx^{\nu} = g'_{\mu\nu} Dx'^{\mu} Dx'^{\nu} = (Ds')^2 ,$$

where the symbol D denotes the total differential in curved spacetime.

Spatio-temporal measurement and fixing of the metric properties are carried out usually by means of electromagnetic waves whose speed may vary depending on position and time in the frame of reference, but not on the velocity of the radiating bodies. For the electromagnetic wave interval is always zero: $Ds = 0$.

4. Physical properties of matter and fields except the gravitational field are dependent from the corresponding tensors of energy density and momentum. There is a mathematical function of the metric tensor $g_{\mu\nu}$ (e.g. the Hilbert-Einstein tensor on the left side of the equation for the metric) which is proportional to the total energy-momentum tensor of substance and fields on the right side:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\gamma}{c^4}\left(\phi_{\mu\nu} + W_{\mu\nu}\right), \qquad (1)$$

where $R_{\mu\nu}$ − Ricci tensor, R − scalar curvature, Λ − cosmological constant, γ − gravitational constant, c − speed of light, $\phi_{\mu\nu}$ − energy-momentum tensor of substance, $W_{\mu\nu}$ − energy-momentum tensor of electromagnetic field and other nongravitational fields. Using this equation, the connection may be found between the geometric properties of spacetime, on the one hand, and the physical properties of existing substances and non-gravitational fields, on the other hand.

5. There are used additional conditions which are necessary for the calculation of ratios for the shifts and turns of the compared reference frames, the velocity of their motion relative to each other, and taking into account the symmetry properties of reference systems.

To derive the transformations between the differentials of the coordinates and time of any two frames of reference, we use the condition of equality of intervals $Ds = Ds'$ in axiom 3. Interval is invariant to the calculation of which in each frame requires knowledge of the metric tensor specified in axiom 2. In addition, according to axiom 5 there should be additional relationships and connections between these frames of reference. For example, the Lorentz transformation for two inertial reference systems take into account: the location and relative orientation of reference frames, and their velocity relative to each other, the symmetry transformations for the axes perpendicular to the velocity of movement, including the same speed of light.

The principle of equivalence can be attributed to the independence of the metric on the type and properties of test particles and waves, assumed in axiom 1. In accordance with axiom 4 the transition from general relativity to special theory of relativity must be accompanied by approaching to zero density and velocity of test particles, as well as the strengths of non-gravitational fields acting on the particles. Taking in account the axiom 5 it is enough to get all the relations of special relativity.

Axioms of the gravitational field in GR

1. Properties of the gravitational field are given by the velocity of propagation of gravitational interaction, equal to the velocity of light and depends in general on the coordinates and time, as well as by non-degenerate metric tensor of second rank $g_{\mu\nu}$.

2. The gravitational field is reduced to the geometric distortion (strain) of spacetime caused by the source of substance and any nongravitational field. The degree of curvature of spacetime is fixed by the curvature tensor of the Riemann-Christoffel $R_{\rho\mu\sigma\nu}$ which is the function of $g_{\mu\nu}$ and its derivatives of first and second order over coordinates and time. With the help of metric contraction, using the metric tensor, the Ricci tensor $R_{\mu\nu}$ and then scalar curvature R may be found from the tensor $R_{\rho\mu\sigma\nu}$.

3. Gravitational acceleration is reduced to the gradients of the metric tensor $g_{\mu\nu}$, i.e. to the rate of change components of the metric tensor in space and time.

4. Properties of matter, defined as a substance and non-gravitational fields, are given by the energy-momentum tensor $T_{\mu\nu} = \phi_{\mu\nu} + W_{\mu\nu}$.

5. Relationship between the gravitational (metric) field, given by the metric tensor $g_{\mu\nu}$ through the curvature of spacetime and matter is defined by the Hilbert-Einstein equations for the metric:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\gamma}{c^4} T_{\mu\nu} .$$

From Axiom 3 here can be deduced the principle of equivalence. Covariant derivative acting on both sides of the equation for the metric in axiom 5, draws them to zero. It fixes the properties of the Hilbert-Einstein tensor, and simultaneously sets the equation of motion of substance.

Comparison of the theories of relativity

Axioms of the metric theory of relativity (MTR) are [3]:

1. Properties of the spacetime manifold in a given frame depend on the properties of the test bodies and the waves, through which the spacetime measurements are fulfilled in the frame of reference. The most important property of test bodies and the waves is the speed c of their propagation, as it appears in the formulas to measure the velocities of other bodies and delay information in distance measurements.

2. Geometric properties of spacetime are fixed by a relevant mathematical object, which is a function of spacetime coordinate reference system. For a large class of reference systems suitable mathematical object is the non-degenerate four-dimensional symmetric metric tensor of second rank $g_{\mu\nu}$, whose components are scalar products of unit vectors of axes chosen reference system. Tensor $g_{\mu\nu}$ allows finding any invariants associated with 4-vectors and tensors.

3. Square of the interval $(Ds)^2$ between two close events, understood as the tensor contraction of the metric tensor $g_{\mu\nu}$ with the product of differentials of the coordinates $Dx^{\mu} Dx^{\nu}$, is invariant, the measure of its own dynamic (proper) time τ of the moving particle, and does not depend on the choice of the reference system:

$$(Ds)^2 = c^2 (D\tau)^2 = g_{\mu\nu} Dx^{\mu} Dx^{\nu} = g'_{\mu\nu} Dx'^{\mu} Dx'^{\nu} = (Ds')^2 .$$

The interval Ds for two close events is zero, if these events are related to the propagation of test bodies and the waves, through which the spacetime measurements and fixing of metrics are fulfilled.

4. The physical properties of substance and any fields including the gravitational field in some frame of reference are given by the corresponding tensors of energy density and momentum. There is a mathematical function of the metric tensor $g_{\mu\nu}$, found by certain rules and proportional to the total energy-momentum tensor of substance and fields, acting in this frame of reference. In the simplest case, such the function is the Einstein-Hilbert tensor, in the left part of the equation for the metric:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\gamma\beta}{c_g^4} \left(\phi_{\mu\nu} + U_{\mu\nu} + W_{\mu\nu} \right), \tag{2}$$

where β − constant depending on the type of test particles or waves, determined by comparison with experiment or with the formulas of classical physics in the weak-field or low-velocity limit,

c_g − propagation speed of gravity, presumably equal to the speed of light,

$U_{\mu\nu}$ − tensor of energy-momentum density of gravitational field.

Equation (2) provides the link between the geometric properties of used spacetime manifold, on the one hand, and the physical properties of available substance and existing fields, on the other side. Covariant derivative acting on both sides of the equation for the metric (2), draws them to zero. It fix the properties of the Hilbert-Einstein tensor (or equivalent tensor), and simultaneously sets the equation of motion of substance under the influence of fields.

5. There are used additional conditions which are necessary for the calculation of ratios for the shifts and turns of the compared reference frames, the velocity of their motion relative to each other, and taking into account the symmetry properties of reference systems.

Equivalence of the acceleration due to gravitation and inertial acceleration under the action of uniformly distributed over the volume of the test body non-gravitational forces of the same quantity leads to the equality of gravitational and inertial masses. The homogeneity of the applied force means that in system of small size all part of the system are accelerated equally and the relative internal acceleration is absent. In this case, the individual elements of the test body does not put pressure on each other and behave as if the test body moving by inertia in the absence of forces. Masses of bodies may be weighed in relation to the standard mass in a gravitational field, and the masses are proportional to the gravitational forces. This implies the independence of the forms of motion of falling bodies from the mass and composition of these bodies. Because at any point in the gravitational field a falling body behaves in the same way as moving by inertia (but with a change in velocity), it is assumed that in the falling body take place Lorentz invariance. Then the Lorentz invariance should be at any point in the trajectory of the falling body and does not depend on the velocity, and the falling observer does not have to reveal by inner experience acceleration of the movement. As a result, the equivalence principle leads to the identification of the effect of the gravitational field of a massive body with the effect of deformation of spacetime around the massive body. Such are the consequences of the equivalence principle in general relativity.

In the metric theory of relativity (MTR), instead of the principle of equivalence of forces considers the principle of equivalence of energy and momentum. Indeed, from Einstein-Hilbert equation (2) for the metric in the MTR can be seen that the metric is completely determined by the sources in the form of tensors of the density-energy-momentum of substance and fields including the gravitational field itself [3]. Only the energy-momentum of the system is needed to determine the metrics and the equations of motion of a test body. If two different interactions have the same dependence of the energy-momentum, then the metric and the law of motion in both cases coincide. The equation of general relativity for the metric (1) differs from equation (2) for the metric MTR that the right-hand side of (2) contains the tensor density of the energy-momentum of gravitational field. The contribution of this tensor in weak fields is small, and the MTR metric is slightly different from the metric of general relativity. However, in strong gravitational fields the tensor can not be ignored, since there is a significant self action of field on the source of field.

From the comparison of the axioms of general relativity in GR with the axioms of the metric theory of relativity follows the features of these theories are listed in Table 1.

Table 1

Features of theories	General relativity in GR	Metric theory of relativity
Metric properties of spacetime:	Do not depend on the type of test particles and waves	Depend on the type of test particles and waves
Interval is equal to zero: $Ds = 0$	Only for electromagnetic waves	For all test particles and waves, which are used for the space-time measurement and fixing of metrics
Sources of energy and momentum that define metric:	Substance and any non-gravitational field	Substance and any field including the gravitational field
The principle of equivalence is understood as:	Equivalence of phenomena in two reference systems of small size, one of which is accelerated by the gravitational force, while the other receives the same acceleration under the action of uniformly distributed non-gravitational forces of the same magnitude	Equivalence of energy and momentum: "In the accelerated frame the metric is locally does not depend on the type of the current force causing this acceleration, but depend on the configuration of forces in space-time reference system defined by the energy-momentum tensor"

Comparison of the theories of gravitational field

Axioms of a covariant theory of gravitation (CTG) in 4-dimensional vector-tensor formalism are given by [3]:

1) The properties of the gravitational field are given by the velocity of propagation of gravitational interaction c_g, as well as the scalar potential ψ and vector potential $\boldsymbol{D}$.

2) The potentials of the gravitational field can be combined into 4-vector of gravitational potential with covariant index:

$$D_\mu = \left(\frac{\psi}{c_g}, -\mathbf{D} \right).$$

The rate of change of potentials in spacetime of chosen reference system is given by the tensor of gravitational field, made up of derivatives from components of 4-vector gravitational potential:

$$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu ,$$

where ∇_μ denotes the covariant derivative , μ,ν – the usual 4-indices, so that

$$\partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{c\partial t}, \quad \partial_1 = \frac{\partial}{\partial x^1} = \frac{\partial}{\partial x}, \quad \partial_2 = \frac{\partial}{\partial x^2} = \frac{\partial}{\partial y}, \quad \partial_3 = \frac{\partial}{\partial x^3} = \frac{\partial}{\partial z}.$$

With an appropriate choice of field potentials, there is the relation of symmetric potentials:

$$\nabla_\rho \Phi_{\mu\nu} + \nabla_\mu \Phi_{\nu\rho} + \nabla_\nu \Phi_{\rho\mu} = \partial_\rho \Phi_{\mu\nu} + \partial_\mu \Phi_{\nu\rho} + \partial_\nu \Phi_{\rho\mu} = 0. \qquad (3)$$

3) The properties of substance are given its density ρ_0 in the comoving frame of reference and velocity $\mathbf{V}$.

4) The quantities ρ_0 and $\mathbf{V}$ come in 4-vector density of mass current or momentum density:

$$J^\mu = \rho_0 u^\mu = \left(\frac{c_g \rho_0}{\sqrt{1 - V^2/c_g^2}}, \frac{\mathbf{V} \rho_0}{\sqrt{1 - V^2/c_g^2}} \right) = (c_g \rho, \mathbf{J}),$$

where $u^\mu = \left(\frac{c_g}{\sqrt{1 - V^2/c_g^2}}, \frac{\mathbf{V}}{\sqrt{1 - V^2/c_g^2}} \right)$ – 4-velocity of an element of substance,

$\rho = \frac{\rho_0}{\sqrt{1 - V^2/c_g^2}}$ – density of moving substance,

$\mathbf{J}$ – 3-vector density of mass current.

5) The relation between the gravitational field and substance can be expressed through the relationship of 4-vector gravitational potential D^μ and 4-vector density of mass current J^μ, or through connection between the tensor $\Phi^{\mu\nu}$ and J^μ:

$$\square^2 D^\mu = \frac{\partial^2 D^\mu}{c_g^2 \partial t^2} - \nabla^2 D^\mu + R^\mu{}_\nu D^\nu = -\frac{4\pi\gamma J^\mu}{c_g^2} = -\nabla_\nu \Phi^{\mu\nu}, \quad (4)$$

where $\square^2$ means four-dimensional D'Alembert operator in curved Riemannian space, acting on 4-vector D^μ, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the 3-Laplace operator, $R^\mu{}_\nu$ is the Ricci tensor with mixed indices.

Features of the gravitational field in GR and in the covariant theory of gravity stemming from their axioms are given in Table 2.

Table 2

Features of theories	**The theory of the gravitational field in GR**	**Covariant theory of gravitation**
Gravitational field is:	Metric tensor field, which is characterized by the tensor $g_{\mu\nu}$ and its gradients in the form of Christoffel symbols	Physical vector field, characterized by the 4-vector potential and its gradients in the form of the antisymmetric tensor of gravitational field strengths
Properties:	Contraction of the metric tensor in the form $g_{\mu\nu} g^{\rho\nu} = \delta^\rho_\mu$ gives the Kronecker delta δ^ρ_μ	The components of the 4-vector potential D^μ are calibrated so that there are condition of symmetry of the potentials (3) and the wave equation (4)
The speed of the gravitational field is:	The speed of light	The speed of propagation of gravitation (about the speed of light)
The connection between gravitational field and substance in the absence of other fields:	Through the Hilbert-Einstein tensor equations for the metric (1), linking the function of the metric tensor and the tensor of density energy-momentum of substance	Through equation (4) for the potentials and strengths of the gravitational field, and a 4-vector density of mass current J^μ
Sources of energy and momentum that define a metric:	Substance and any non-gravitational fields	Substance and any fields including the gravitational field

Despite the difference in systems of axioms of the gravitational field in GR and in CTG, we can show that the equation of motion of general relativity is a special case of equations of motion of the CTG. As was found in [3], the material derivative on proper time in the general case can be written in the form of an operator using 4-velocity u^{μ} of elements of substance :

$$\frac{D}{D\tau} = u^{\mu} \nabla_{\mu}, \tag{5}$$

where the symbol D denotes the total differential in curved spacetime, and ∇_{μ} is the covariant derivative.

By definition in KTG, the force density is the total rate of change in the 4-vector density of mass current on the proper time in a Riemannian space-time:

$$f^{\nu} = \frac{DJ^{\nu}}{D\tau} = u^{\mu} \nabla_{\mu} J^{\nu} = u^{\mu} (\partial_{\mu} J^{\nu} + \Gamma^{\nu}_{\mu\rho} J^{\rho}) = \frac{dJ^{\nu}}{d\tau} + \Gamma^{\nu}_{\mu\rho} u^{\mu} J^{\rho}, \tag{6}$$

where $\Gamma^{\nu}_{\mu\rho}$ – Christoffel symbol.

On the other hand, the expression for the force acting on the element of substance by gravitational and electromagnetic fields is obtained by taking the covariant derivative in equation (2), written in contravariant indices. Then the left side of the equation for the metric (2) gives zero, and from the right side of this equation follows:

$$f^{\mu} = \nabla_{\nu} \phi^{\mu\nu} = -\nabla_{\nu} U^{\mu\nu} - \nabla_{\nu} W^{\mu\nu} = g^{\mu\rho} (\Phi_{\rho\nu} J^{\nu} + F_{\rho\nu} j^{\nu}). \tag{7}$$

where $F_{\rho\nu}$ – tensor of electromagnetic field strengths,

$j^{\nu} = \rho_{0q} u^{\nu}$ – 4-vector of electromagnetic current density,

ρ_{0q} – electric charge density of the element of substance in its rest system.

Comparing (6) and (7) gives the equation of motion of the element of substance in the CTG under the influence of gravitational and electromagnetic forces:

$$\frac{dJ^{\nu}}{d\tau} + \Gamma^{\nu}_{\mu\rho} u^{\mu} J^{\rho} = g^{\nu\rho} (\Phi_{\rho\mu} J^{\mu} + F_{\rho\mu} j^{\mu}). \tag{8}$$

Equation (8) make it possible fully account for reactive force of Meshcherskiy [5], appears due to changes in the density of the element of substance. The density of matter is part of the 4-vector density of mass current J^{ν}, from which in (8) is taken the derivative with respect to proper time, which characterizes the reaction force in the mechanics of bodies with variable mass.

To move to the formula for the force in general relativity (8) one should make the following simplification: assume equal $\Phi_{\rho\mu}$ to zero (in general relativity the gravitational field is the metric field does not possess the property of self action, and therefore the gravitational field in the right-hand side of equation (1) as a source of distortion spacetime is absent), and assume density of substance ρ_0 constant over time and volume of test particle. Then the quantity ρ_0 in the left-hand side of (8) can be reduced, and from the 4-vector J^{ν} it is possible to pass to the 4- vector of velocity u^{ν}:

$$\frac{du^{\nu}}{d\tau}+\Gamma^{\nu}_{\mu\rho}u^{\mu}u^{\rho}=\frac{1}{\rho_0}g^{\nu\rho}F_{\rho\mu}j^{\mu}. \tag{9}$$

Further, it should be noted that in view of (5) 4-vector of velocity is determined by the total derivative of the coordinates on the proper time:

$$u^{\nu}=\frac{Dx^{\nu}}{D\tau}=u^{\mu}\nabla_{\mu}x^{\nu}=u^{\mu}(\partial_{\mu}x^{\nu}+\Gamma^{\nu}_{\mu\rho}x^{\rho})=\frac{dx^{\nu}}{d\tau}+\Gamma^{\nu}_{\mu\rho}u^{\mu}x^{\rho}. \tag{10}$$

Substituting 4-vector of velocity in (9), in view that the interval can be expressed through the differential of proper time in the form $Ds=c\,D\tau$ or $ds=c\,d\tau$, one obtain:

$$\frac{d}{ds}\left(\frac{dx^{\nu}}{ds}+\frac{1}{c}\Gamma^{\nu}_{\mu\rho}u^{\mu}x^{\rho}\right)+\Gamma^{\nu}_{\mu\rho}\left(\frac{dx^{\mu}}{ds}+\frac{1}{c}\Gamma^{\mu}_{\lambda\beta}u^{\lambda}x^{\beta}\right)\left(\frac{dx^{\rho}}{ds}+\frac{1}{c}\Gamma^{\rho}_{\delta\varepsilon}u^{\delta}x^{\varepsilon}\right)=\frac{1}{\rho_0c^2}g^{\nu\rho}F_{\rho\mu}j^{\mu}. \tag{11}$$

In the simplest case the situation of motion of substance in the absence of electromagnetic fields is considered: $F_{\rho\mu}=0$, or in the absence of charges for the particles of matter: $j^{\mu}=0$. Then the right side of the equation of motion (11) will be zero. If to ignore the products of Christoffel symbols and derivatives from them as second-order terms due to the factors $\frac{1}{c}$, we obtain the standard expression of GR equations of motion for the substance in a gravitational field:

$$\frac{d}{ds}\left(\frac{dx^{\nu}}{ds}\right)+\Gamma^{\nu}_{\mu\rho}\frac{dx^{\mu}}{ds}\frac{dx^{\rho}}{ds}=0. \qquad (12)$$

For the propagation of light must be: $ds=c\,d\tau=0$. Consequently, in (9) differential $d\tau$ must tend to zero. Further, in view of (10) and after multiplication on $(d\tau)^2$ we have:

$$d\tau\, d\left(\frac{dx^{\nu}}{d\tau}+\Gamma^{\nu}_{\mu\rho}u^{\mu}x^{\rho}\right)+\Gamma^{\nu}_{\mu\rho}\left(dx^{\mu}+d\tau\,\Gamma^{\mu}_{\lambda\beta}u^{\lambda}x^{\beta}\right)\left(dx^{\rho}+d\tau\,\Gamma^{\rho}_{\delta\varepsilon}u^{\delta}x^{\varepsilon}\right)=\frac{(d\tau)^2}{\rho_0}g^{\nu\rho}F_{\rho\mu}j^{\mu}. \qquad (13)$$

For the first term on the left side (13) can be written:

$$d\tau\, d\left(\frac{dx^{\nu}}{d\tau}\right)=d\tau\cdot\lim_{\tau_2\to\tau_1}\left(\frac{dx^{\nu}(2)-dx^{\nu}(1)}{d\tau}\right)=\lim_{\tau_2\to\tau_1}\left(dx^{\nu}(2)-dx^{\nu}(1)\right).$$

In the weak-field approximation we can neglect by some quantities in parentheses such as $d\tau\,\Gamma^{\mu}_{\lambda\beta}u^{\lambda}x^{\beta}$ compared with the 4-vectors dx^{μ}, as they relate to each other about as much as the potential of the gravitational field with the square of the speed of light. Setting now in (13) $d\tau=0$, we get zero in right-hand side and arrive to the following:

$$\lim_{\tau_2\to\tau_1}\left(dx^{\nu}(2)-dx^{\nu}(1)\right)+\Gamma^{\nu}_{\mu\rho}\,dx^{\mu}dx^{\rho}=0.$$

We choose as the proper time for the light quantum parameter of time λ along the path, marking the location of the quantum in space, and divide the above equation by the square of the differential $d\lambda$:

$$\frac{d}{d\lambda}\left(\frac{dx^{\nu}}{d\lambda}\right)+\Gamma^{\nu}_{\mu\rho}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\rho}}{d\lambda}=0. \qquad (14)$$

Equation (14) represents the standard equation of motion for the light quantum in general relativity. As was seen in the derivation of (12) and (14) from the equation of CTG (8), the equations of motion of general relativity for particles and light are valid only in weak field approximation, and are a consequence of CTG. In this regard, again the question arises, why in the solar system detects such unexplained phenomena with general relativity, as the Pioneer anomaly [6] and flyby anomaly [7]? One explanation is given in [3], where the difference between the equations of motion (12) general relativity and the equations of motion (8) CTG is underlined. Now we again point out that instead of defining the 4-vector of velocity $u^{\nu} = \frac{dx^{\nu}}{d\tau}$ as adopted in GR, there should be used a more precise definition (10), adopted in CTG. As a result, in the calculations with the help of general relativity in the formulas appear previously unaccounted terms related to the curvature of spacetime.

Thus, from the system of axiom for general relativity in GR, and the system of axiom for the gravitational field in GR displays all the basic features of general theory of relativity. The axioms of general theory of relativity are given in the form that allows to compare them with the axioms of covariant theory of gravitation (CTG) and the metric theory of relativity (MTR). As a consequence, it turns out [4] that general relativity in GR is a special case of the MTR. With regard to the axioms of the gravitational field, in GR principle of geometrization of gravitation and the equivalence principle lead to the concept of metric tensor field as the field of gravitation. In CTG gravitational field is characterized by the vector field of 4-potential and built with the help of the antisymmetric tensor field strengths of the gravitational field, which consists of two components – the gravitational acceleration and the torsion field. The principle of determining the gravitational field in the CTG is similar to the definition of the electromagnetic field, so that the gravitational field of CTG is no less real than the electromagnetic field, with whom it refers to the fundamental fields. The latter means that the electromagnetic and gravitational fields exist not only in research that are available to modern science, but according to the theory of infinite nesting of matter act at different levels of matter. In this case, the gravitational field at the level of elementary particles leads to strong gravity, and at the macro level – to the normal gravity [8].

Analysis of the equivalence principle in general relativity shows that it is valid only in the infinitely small regions, in which it is possible approximation of Lorentz invariance. However, this approximation becomes inaccurate in large enough areas where we can not neglect the curvature of spacetime. For example, if the test particle is massive, its own gravitational field should be considered in the equation of motion of the particle in an external gravitational field. This is because the metric of the two interacting bodies in a nonlinear manner depending on the values of the metrics of these bodies, taken separately from each other. Therefore general relativity, which use in the calculation principle of equivalence and the geometrization of the gravitational field is only an intermediate theory on the way of building more complete theory of relativity and more deep theory of gravitational field, fully taking into account the interaction of the gravitational field with substance and other fields.

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