**Brocard`s Problem. Variants of Brocard`s Problem**

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## Abstract

In this article we considered an open problem. One of the problems in the list of open problems of General Number Theory, existing in [1], [2] is the Brocard`s Problem, asking to find integer values of $n$, for which $n!+1 = m^2$. 'Introduction' section is dedicated to the statement of the main problem. We presented some historical overview and known facts about this problem in the 'Historical overview and known facts' section , based on information presented in the web [1], [2]. In the section 'Variants of the Problem' several variants of the Problem are presented by author based on more general $n!+A=k^2$ [4] equation and asked to find solutions for them.

## Short Introduction to the problem

One of the open problems in General Number Theory as well as in Mathematics is the Brocard`s Problem. Brocard`s Problem asks to find integer values of $n$, for which $n!+1 = m^2$, where n! is the factorial. It was posed by Henri Brocard in a pair of articles in 1876 and 1885, and independently in 1913 by Ramanujan.

More generally the problem has the following form

**Do integers, $\mathbf{n},\mathbf{m}$, exist such that $\mathbf{n}!+\mathbf{1}=\mathbf{m}^{\mathbf{2}}$ , other than $\mathbf{n}=\mathbf{4},\mathbf{5},\mathbf{7}$ ?**

## Historical overview and known facts

Pairs of the numbers (*n*, *m*) that solve Brocard's problem are called **Brown numbers**. The only known solutions are $n = 4,5 \; and \; 7$ .There are only three known pairs of Brown numbers: $(4,5), (5,11), and \; (7,71)$.

1. In 1906, Gérardin claimed that, if $n > 71$, then $m$ must have at least 20 digits.
2. In 1935, Gupta stated that calculations of $n!$ up to $n = 63$ gave no future solutions.
3. In 1986, Wells claimed, that there are no other solutions with $n < 10^7$.
4. Paul Erdős conjectured that no other solutions exist.
5. In 1993, Overholt showed that there are only finitely many solutions provided that the $abc$ conjecture [3] is true.
6. In 1994, Guy claimed, that it is virtually certain that there are no more solutions

7. In 2000, Berndt and Galway [6] performed calculations for $n$ up to $10^9$ and found no further solutions.

Wilson has also computed the least $k$ such that $n! + k^2$ is square starting at $n = 4$, giving 1, 1, 3, 1, 9, 27, 15, 18, 288, 288, 420, 464, 1856... (Sloane's A038202)

**Variants of the Problem**

It is natural to consider the more general diophantine equation $n! + A = k^2$ $(1)$. An information about solutions and properties of general equation can be found in [4].

From this poin, We would like to present some new versions of equation for the Brocard`s Problem.

It seems, from generel point of view, that equations presented bellow, are also has right to be considered as an equation for Brocard`s problem

1. $(n+\alpha)! + A = k^2$. This equation is $\alpha$-Brocard`s Problem, where $\alpha > 0$ $or$ $\alpha < 0$ an integer, parameter. When $\alpha = 0$ we have Brocard`s problem. Find solutions for this problem.
2. $n! + A = (k+\beta)^2$. This equation is $\beta$-Brocard`s Problem, where $\beta > 0$ or $\beta < 0$ an integer, parameter. When $\beta = 0$ we have Brocard`s problem. Find solutions for this problem.
3. $(n+\alpha)! + A = (k+\beta)^2$. This equation can be considered as $\alpha, \beta$- Brocard`s Problem. $\alpha, \beta$ are integers. Find solutions for this problem.
4. Find solutions for $(n+\alpha)! = (k+\beta)^2$ equation, i.e. $A = 0$.
5. Find solutions for $(n! + A)! + (k+\beta) = (k+\beta)^{(n!+A)}$ equation. It is clear, that when we have $(n! + A) = 2$, our equation comes to $2! + (k+\beta) = (k+\beta)^2$, which in its turn can be considered as Brocard`s Problem, when $A = k$ in $(1)$.

## References


1. http://en.wikipedia.org/wiki/Brocard%27s_problem

2. http://mathworld.wolfram.com/BrocardsProblem.html

3. http://en.wikipedia.org/wiki/Abc_conjecture

4. Dabrowski, A. (1996), "On the Diophantine Equation $x! + A = y^2$", *Nieuw Arch. Wisk.* **14**: 321–324.

5. Berndt, Bruce C.; Galway, William F. (2000), "The Brocard–Ramanujan diophantine equation $n! + 1 = m^2$", *The Ramanujan Journal* **4**: 41–42,