

# One-Way Speed of Light Relative to a Moving Observer

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**Abstract.** The one-way speed of light relative to a moving observer is determined using the range measurement equation of the Global Positioning System. This equation has been rigorously tested and verified in the Earth-Centred Inertial frame where light signals propagate in straight lines at constant speed  $c$ . The result is a simple demonstration of light speed anisotropy that is consistent with light speed anisotropy detected in other experiments and inconsistent with the principle of light speed constancy. This light speed anisotropy was not observed before because there has been no direct one-way measurement of light speed relative to a moving observer.

**Keywords:** special theory of relativity, principle of light speed constancy, GPS, range measurement equation, one-way speed of light, moving observer, ECI frame.

## 1. Introduction

The GPS is a modern timing-ranging system with accurate synchronized atomic clocks that enable precise navigation on the Earth [1]. It has been hailed as a working example of the application of special relativity [1-3]. According to the IS-ICD-200 GPS Interface Specification, GPS signals propagate in straight lines at the constant speed  $c$  in an Earth-Centered Inertial (ECI) frame, a frame that moves with the Earth but does not share its rotation. Wolf and Petit [4] tested the isotropy of the speed of light in the ECI frame and published a limit of  $\delta c / c < 5 \times 10^{-9}$ . This light speed isotropy in the ECI frame is utilized in the GPS range measurement equation given by [1, 2]

$$\left| \overline{r}_r(t_r) - \overline{r}_s(t_s) \right| = c(t_r - t_s) \quad (1)$$

where  $t_s$  is the time of transmission of an electromagnetic signal from a source,  $t_r$  is the time of reception of the electromagnetic signal by a receiver,  $\overline{r}_s(t_s)$  is the position of the

source at the time of transmission of the signal and  $\bar{r}_r(t_r)$  is the position of the receiver at the time of reception of the signal. Using elapsed time measurements determined by the GPS clocks and the light speed value  $c$ , this equation allows accurate determination of position on the Earth. It has been extensively and rigorously tested and verified and the system's very successful operation has resulted in the world-wide proliferation of GPS technology.

Light speed isotropy in the ECI frame and the appearance of  $c$  in the range measurement equation are sometimes interpreted as demonstration of the principle of light speed constancy [2]. This principle from Einstein's special theory of relativity holds that light speed is constant in all inertial frames [5-7]. It should be noted that even though the principle is stated as applying in inertial frames, for reasons of practicality the vast majority of light speed experiments claiming confirmation of the principle and its several important applications such as clock synchronization and the SI unit of length involve the non-inertial frame of the rotating Earth [8-14]. Zhang [8] has pointed out that two-way light speed isotropy in this frame has been confirmed while one-way light speed isotropy has not been legitimately examined in the few experiments [15-17] in which such a test has been attempted.

In this regard, light speed anisotropy induced by the Earth's rotational motion was directly demonstrated for East-West light travel using the synchronized clocks of the GPS [18]. Light speed variation arising from the Earth's orbital motion was also detected in the Roemer experiment involving occultations of Jupiter's satellite Io observed from the moving Earth [19] and by Shtyrkov in the tracking of geostationary satellites [20]. In this paper we supplement these findings by demonstrating light speed anisotropy in the ECI frame for a relatively moving observer utilizing the range measurement equation of the GPS. Previously Wang [21] used the range measurement equation to determine elapsed time and inferred without direct calculation that the speed of light is dependent on the observer's uniform motion relative to the ECI frame. Here we use the range measurement equation to directly evaluate the one-way speed of light relative to an observer moving uniformly in the ECI frame, the light emanating from a source fixed in that frame.

## 2. Light Speed Relative to a Moving Observer in the ECI Frame

Consider a GPS station A fixed in the ECI frame and a GPS station B moving at a constant speed  $v$  relative to the ECI frame. On an axis fixed in the ECI frame along the line joining the two stations with station A closer to the origin O than station B and taking positive values, let  $x_A$  be the fixed position of station A and  $x_B(t)$  be the position of station B at time  $t$ . At time  $t_I$  let the distance between the two stations be  $L$  given by

$$x_B(t_I) - x_A = L \quad (2)$$

### 2.1 Observer moving away from fixed source

Let station B move directly away from fixed station A at a speed  $v$  relative to the ECI frame. At time  $t_I$  let station A transmit a signal directly to station B which receives it at time  $t_F$ . Then from the range measurement equation (1),

$$x_B(t_F) - x_A = c(t_F - t_I) \quad (3)$$

where  $x_B(t_F)$  is the position of station B at time  $t_F$ . Since station B is moving uniformly away from station A at speed  $v$  relative to the ECI frame, it follows that the relation between the position  $x_B(t_F)$  of station B at the time of reception of the signal and its position  $x_B(t_I)$  at the time of emission of the signal is given by

$$x_B(t_F) = x_B(t_I) + v(t_F - t_I) \quad (4)$$

Substituting for  $x_B(t_F)$  from (4) in (3) yields

$$x_B(t_I) - x_A + v(t_F - t_I) = c(t_F - t_I) \quad (5)$$

Using (2) this becomes

$$x_B(t_I) - x_A = L = (c - v)(t_F - t_I) \quad (6)$$

Hence for an observer on station B the range measurement equation gives the elapsed time as

$$(t_F - t_I) = \frac{L}{c - v} \quad (7)$$

Therefore the speed  $c_{AB}$  of the light relative to station B for the light traveling from the fixed station A to the moving station B is given by initial separation  $L$  divided by elapsed time  $(t_F - t_I)$  which using (7) is

$$c_{AB} = \frac{L}{(t_F - t_I)} = \frac{L}{L/(c - v)} = c - v \quad (8)$$

This result (8) is unaffected by “relativistic” changes in the direct measurement of  $L$  by the moving observer which are second-order.

## 2.2 Observer moving toward fixed source

Let station B move directly toward fixed station A at a speed  $v$  relative to the ECI frame. At time  $t_I$  let station A transmit a signal directly to station B which receives it at time  $t_F$ . Then using the range measurement equation (1) and noting that  $x_B(t_F) > x_A$ ,

$$x_B(t_F) - x_A = c(t_F - t_I) \quad (9)$$

Since station B is moving uniformly toward station A at speed  $v$  relative to the ECI frame, it follows that the relation between the position  $x_B(t_F)$  of station B at the time of reception of the signal and its position  $x_B(t_I)$  at the time of emission of the signal is given by

$$x_B(t_F) = x_B(t_I) - v(t_F - t_I) \quad (10)$$

Substituting for  $x_B(t_F)$  from (10) in (9) yields

$$x_B(t_I) - x_A - v(t_F - t_I) = c(t_F - t_I) \quad (11)$$

Using (2) this becomes

$$x_B(t_I) - x_A = L = (c + v)(t_F - t_I) \quad (12)$$

Hence for an observer on station B the range measurement equation yields the elapsed time as

$$(t_F - t_I) = \frac{L}{c + v} \quad (13)$$

Therefore the speed  $c_{AB}$  of the light relative to station B for the light traveling from the fixed station A to the moving station B is given by initial separation  $L$  divided by elapsed time  $(t_F - t_I)$  which using (13) is

$$c_{AB} = \frac{L}{(t_F - t_I)} = \frac{L}{L/(c + v)} = c + v \quad (14)$$

Again, this result (14) is unaffected by “relativistic” changes in the direct measurement of  $L$  by the moving observer which are second-order.

### 2.3 Discussion

The elapsed times determined in equations (7) and (13) indicate that the light transmitted from station A takes longer to arrive at station B when station B is moving away from A than towards it. Following from this equations (8) and (14) indicate that for an observer moving at a constant speed  $v$  relative to the ECI frame, the speed of light from a source fixed in the ECI frame relative to that moving observer is  $c - v$  for the observer moving away from the source and  $c + v$  for the observer moving towards the source. Similar light speed changes  $c \pm w$  for a moving observer were reported in the Roemer [19] and Shtyrkov [20] experiments involving the orbital movement of the Earth at speed  $w$  in its approximately linear motion around the Sun.

The light speeds  $c \pm v$  for a moving observer determined using the GPS range measurement equation in the ECI frame were not detected by the many light speed experiments [8-17] conducted in the terrestrial frame which all give light speed  $c$ . These changed light speed values  $c \pm v$  observed in an inertial frame directly contradict the principle of light speed constancy which requires constant light speed  $c$  that is independent of the motion of the observer.

Additionally the operational accuracy of the range measurement equation in the ECI frame and the detection of light speed anisotropy  $c \pm v$  in inertial frames moving relative to the ECI frame indicate that light travels at speed  $c$  in the ECI frame only and travels at different speeds in frames moving relative to the ECI frame. The ECI frame therefore behaves as a preferred frame for the propagation of light though considerations of clock bias [22] suggest that this may only be an illusion! In any event the observed light speed changes in this and previous experiments are confirmation of the existence of a preferred frame and it is ironic that the “incredible accuracy” of this “modern radio navigational system” [GPS] is cited by Rindler as an indication of the absence of a preferred frame [Ref.23p10].

Why was this light speed anisotropy, so easily revealed by GPS technology, not observed before considering the numerous light speed experiments that have been conducted over the past century? The simple answer is that there has previously been no direct one-way measurement of light speed with respect to a moving observer among the many that have been performed [8, 24]. The anisotropy result confirmed by this and

previous work [18-20] therefore represents an important development in space-time research that deserves serious attention by the scientific community.

### 3. Conclusion

In this paper, elementary analysis involving the rigorously verified range measurement equation of the GPS was used to determine the one-way speed of light in the ECI frame between a point fixed in the ECI frame and an observer moving at constant speed  $v$  relative to the ECI frame. The detected light speed anisotropy  $c \pm v$  is a clear demonstration of light speed variation in the ECI frame arising from the movement of the observer and is consistent with anisotropy results previously obtained in the non-inertial terrestrial frame [18-20]. It contradicts the principle of light speed constancy which is formulated in inertial frames and is today routinely applied in the frame of the surface of the Earth. This particular source of light speed anisotropy was not observed before because there has been no direct one-way measurement of light speed with respect to a moving observer.

This significant result obtained using accurate GPS technology further supports the finding [25] that the Absolute Space Theory of the Selleri (Inertial) Transformations is the best description of space and time, and that Special Relativity Theory based on the Lorentz Transformations along with all theories that are derived from members of the complete set of “equivalent” space-time transformations [26] are invalid representations of the physical world.

## References

1. Ashby, N., Relativity in the Global Positioning System, Living Reviews in Relativity, 6, 1, 2003.
2. Ashby, N., Relativity in the Future of Engineering, IEEE Transactions on Instrumentation and Measurement, 43, 505, 1994.
3. Will, C.M., Special Relativity: A Centenary Perspective, Progress in Mathematical Physics, 47, 33, 2006.
4. Wolf, P. and Petit, G., Satellite Test of Special Relativity Using the Global Positioning System, Physical Review A, 56, 4405, 1997.
5. French, A.P., Special Relativity, Nelson, London, 1968.
6. Rindler, W., Introduction to Special Relativity, Clarendon Press, Oxford, 1991.
7. Williams, W.S.C., Introducing Special Relativity, Taylor and Francis, London, 2002.
8. Zhang, Y.Z., Special Relativity and its Experimental Foundations, World Scientific, Singapore, 1997.
9. Muller, H., Herrmann, S., Braxmaier, C and Peters, A., Modern Michelson-Morley Experiment using Cryogenic Optical Resonators, Phys. Rev. Lett. 91, 020401, 2003.
10. Wolf, P. et al., Improved Test of Lorentz Invariance in Electrodynamics Physical Review D 70, 051902, 2004.
11. Hermann, S., A. Senger, E. Kovalchuk, H. Muller and A. Peters, Test of the Isotropy of the Speed of Light Using a Continuously Rotating Optical Resonator, Phys. Rev. Lett, 95, 150401, 2005.
12. Antonini, P., M. Okhapkin, E. Goklu and S. Schiller, Test of Constancy of Speed of Light With Rotating Cryogenic Optical Resonators, Phys. Rev. A 71, 050101, 2005.
13. Eisele, C, Nevsky, A. and Schiller, S., Laboratory Test of the Isotropy of Light Propagation at the  $10^{-17}$  Level, Physical Review Letters, 103, 090401, 2009.

14. Herrmann, S. et al., Rotating optical cavity experiment testing Lorentz invariance at the  $10^{-17}$  level, *Physical Review D* 80, 105011, 2009.
15. Gagnon, D.R., Torr, D.G., Kolen, P.T. and Chang, T., Guided-Wave Measurement of the one-way Speed of Light, *Physical Review A*, 38, 1767, 1988.
16. Krisher, T.P., Maleki, L., Lutes, G.F., Primas, L.E., Logan, R.T., Anderson, J.D. and Will, C.M., Test of the Isotropy of the One-way Speed of Light Using Hydrogen-Maser Frequency Standards, *Phys. Rev. D* 42, 731, 1990.
17. Riis, E., Lars-Ulrik, A.A., Bjerre, N and Poulsen, O., Test of the Speed of Light Using Fast-Beam Laser Spectroscopy, *Physical Review Letters*, 60, 81, 1988.
18. Gift, S.J.G., One-Way Light Speed Measurement Using the Synchronized Clocks of the Global Positioning System (GPS), *Physics Essays*, 23, 271, 2010.
19. Gift, S.J.G., Light Speed Invariance is a Remarkable Illusion, *Physics Essays*, 23, 1, 2010.
20. Shtyrkov, E.I., Observation of Ether Drift in Experiments with Geostationary Satellites, *Proceedings of the Natural Philosophy Alliance*, pp201-205, 12<sup>th</sup> Annual Conference, Storrs CT, 23-27 May 2005.
21. Wang, R., Successful GPS Operations Contradict the Two Principles of Special Relativity and Imply a New Way for Inertial Navigation-Measuring Speed Directly, *Proceedings of the IAN World Congress in Association with the U.S. ION Annual Meeting*, 26-28 June 2000, San Diego, CA.
22. Hatch, R.R., Those Scandalous Clocks, *GPS Solutions*, 8, 67, 2004.
23. Rindler, W., *Relativity Special, General and Cosmological*, 2<sup>nd</sup> edition, Oxford University Press, Oxford, 2006.
24. Gezari, D. Y., Experimental Basis for Special Relativity in the Photon Sector, arXiv: 0912.3818.
25. Gift, S.J.G., Separating Equivalent Space-Time Theories, *Apeiron*, 16, 408, 2009.
26. Selleri, F., Recovering the Lorentz Ether, *Apeiron*, 11, 246, 2004.