Symmetric equations that reproduce the fine structure constant and the muon-, neutron-, and proton-electron mass ratios

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Symmetric equations are introduced that reproduce the fine structure constant inverse and the muon-, neutron-, and proton-electron mass ratios near their experimental limits.

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The fine structure constant (FSC) and the muon-, neutron-, and proton-electron mass ratios can be accurately and economically reproduced as follows. Assume $M$ and $N$ are positive integers. Then define

\[ l_0 = \frac{1}{M^2}, \quad l_1 = \frac{[M - l_0/3M^2]^2}{N^2}, \quad q_0 = \frac{1}{M^3}, \quad q_1 = \frac{M^2 - q_0}{N^2}. \]

Similarly, define

\[ l_2 = \frac{M^3 - l_0}{N}, \quad q_2 = \frac{M^3 - q_0}{N}, \]
\[ l_3 = \frac{[M - l_0/3M^2]^3}{N}, \quad q_3 = \frac{[M - q_0/3M^2]^3}{N}, \]

which are symmetric under $l \leftrightarrow q$, so that for

\[ M = 10 \quad \text{and} \quad N = 3 \]

the FSC inverse can be approximated four ways

\[ \frac{l_1 + l_2}{N^2} = 137.036 \ 000 \ 001 \ 111, \quad \frac{q_1 + q_2}{N^2} = 137.036, \]
\[ \frac{l_1 + l_3}{N^2} = 137.036 \ 000 \ 002 \ 346, \quad \frac{q_1 + q_3}{N^2} = 137.036 \ 000 \ 000 \ 012, \]

which are also symmetric under $l \leftrightarrow q$. Also define

\[ L = \frac{4.13}{l_0 q_0}, \quad Q = \frac{6}{l_0 q_0}, \]
\[ L' = \frac{L}{1 - l_0}, \quad Q' = \frac{Q}{1 + l_0}, \]
\[ L'' = \frac{L'}{1 - l_0}, \quad Q'' = \frac{Q'}{1 - l_0}, \]

so that

\[ \frac{l_0 L - 1}{l_2 - l_0} = 206.768 \ 270 \ 731, \quad \frac{q_0 L + Q'}{q_2 - q_0} = 1838.683 \ 654 \ 735, \]
\[ \frac{l_0 L - 1}{l_3 - l_0} = 206.768 \ 270 \ 724, \quad \frac{q_0 L + Q'}{q_3 - q_0} = 1838.683 \ 654 \ 734, \]

which are nearly symmetric under $l \leftrightarrow q$; also note that

\[ l_0 q_0 N \left[ \frac{Q''}{l_0} - q_0 L'' \right] = 1836.152 \ 675 \ 237. \]

These reproduce, respectively, the muon-, neutron-, and proton-electron mass ratios and follow [1]. With the exception of the less precisely measured muon-electron mass ratio, which above is reproduced at its experimental limit, all of these values, including the FSC inverse, are within just a few parts per billion of their 2006 CODATA values [1, 2].
Analysis of the above definitions gives

\[(q_1 + q_2) - (l_1 + l_2) = \frac{(M - N^3/3 - 1) - N^3/9M^5}{NM^3}. \quad (1)\]

It follows that if

\[M = N^3/3 + 1\]

and

\[N^3/9M^5 \ll 1\]

then \(q_1 + q_2\) will closely approximate \(l_1 + l_2\), making two of the above FSC approximations nearly equal. Inspection reveals that the smallest positive integers fulfilling Eq. (2) are \(M = 10\) and \(N = 3\). These, as already shown, actually bring all four FSC approximations into numerical alignment. Moreover, this alignment takes place at a value that (purely coincidentally?) is nearly an exact match for the experimental FSC. The economy with which the definitions of \(l_0\), \(q_0\), etc. unambiguously single out the precisely known FSC (via Eq. (2)) provides good evidence for a non-coincidental—i.e., physical—origin for \(l_0\), \(q_0\), etc. This evidence is stronger still when one also takes into account the efficiency with which these same definitions help to reproduce three precisely known mass ratios.

Further evidence for non-coincidence is supplied:

- by [3], which shows how a brute-force computer search for approximations of the FSC automatically finds \((q_1 + q_2)/N^2\).
- by [4], which accurately models the observed quark and lepton mixing angles with the aid of \((l_1 + l_3)/N^2\) (the mixing model nexus), while requiring no help from free variables “adjusted to fit experiment.”
- by [5], which shows that the relation that \((q_1 + q_2)/N^2\) has with \((l_1 + l_3)/N^2\) derives from a general case involving the “broken symmetry” of simple algebraic identities.
- by [6], which exploits cuboctahedral symmetry, and 10, 3, and 4.1, to specify the quark and lepton masses, charges, and generations.

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