Geometry: Problems of dividing objects.

Thales’s Theorem and an idea which can arose when You applied the theorem, for solving an interesting and simple problem in Geometry.

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Abstract

Geometry it is not a word, moreover it is not just mathematical research area. It is art, it is the base of our Nature, it is language of Nature. The aim of this article is to present how Thales’s theorem is working for simple cases, when we need to divide a geometrical object into equal parts: mainly, we considered the problem of dividing a straight segment of length $N$ into $n$ equal parts. On the base of this simple case, we proposed a generalizations of the problem. We presented they as questions. Purpose of this article is to ask to find solutions for the questions. It seems, that for the positive answer, here must be developed geometrical techniques.

The problem and how the Thales`s Theorem is working for this problem?

There are a lot of interesting Geometrical problems as well as may be You had opportunity to meet some of them, even more, You tried to solve them. Even, there can be a situation, when the solving of such problems was enjoyable for You. For instance, some of such problems are problems which demand to divide a 2D geometrical object into equal parts, or divide it into such parts wich are similar to the object. Let for a simplicity, consider a problem, where we need to divide a 1D $AB$ straight segment of length $N$ into $n$ equal parts.

There are, at least, two situations. Let’s consider them!

**Situation 1 and Solution 1**

When, the length of the straight segment $N$ is divisible to $n$, there is not to need to be worried, because we need just use a ruler.
**Situation 2 and Solution 2**

When, the length of the straight segment $N$ is not divisible to $n$, for instance, when $N = 25$ and $n = 7$. What we can to do?

It seems, that the solution of this situation is not exists. The joy of us, this issue has a solution, which follows from Thales`s Theorem and at least, it is known to each amenian school children. Try Yourself to find more information about this theorem!

**Steps for solving the problem**

1. **Step 1.** Draw an arbitrary $AD$ straight segment from top $A$ of $AB$ segment
2. **Step 2.** Divide the new $AD$ segment in to arbitrary equal to each other $n$ parts
   
   $$AA_1 = A_1A_2 = \ldots = A_{n-1}D$$
3. **Step 3.** Draw a straight line and connect $D$ to $B$
4. **Step 4.** Draw straight lines paralel to $BD$ from $A_1, A_2, \ldots, A_{n-1}$ and intersect them with $AB$
5. **Step 5.** Check, that obtained segments on $AB$ are equal each to other.

Let before going forward, start to analysis of this problem from general point of view and explain the motivations for the new problem, which will be described in coming part of the work.

**Problems for future works**

From first look we see, that we have a straight segment and we use the second straight segment for dividing our segment in to arbitrary equal parts. From analytical geometry we know that the equation of straight segment passing between two points in Descartes`s coordination system is given by

$$y_i = k_ix + b_i \quad (i \in \{1,2\}, y_i = \{f(x), g(x))\).$$ (1)

What follows from the definition and from the solution of our problem?

*We use $g(x)$ for dividing $f(x)$ into equal parts, where $f(x)$ and $g(x)$ are described by (1).*

The last interpretation of the problem, from our point of view, allows us to pose the following questions as the natural generalizations of the initial problem.
Problem 1.
Suppose that \( f(x) \) already is not described by (1) i.e. it is not straight segment, but \( g(x) \) is a straight segment. Is it possible to divide \( f(x) \) into equal parts by using \( g(x) \)?
Develop geometrical techniques.

Problem 2.
Suppose that \( f(x) \) and \( g(x) \) are not at all straight segments described by (1). Is it possible to divide \( f(x) \) into equal parts by using \( g(x) \)?
Develop geometrical techniques.

Generally, we can consider functions of \( F(x, y, z, \ldots) \) form. In respect to this situation we propose the following problem.

Problem 3
Develop a geometrical techniques which will allow us to do the following

1. Use \( F(x, y, z, \ldots) \) to divide \( G(x, y, z, \ldots) \) into equal parts

2. Use \( F(x, y, z, \ldots) \) curve from \( n - 1 \) dimensional space to divide \( G(x, y, z, \ldots) \) from \( n \) dimensional space into equal parts

We would like to thank to all, who will read this work and will try to solve the problems