The Fictitious 'Interior' of Schwarzschild Spacetime

(and a couple of other fallacies)

by

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All arguments for the black hole are based upon the same fundamental idea in that they conceive of a region that in actual fact does not exist. This fictitious region the relativists call the 'interior', i.e. the region they think is contained by their a spherically symmetric surface they call the 'event horizon'. But there is no such region. The notion comes from a failure to recognise that the centre of spherical symmetry of the problem at hand is not located where they think it is, at their $r = 0$. I shall discuss this now in more detail.

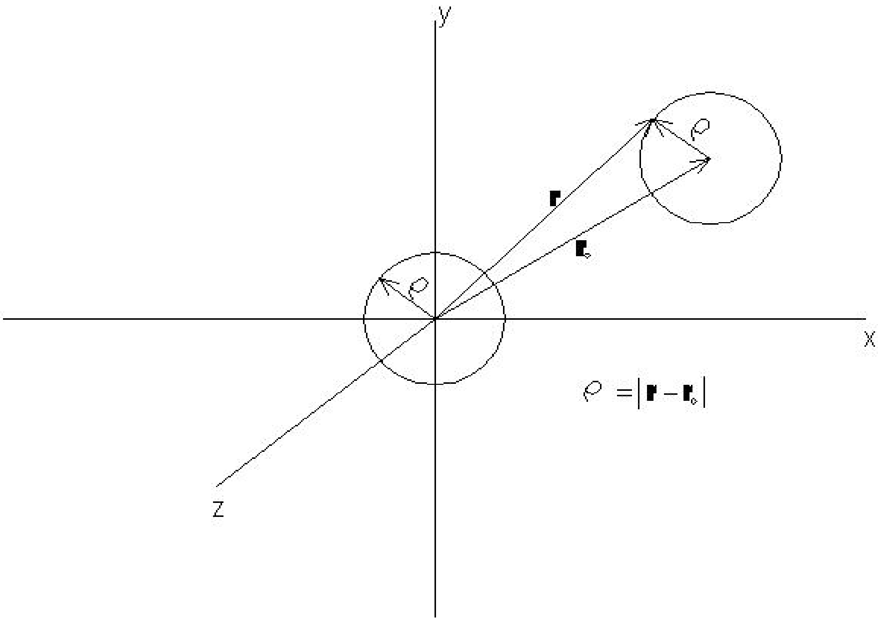

Figure 1
Translation of a sphere.

Consider the sphere in figure 1, radius ρ, centred at the origin of the system of coordinates. The intrinsic geometry of this sphere is independent of position in the space described by the system of coordinates. Shift the sphere to some arbitrary point in the space, away from the origin of the coordinate system as depicted. The centre of the shifted sphere is now located at the extremity of the fixed vector $\mathbf{r}_o$, relative to the origin of coordinates. The surface of the shifted sphere is the locus of points at the extremity of the variable vector $\mathbf{r}$, and the radius of the sphere, which is unaltered by this translation, is $\rho = | \mathbf{r} - \mathbf{r}_o |$. The centre of the translated sphere is no longer at the origin of coordinates. Consider a point on the surface of the shifted sphere. It is at a distance ρ from the centre of the sphere. Let this point approach the centre of the sphere along any radius of the sphere. This can be described by the scalar $\rho \to 0$. The direction of approach is immaterial, since only radial distance is considered. Relative to the centre of the shifted sphere, the metric describing the sphere can be written:

$$\mathrm{ds}^2 = d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{1}$$
$$0 \le \rho < \infty$$

And relative to the origin of coordinates depicted, can be written:

$$\mathrm{ds}^2 = (d\,|\,\mathbf{r} - \mathbf{r}_o\,|\,)^2 + |\,\mathbf{r} - \mathbf{r}_o\,|^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{2}$$
$$0 \le |\,\mathbf{r} - \mathbf{r}_o\,| < \infty$$

Now the centre of the shifted sphere is shifted with the sphere. It would be quite absurd to suggest that although the sphere has been shifted away from the origin of the coordinate system, the centre of the sphere is still located at the origin of the coordinate system. Now if one shifted the sphere away from the origin of the coordinate system, without realising it, and so treated the origin of the coordinate system as still the centre of the shifted sphere, then the resulting analysis would quickly lead to erroneous conclusions. Well, the black hole is precisely a result of this very misconception.

Consider again the situation in figure 1, except that the vectors $\mathbf{r}$ and $\mathbf{r}_o$ are always collinear, as shown in figure 2.

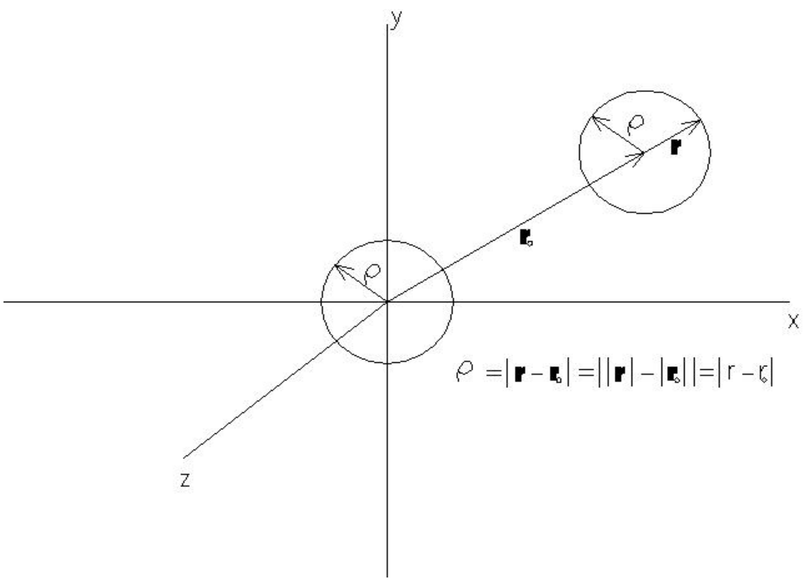

Figure 2
$\mathbf{r}$ and $\mathbf{r}_o$ collinear with the origin of the coordinate system.

Relative to the centre of the shifted sphere the metric describing the sphere is unaltered, thus:

$$\mathrm{ds}^2 = d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
$$0 \le \rho < \infty$$

Relative to the origin of the coordinate system, owing to the imposed collinearity, the vector notation can be dropped, to give the expression

$$ds^2 = (d\,|\,r - r_o\,|\,)^2 + |\,r - r_o\,|^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$= dr^2 + |\,r - r_o\,|^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \qquad (3)$$

$$0 \le |\,r - r_o\,| < \infty$$

wherein the point on the surface of the shifted sphere approaches the centre of the shifted sphere along the collinear vector line, so that $|\,\mathbf{r} - \mathbf{r}_o\,| = |\,r - r_o\,|$, as shown in figure 2. The absolute value signs are necessary because r may be greater than or less than r_o.

Consider now figure 3.

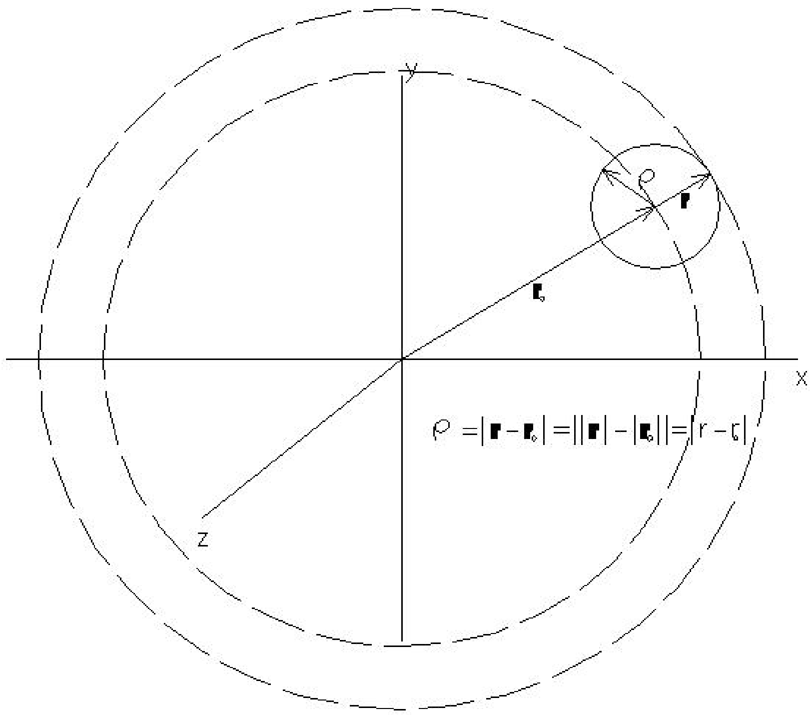

Figure 3
Meaning of r_o - it is the point at the centre of spherical symmetry in the spatial section of Minkowski space.

If one now studies the intrinsic geometry of the shifted sphere without realising that it is no longer at the origin of the coordinate system, how would one interpret $r_o = |\,\mathbf{r}_o\,|$? In other words, as a point on the surface of the shifted sphere approaches the centre of the sphere along the collinear radial line, so that $r \to r_o$, how would this be interpreted when ignorant of the fact that the sphere has been shifted away from the origin of the coordinate system ? In the case of the black hole, it has been misinterpreted as the said point approaching the surface of a sphere of radius r_o (broken line in figure 3), and so the spherical space contained by that radius is misinterpreted as the interior of the black hole, with the event horizon at radius r_o. Under this misconception the relativists construct their Kruskal-Szekeres coordinates, and their Eddington-Finkelstein coordinates, thinking, erroneously, that there is an 'interior' region. The conception is utter nonsense.

Consider now the usual metric for Minkowski spacetime,

$$\mathrm{ds}^2 = \mathrm{dt}^2 - \mathrm{dr}^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{4}$$
$$0 \le r < \infty$$

Minkowski spacetime actually acts as a parametric space for the generalisation to Einstein's gravitational field. There is a mapping of distance between two points in the spatial section of Minkowski space into the components of the metric tensor for Einstein's gravitational field.

Now the situation depicted in figure 3 is what is introduced unwittingly by the relativists when they write the 'generalised' metric,

$$\mathrm{ds}^2 = A(r)\mathrm{dt}^2 - B(r)\mathrm{dr}^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{5}$$

In other words, what they have unwittingly done by writing this metric is shift the parametric sphere in the spatial section of Minkowski spacetime away from the origin of coordinates in Minkowski spacetime (as described by eq. (3) above), and think that the centre of the shifted parametric sphere is still located at $r = 0$ in Minkowski spacetime. This is compounded by their misinterpretation of r in their general metric as the radius in the spatial section of their general metric, simply because it is the radius in the spatial section of the usual Minkowski metric from which they start, and with that they think that $0 \le r < \infty$ in their general metric, which they determine finally as their so-called 'Schwarzschild' solution,

$$\mathrm{ds}^2 = \left(1 - \frac{2m}{r}\right)\mathrm{dt}^2 - \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{dr}^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{6}$$
$$0 \le r < \infty$$

where they claim m is the mass of the source of the alleged associated gravitational field. The relativists do not even realise that Minkowski spacetime is a parametric space for the generalised metric leading up to their so-called 'Schwarzschild' solution, and so they don't realise that by driving their r down to zero in their 'Schwarzschild' solution they are driving the centre of the shifted parametric sphere in the spatial section of Minkowski spacetime, as depicted in figure 3, down to the origin of the parametric coordinate system, when in fact the centre of the parametric sphere is located not at $r = 0$ but at $r = r_o = 2m > 0$.

The relativists delude themselves, following Hilbert's original lead, by thoughtlessly applying implicit transformations of coordinates in the determination of their generalised metric, eq. (5) above. To see this let's carry out the overall procedure.

The relativists start with eq. (4) above and write the generalisation,

$$\mathrm{ds}^2 = A(r)\mathrm{dt}^2 - B(r)\mathrm{dr}^2 - C(r)r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{7}$$
$$A,\, B,\, C > 0$$
$$0 \le r < \infty$$

$A,B,C > 0$ by construction, in order to preserve the signature of Minkowski spacetime at (+,-,-,-). Then they write $(r^*)^2 = C(r)r^2$ to get,

$$\mathrm{ds}^2 = A^*(r^*)\mathrm{dt}^2 - B^*(r^*)\mathrm{dr}^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{8}$$
$$A^*,\ B^* > 0$$
$$0 \le r^* < \infty$$

Then they simply drop the * to get,

$$\mathrm{ds}^2 = A(r)\mathrm{dt}^2 - B(r)\mathrm{dr}^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{9}$$
$$A,\ B > 0$$
$$0 \le r < \infty$$

Now in going from (7) to (9), since $C(r)$ is an *a priori* unknown function, and since $r^*=C(r)\,r^2$, it is impossible to know that $r^*(r=0) = 0$, but that is precisely what the relativists do. It is this step that subtly introduces the shift of the centre of the parametric sphere in the spatial section of the parametric Minkowski spacetime, because the r appearing in (9) is not the same r appearing in (6) and (7). Then, in ignorance of all this, the relativists think that their 'Schwarzschild' solution, eq. (6) above, contains an 'interior' region that constitutes their black hole. And so they produce the nonsense of a shifted sphere with 'centre' at $r = 0$, as depicted in figure 3, thinking that their $r = r_o = 2m$ is the 'radius' of a sphere the surface of which they call an event horizon of a black hole, with infinitely dense point-mass singularity (i.e. centre) at $r = 0$. Moreover, their r is not even a distance let alone a radial one in the 'Schwarzschild' solution. So they are blind to both radii in their solution and the parametric nature of Minkowski spacetime, mixing up both, with a concomitant *ad hoc* introduction of Newton's escape velocity, into a nonsensical amalgum that defies belief.

There is no interior region associated with Schwarzschild spacetime, as Schwarzschild's actual solution testifies, as also does Droste's solution and the solution by Brillouin, and the situation depicted in figure 3. My generalisation of their solutions, in accordance with the intrinsic geometry of the line-element, amplies this fact, viz,

$$\mathrm{ds}^2 = \left(1 - \frac{\alpha}{R_c}\right)\mathrm{dt}^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1}\mathrm{dR}_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{10}$$
$$R_c = R_c(r) = \left(\,|\,r - r_o\,|^{\,n} + \alpha^n\right)^{1/n}$$
$$r \in \Re, \quad n \in \Re^+, \quad r \ne r_o$$

wherein α is an indeterminable constant and both r_o and n are entirely arbitrary constants.

Now consider general situation depicted in figure 4.

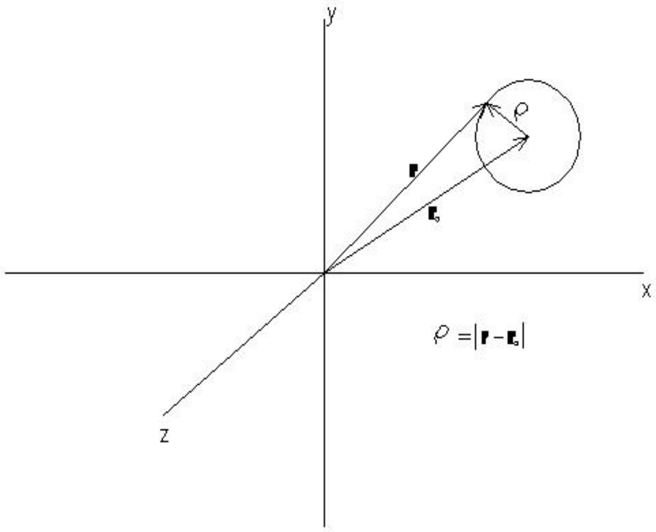

Figure 4: Spatial section of Minkowski spacetime
(ρ is mapped under Ric = 0 into the $g_{\mu\nu}$ of Schwarzschild spacetime)

Let the variable point approach the fixed point directly, i.e. along a radial path. Then the distance ρ between any two points, one of them fixed, in the spatial section of Minkowski spacetime, is mapped into the components of the metric tensor, $g_{\mu\nu}(\rho)$, of Schwarzschild spacetime, by a function, $R_c(\rho)$, subject to the 'field' equations Ric = 0:

$$\text{(Minkowski spacetime)} \qquad \rho \qquad \overset{\text{Ric=0}}{\overrightarrow{R_c(\rho)}} \qquad g_{\mu\nu}(\rho) \qquad \text{(Schwarzschild spacetime)}$$

Consequently, as the radial distance ρ in Minkowski spacetime approaches zero, the proper radius R_p of Schwarzschild spacetime approaches zero (and vice-versa):

$$\rho \to 0 \Longleftrightarrow R_p \to 0 \tag{11}$$

Clearly, it does not matter where the two points are located in the spatial section of Minkowski spacetime (which acts as a parametric space). If the two points are distinct from and collinear with the origin of coordinates of the spatial section of Minkowski spacetime, then relations (1) and (3) above, as depicted in figure 3, applies, and so expression (11) becomes,

$$|\, r - r_o \,| \to 0 \Longleftrightarrow R_p \to 0 \tag{12}$$

If r_o = 0, and if it is taken (as is usual) that $r \geq 0$, then expression (12) reduces to

$$r \to 0 \Longleftrightarrow R_p \to 0 \tag{13}$$

But in all cases, $R_c(\rho)$, the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of Schwarzschild spacetime, is never zero. The reason is that range on $R_c(\rho)$ is fixed by the form of the line-element and, thereby, the relations (11) through (13). Curvatures do not determine a geometry; they are consequents of a geometry; a geometry being determined in full by the associated line-element. Given a line-element (i.e. a metric), one can calculate things, such as curvatures, that are intrinsic to the geometry. Thus, in relation to $R_c(\rho)$,

$$\rho \to 0 \Longleftrightarrow R_c \to \alpha \tag{11b}$$

$$|\, r - r_o \,| \to 0 \Longleftrightarrow R_c \to \alpha \tag{12b}$$

$$r \to 0 \Longleftrightarrow R_c \to \alpha \tag{13b}$$

where α is a positive constant. Consequently, all attempts to account for an 'interior' region of the so-called 'Schwarzschild' solution, are quite meaningless, because the region does not exist - it is just a delusion, as figure 3 depicts and metric (10) demonstrates.

In summary, the situation is this, as depicted in figure 5.

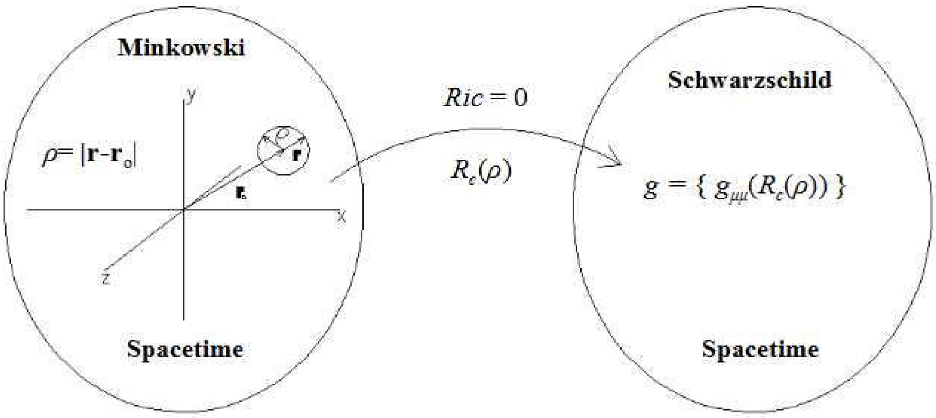

Figure 5: Mapping between Minkowski spacetime and Schwarzschild spacetime
(ρ is mapped by $R_c(\rho)$ under Ric = 0 into the $g_{\mu\nu}$ of Schwarzschild spacetime)

The distance ρ is mapped by $R_c(\rho)$ into the components of the metric tensor $g_{\mu\nu}$ of Schwarzschild spacetime.

In addition to the geometric errors committed by the relativists, contrary to the allegations of teh relativists, there is actually no mass present in their expression (6) above. First, expression (6) is a solution for Ric = 0, which is a spacetime that by construction contains no matter, and so no sources of a gravitational field. Second, the introduction of mass to get expression (6) above is by means of a *post hoc* introduction of the Newtonian escape velocity expression, which is a relation that involves two bodies - one body escapes from another. In other words, the relativists arbitrarily introduce a two-body relation from Newton's theory into what they allege is a one-body configuration in General Relativity. Then they proclaim that they got themsleves a Newtonian approximation ! Not surprising, since they put in Newton's escape velocity deliberately in order to make it so. It is nonsense - it is impossible for a two-body relation to appear in an alleged one-body configuration of matter. The introduction of Newton's escape velocity is disguised by the relativists owing to their setting both c and G to unity. To see this, write metric (6) emphatically with c and G, thus

$$\mathrm{ds}^2 = \left(c^2 - \frac{2\mathrm{Gm}}{r}\right)\mathrm{dt}^2 - c^2\left(c^2 - \frac{2\mathrm{Gm}}{r}\right)^{-1}\mathrm{dr}^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{6b}$$

from which it is immediately apparent that Newton's escape velocity expression appears, viz.

$$v_e = \sqrt{\frac{2\mathrm{Gm}}{r}} \tag{14}$$

Then when $v_e = c$, the alleged black hole 'escape velocity', the corresponding radius (which is a Newtonian radius) is $r_s = 2Gm/c^2$, which the relativists call the 'Schwarzschild radius', the 'radius' of their alleged event horizon of their black hole (see figure 3 again). But r is not even a distance in Schwarzschild spacetime, and so, certainly not the radius of anything therein. And there is no 'escape velocity' for a black hole since it is allegedly a lone body in an otherwise empty universe, and so there is nothing present that can escape from it or with which it can interact in any way. *Ric* = 0 is not a two-body problem !

The alleged black hole with its 'interior' is nonsense on a deeper level. Einstein and his followers maintain that his Principle of Equivalence and his 'laws' of Special Relativity must manifest in sufficiently small regions of his gravitational field. Such regions can be located anywhere in Einstein's gravitational field. Now both the Principle of Equivalence and the 'laws' of Special Relativity are defined in terms of the *a priori* presence of multiple arbitrarily large finite masses, and so they cannot possibly manifest in a spacetime that by construction contains no matter. *Ric* = 0 is a spacetime that by construction contains no matter, and so it necessarily precludes the Principle of Equivalence and the 'laws' of Special Relativity, and so *Ric* = 0 violates the physical principles of General Relativity. Consequently it is inadmissible. Black holes were conjured up from *Ric* = 0. Finally, since *Ric* = 0 is inadmissible, Einstein's field equations are of such a form that the total gravitational energy is always zero, putting them into direct conflict with the usual conservation of energy and momentum and thus experiment. If the usual conservation of energy and momentum, so well determined by experiment, is valid, then Einstein's theory is invalid. I conclude that General Relativity is invalid. That takes black holes, big bangs, Einstein gravitational waves, etc., with it into the dustbin of scientific history, all at once.

Another error the relativists commit is this: although they obtain their 'Schwarzschild' solution from a generalised static metric of fixed signature (+,-,-,-), as noted in relation to expression (7), they nonetheless permit their 'solution', expression (6), to change signature to (-,+,-,-), in direct violation of their initial construction. There is no Minkowski spacetime with signature (-,+,-,-), and so their 'solution' with signature (-,+,-,-) is not a generalisation of Minkowski spacetime, since Minkiwski spacetime cannot change signature to (-,+,-,-). So it is really no wonder that the change of signature to (-,+,-,-) results in a non-static solution to a static problem !

Thus, the Kruskal-Szekeres coordinates and the 'interior' region they allegedly extend to, do not deal with any region that exists in Schwarzschild spacetime. They are based upon misconceptions as to radii, mappings, parameters, Newtonian relations, and the magical presence of mass in a spacetime that by construction contains no matter. A complete manifold cannot be extended. Schwarzschild spacetime cannot be 'extended', because it is complete.

Then there is the major fact that "Schwarzschild's solution" is not Schwarzschild's solution at all. Schwarzschild's actual solution does not permit black holes! But the relativists never let the facts get in the way of a good story.