THE PARALLEL POSTULATE IS DEPENDED ON THE OTHER AXIOMS.

This article was sent to some specialists in Euclidean Geometry for criticism. The geometrical solution of this problem is based on the four Postulates for Constructions in Euclid geometry.

EUCLID ELEMENTS FOR A PROOF OF THE PARALLEL POSTULATE (AXIOM)

The first Definitions (D) of Terms in Geometry:

1. D1. A point is that which has no part. (Position)
2. D2. A line is a breathless length. (For straight line, The Whole is equal to the Parts)
3. D3. The extremities of lines are points. (Equation).
4. D4. A straight line lies equally with respect to the points on itself. (Identity)
5. D. A midpoint C divides a Segment AB (of a straight line) in two. CA = CB
   Any point C divides all straight lines through this in two.
6. D. A straight line AB divides all Planes through this in two.
7. D. A Plane ABC divides all Spaces through this in two.

Common Notions.

CN1. Things which equal the same thing also equal one another.
CN2. If equals are added to equals, then the wholes are equal.
CN3. If equals are subtracted from equals, then the remainders are equal.
CN4. Things which coincide with one another equal one another.
CN5. The whole is greater than the part.

The five Postulates (P) for construction are:

1. P1. To draw a straight line from any point A to any other B.
2. P2. To produce a finite straight line AB continuously in a straight line.
3. P3. To describe a circle with any centre and distance. \( P_1, P_2 \) are Unique.
4. P4. That, all Right angles are equal to each other.
5. P5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a Plane)
5a. The same is Player’s Postulate which states that, From any point M, not on a straight line AB, only one line MM’ can be drawn Parallel to AB.

Since a straight line passes through two points only and because point M is the third then The Parallel Postulate it is valid on a Plane (three points only).
**AB** is a straight line through points A, B. M is a point not on AB since MA + MB > AB.

To prove that, one and only one line MM’ can be drawn Parallel to AB.

To prove the above Axiom is necessary to show:

a. The parallel to AB is the locus of all points at a constant distance h from the line AB, and for point M is MA1,

b. The locus of all these points is a straight line.

**METHOD**

**STEP 1.**

Draw the circle (M, MA) be joined meeting line AB in C.

Since MA = MC, point M is on mid-perpendicular of AC. Let A1 be the midpoint of AC, (it is A1A + A1C = AC because A1 is on the straight line AC. Triangles MAA1, MCA1 are equal because the three sides are equal, therefore angle < MA1A = MA1C (CN1) and since the sum of the two angles < MA1A + MA1C = 180° (CN2, 6D) then angle < MA1A = MA1C = 90° (P4) so, MA1 is the minimum fixed distance h of point M to AC.

**STEP 2.**

Let B1 be the midpoint of CB (it is B1C + B1B = CB because B1 is on the straight line CB) and draw B1M’= h equal to A1M on the mid-perpendicular from point B1 to CB.

Draw the circle (M’, M’B = M’C) intersecting the circle (M, MA = MC) at point D. (P3)

Since M’C = M’B, point M’ lies on mid-perpendicular of CB. (CN1)
Since M’C = M’D, point M’ lies on mid-perpendicular of CD. (CN1)
Since MC = MD, point M lies on mid-perpendicular of CD. (CN1)

Because points M and M’ lie on the same mid-perpendicular (This mid-perpendicular is drawn from point C’ to CD and it is the midpoint of CD), and because only one line MM’ passes through points M and M’, then line MM’ coincides with this mid-perpendicular (CN4)

**STEP 3.**

Draw the perpendicular of CD at point C’. (P3, P1)
Because \( MA_1 \perp AC \) and also \( MC' \perp CD \) then angle \( < A_1MC' = A_1CC' \). (CN2, CN3, E.I.15)

Because \( MB_1 \perp CB \) and also \( MC' \perp CD \) then angle \( < B_1MC' = B_1CC' \). (CN2, CN3, E.I.15)

b The sum of angles \( A_1CC' + B_1CC' = 180^\circ = A_1MC' + B_1M'C' \). (6.D), and since Point \( C' \) lies on straight line \( MM' \), therefore the sum of angles in shape \( A_1B_1M'M' \) are \(< MA_1B_1 + A_1B_1M' + [B_1M'M + M'MA_1] = 90^\circ + 90^\circ + 180^\circ = 360^\circ \). i.e. The sum of angles in a Quadrilateral is \( 360^\circ \) and in Rectangle all equal to \( 90^\circ \). (m)

c The right-angled triangles \( MA_1B_1, M'B_1A_1 \) are equal because \( A_1M = B_1M' \) and \( A_1B_1 \) common, therefore side \( A_1M' = B_1M \). Triangles \( A_1MM', B_1M'M \) are equal because have the three sides equal each other, therefore angle \( < A_1MM' = B_1M'M \), and since their sum is \( 180^\circ \) as before (6D), so angle \( < A_1MM' = B_1M'M = 90^\circ \). (CN2).

d Since angle \( < A_1MM' = A_1CC' \) and also angle \( < B_1M'M = B_1CC' \) (P4), therefore quadrilaterals \( A_1CC'M', B_1CC'M', A_1B_1M'M' \) are Rectangles (CN3). From the above three rectangles and because all points \( (M, M' \text{ and } C') \) equidistant from \( AB \), this means that \( C'C \) is also the minimum equal distance of point \( C' \) to line \( AB \) or, \( h = MA_1 = M'B_1 = CD / 2 = C'C \) (CN1)

Namely, line \( MM' \) is perpendicular to segment \( CD \) at point \( C' \) and this line coincides with the mid-perpendicular of \( CD \) at this point \( C' \) and points \( M, M', C' \) are on line \( MM' \). Point \( C' \) equidistant, \( h \), from line \( AB \), as it is for points \( M, M' \), so the locus of the three points is the straight line \( MM' \), so the two demands are satisfied, \( (h = C'C = MA_1 = M'B_1 \text{ and also } C'C \perp AB, MA_1 \perp AB, M'B_1 \perp AB) \). (o.e.δ.)

e The right-angle triangles \( A_1CM, MCC' \) are equal because side \( MA_1 = C'C \) and \( MC \) common so angle \( < A_1CM = C'MC \), and The Sum of angles \( C'MC + MCB_1 = A_1CM + MCB_1 = 180^\circ \)

All above is a Proof of the Parallel Postulate due to the fact that the parallel postulate is dependent of the other four axioms (now is proved as a theorem from the other four). Since \( AB \) is common to \( \infty \) Planes and only one Plane is passing through point \( M \) (Plane \( ABM \) from the three points \( A, B, M \), then the Parallel Postulate is valid for all Spaces which have this common Plane, as Spherical, \( n \)-dimensional Spaces geometry. It was proved that it is a necessary logical consequence of the other axioms, agree also with the Properties of physical objects, \( d + 0 = d, d * 0 = 0 \), now is possible to decide through mathematical reasoning, that the geometry of the physical universe is Euclidean. Since the essential difference between Euclidean geometry and the two non-Euclidean geometries Spherical and hyperbolic geometry, is the nature of parallel line, the parallel postulate so,

The consistent System of the < non-Euclidean geometry > have to decide the direction of the existing mathematical logic.

The above consistency proof is applicable to any line Segment \( AB \) on line \( AB \), (segment \( AB \) is the first dimensional unit, as \( AB = 0 \to \infty \)) and \( M \) any Point not on line \( AB \), (\( MA + MB > AB \) on three points only which consist the Plane), and answers to the cry about the < crisis in the foundations of Euclid geometry >.

TYPES OF GEOMETRY

<table>
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<th>Hyperbolic</th>
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<td>[ h \sim h' ]</td>
<td>[ h \perp AB ]</td>
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1 Any single point \( A \) constitutes a Unit without any Position (non-dimentional= Empty Space). Simultaneously zero, finite and infinite. The unit meter of Point is equal to 0.

2 Any single point \( B \), not coinciding with \( A \), constitutes another one Unit which has also dimension zero. Only one straight line (i.e. the Whole is equal to the Parts) passes through points \( A \) and \( B \), which consists another un-dimensional Unit, since is consisted of infinite points with dimension zero. A line Segment \( AB \) between points \( A \) and \( B \) (either points \( A \) and \( B \) are near zero or are extended to the infinite), consists the first Unit with one dimensional, the length \( AB \), beginning from Unit \( A \) and a regression ending in Unit \( B \).

\[ AB = 0 \rightarrow \infty, \text{ is the one-dimensional Space. The unit meter of } AB \text{ is } m = 2. (AB/2) = AB \]

3 Adding a third point \( C \), outside the straight line \( AB \), then is constituted a new Unit (the Plane) without position, since is consisted of infinite points, without any position. Shape \( ABC \) enclosed between parts \( AB \), \( AC \), \( BC \) is of two dimensional, the enclosed area \( ABC \). Following harmony of unit meter \( AB=AC=BC \), then Area \( ABC = 0 \rightarrow \infty \), is the two-dimensional Space with unit meter equal to \( m = 2.\pi.\frac{AB}{\sqrt{2}} = \pi.\frac{AB^2}{2}, \text{ i.e. one square equal to the area of the unit circle.} \)

4 Four points \( A,B,C,D \ldots \) not coinciding, consist a new Unit (the Space or Space Layer) without position also, which is extended between the four Planes and all included, forming Volume \( ABCD \). Following the same harmony of the first Unit, shape \( ABCD \) is the Regular Tetrahedron with volume \( ABCD = 0 \rightarrow \infty \), and it is the three-dimensional Space. The dimension of Volume is \( 4 - 1 = 3 \). The unit measure of volume is the side \( X \) of cube \( X^3 \) twice the volume of another random cube of side \( a=AB \) such as \( X^3 = 2.a^3 \) and \( X = \sqrt[3]{2}a \). Geometry measures Volumes with side \( X \) related to the problem of doubling of the cube.

In case that point \( D \) is on a lower Space Layer, then all Properties of Space, or Space Layer are transferred to the corresponding Unit, i.e. to Plane or to the Straight line or to the Point. This Concentrated (Compact) Logic of geometry [CLG] exists for all Space-Layers and is very useful in many geometrical and physical problems. (exists, Quality = Quantity, since all the new Units are produced from the same, the first one, dimensional Unit \( AB \)).

5 \( N \) points represent the \( N \)-dimensional Space or the \( N \)-Space Layer, \( DL \), and has analogous properties and measures. Following the same harmony for unit \( AB \), \( AB = 0 \rightarrow \infty \) then shape \( ABC..M \) (i.e. the \( \infty \) spaces \( AB = 1, 2, \ldots, \text{nth} \)) is the Regular Solid in Sphere \( ABC..M = 0 \rightarrow \infty \). This \( N \) Space Layer is limiting to \( \infty \) as \( N \rightarrow \infty \).

Proceeding inversely with roots of any unit \( AB = 0 \rightarrow \infty \) (i.e. the Sub-Spaces are the roots of \( AB, \sqrt{AB}, \sqrt[3]{AB}, \ldots, \sqrt[n]{AB} \) then it is \( \sqrt[n]{AB} = 1 \) as \( n \rightarrow \infty \)), and since all roots of unit \( AB \) are the vertices of the Regular Solids in Spheres then this \( n \) Space Layer is limiting to \( 0 \) as \( n \rightarrow \infty \).

The dimensionality of the physical universe is unbounded (\( \infty \)) but simultaneously equal to \( (0) \) as the two types of Spaces and Sub-Spaces show.

Because the unit-meters of the \( N \)-dimensional Space Layers coincide with the vertices of the \( n \)-th roots of the first dimensional unit segment \( AB \) as \( AB = \infty \rightarrow 0 \), (the vertices of the \( n \)-sided Regular Solids), therefore the two Spaces are coinciding (the Space Layers and the Sub-Space Layers are in superposition).

That is to say, Any point on the \( N \)-th Space or Space-Layer, of any unit \( AB = 0 \rightarrow \infty \), jointly exists partly or whole, with all Subspaces of higher than \( N \) Spaces, \( N = (N+1) - 1 = (N+2) - 2 = (N+N) - N \ldots = (N+\infty) - \infty \), where \( (N+1) \ldots (N+\infty) \) are the higher than \( N \) Spaces, and with all Spaces of lower than \( N \) Subspaces, \( N = (N-1) + 1 = (N-2) + 2 = (N-N) + N = (N-\infty) + \infty \), where \( (N-1), (N-2), (N-N), (N-\infty) \) are the lower than \( N \) Spaces.
The boundaries of \( N \) points, corresponding to the Space, have their unit meter of the Space and is a Tensor of \( N \) dimension (i.e. the unit meters of the \( N \) roots of unity \( AB \)), simultaneously, because belonging to the Sub-Space of the Unit Segments \( > N \), have also the unit meter of all spaces.

1. The Space Layers (Regular Solids) with sides equal to \( AB = 0 \rightarrow \infty \)
The Increasing Plane Spaces with the same Unit.

![Diagram of N-dimensional plane spaces](image)

2. The Sub-Space Layers (Regular Solids on \( AB \)) as Roots of \( AB = 0 \rightarrow \infty \)
The Decreasing Plane Spaces with the same Unit.

![Diagram of n-dimensional plane sub-spaces](image)

3. The superposition of Plane Space Layers and Sub-Space Layers:
The simultaneously co-existence of Spaces and Sub-Spaces of any Unit \( AB = 0 \rightarrow \infty \), i.e. Euclidean, Elliptic, Spherical, Parabolic, Hyperbolic, Geodesics, metric and non-metric geometries. The Interconnection of Homogeneous and Heterogeneous Spaces, and Subspaces of the Universe.
A linear shape is the shape of N points on a Plane bounded with straight lines. A circle is the shape on a Plane with all points equally distance from a fix point O. A curved line is the shape on a Plane with points not equally distance from a fix point O. Curved shapes are those on a Plane bounded with curved lines.

Rotating the above axial-centrifugally (machine \( \text{AB} \perp \text{AC} \)) is obtained Flat Space, Conics, Sphere, Curved Space, multi Curvature Spaces, Curved Hyperspace etc. The fact that curvature changes from point to point, is not a property of one Space only but that of the common area of more than two Spaces, namely the result of the Position of Points. Euclidean manifold (Point, sectors, lines, Planes, all Spaces etc) and the one dimentional Unit \( \text{AB} \) is the same thing (according to Euclid \( \varepsilon \nu \) to \( \pi \nu \)).

Since Riemannian metric and curvature is on the great circles of a Sphere which consist a Plane, say \( \text{AMA'} \), where the Parallel Postulate is consistent with three points only, therefore the great circles are not lines (this is because it is \( \text{MA} + \text{MA'} > \text{AA'} \)) and the curvature of Space is that of the circle in this Plane, i.e. that of the circle (O, OA).

Because Parallel Axiom is for three points only, which consist a Plane, then the curvature of \(<\text{empty space}>\) is 0, (has not intrinsic curvature).

The physical laws are correlated with the geometry of Spaces and can be seen, using CLG, in Plane Space as it is shown in figures 1, 2, 3. A marvelous Presentation of the method can be seen on Dr Geo-Machine Macro-constructions.

Perhaps, Inertia is the Property of a certain Space Layer, and the Interaction of Spaces at the Commons or those have been called Concentrated Logic, creates the motion.

The parallel axiom (the postulate) on any Segment \( \text{AB} \) is experimentally verifiable, and in this way it is Dependent of the other Axioms and is logically consistent, and since this is true then must be accepted. The Parallel Postulate is a Parallel Axiom, So, all Nature (the Universe) is working according to the Principles (the patterns), the Properties and the dialectic logic of Euclidean geometry.

Hyperbolic and Projective geometry transfers the Parallel Axiom to problem of a point M and a Plane \( \text{AB-CA} \) instead of problem of three points only, which such it is.

Vast (the empty space) is simultaneously \( \infty \) and 0 for every unit \( \text{AB} \), as this is for numbers. Uniformity (Homogenous) of Empty Space creates, all the one dimensional units, the Laws of conservation for Total Impulse, moment of Inertia in Mechanics, independently of the Position of Space and regardless the state of motion of other sources. (Isotropic Spaces) Uniformity (Homogenous) of Empty Time creates, the Laws of conservation of the Total Energy regardless of the state of motion (Time is not existing here), since Timing is always the same and Time Intervals are not existing.
In Special Relativity events from the origin are determined by a velocity and a given unit of time, and the position of an observer is related with that velocity after the temporal unit. Since all Spaces and Substances co-exist, then Past, Present and Future simultaneously exist on different Space Layer. Odd and Even Spaces have common Properties, so Gravity belonging to different Layer as that of particles, is not valid in atom Layer.

Spaces may be simultaneously Flat or Curved or multi-Curved, according to the Concentrated, (Compact) Logic of the Space, so the changing curvature from point to point is possible.

RATIONAL FIGURED NUMBERS OR FIGURES. 20/3/2010

This document is related to the definition of “Heron” that gnomon is as that which, when added to anything, a number or figure, makes the whole similar to that to which it is added. In general the successive gnomonic numbers for any polygonal number, say, of n sides have n-2 for their common difference. The odd numbers successively added were called gnomons. See Archimedes (Heiberg 1881, page 142, ε΄). The Euclidean dialectic logic of an axiom is that which is true in itself. τών δὲ ἄνίσων, ὁμοίων δὲ τὰ κέντρα τῶν βαρέων ὁμοίως ἔσσεται κείμενα. ὁμοίως δὲ λέγομες σμεία κείσθαι ποτί τὰ ὁμοία σχήματα. ἀρ’ φ’ ἐπί τάς ἴσας γωνίας ἀγόμεναι εὐθείαι ποιέοντι γονίας ἴσας ποτί τὰς ὁμολόγους πλευράς. (Επιπέδων ἰσορροπίων ἢ κέντρα βαρών επιπέδουν α’).

This logic exists in nature and is reflected to our minds as dialectic logic of mind. Shortly for ancient Greeks was, (μηδὲν εν τη νοήσει ειμή πρότερον εν τοί αισθήσοι). Since the first dimentional Unit is Segment AB, it is obvious that all Rational Segments are multiples of AB potentially the first polygonal number of any form, and the first is 2AB = AB +AB, which shows that multiplication and Summation is the same action with the same common base, the Segment AB.

To Prove:

The triangle with sides AC1,AB2,C1B2 twice the length of initial segments AC,AB,CB preserves the same angles < A = BAC, B = ABC, C = ACB of the triangle.

Proof:

a. Remove triangle ABC on line AC such that point A coincides with point C (A1). Triangles CB1C1, ABC are equal, so CA’ = AB, C1A’ = CB

b. Remove triangle ABC on line AB such that point A coincides with point B (A2). Triangles BB2C2, ABC are equal, so BC2 = AC, B2C2 = BC

c. The two circles (C,CB1 = AB) and (B,BC2 = AC) determine by their intersection point A’, so triangles CBA’, CBA are equal, and also equal to the triangles CCB1, BB2C2, and this proposition states that sides CBB1 = CA’, BC2 = BA’. Point A’ must simultaneously lie on circles (C1, C1B1), (B2, B2C2), which is not possible unless point A’ coincides with points B1 and C2.

d. This logic exists in Mechanics as follows:

The linear motion of a Figure or a Solid is equivalent to the linear motion of the gravity centre because all points of them are linearly displaced, so
1st Removal ----  BB₁ = AC , CB₁ = AB , BC = BC  
2nd Removal ----  CC₂ = AB , BC₂ = AC , BC = BC  
1st + 2nd Removal ----  CB₁ = AB , BC₂ = AC , BC = BC  

Since all degrees of freedom of the System should not be satisfied therefore points B₁ , C₂ , A’ coincide.

e. Since circles ( C₁ , C₁B₁ = C₁A’ = CB ) , ( B₂ , B₂C₂ = B₂A’ = CB ) pass through one point A’, then C₁A’B₂ is a straight line, this because C₁A’ + A’B₂ = C₁B₂ , and A’ is the midpoint of segment B₂C₁.

f. By reasoning similar to what has just been given, it follows that the area of a triangle with sides twice the initials, is four times the area of the triangle.

g. Since the sum of angles < C₁A’C + CA’B + BA’B₂ = 180° (6D) and equal to the sum of angles CBA + CAB + ACB then the Sum of angles of any triangle ABC is 180°, which is not depended on the Parallel Theorem or else-where.

Verification:
Let be the sides a=5 , b=4 , c=3 of a given triangle and from the known formulas of area S = (a + b + c)/2 = 6, Area = √6.1.2.3 = 6
For a=10 , b=8 , c=6 then S = 24/2 = 12 and Area = √12.2.4.6 = 24 = 4 x 6 (four times)

A GIVEN POINT P , AND ANY CIRCLE (O, OA).

Point P outside the circle. Point P on circle. Point P in circle.

To Prove:

The locus of midpoints M of segments PA, is a circle with center O’ at the middle of PO and radius O’M = OA/2 where,
P is any point on a Plane
A is any point on circle (O, OA)
M is midpoint of segment PA,
Proof:
Let O’ and M be the midpoints of PO, PA. According to the previous given for Gnomon, the sides of triangle POA are twice the size of PO’M, or PO = 2.PO’ and PA = 2.PM therefore as before, OA = 2.O’M, or O’M = OA/2. Assuming M found, and Since O’ is a fixed point, and O’M is constant, then (O’, O’M = OA/2) is a circle.

For point P on the circle:

The locus of the midpoint M of chord PA is the circle (O’,O’M = PO / 2) and it follows that triangles OMP, OMA are equal which means that angle < OMP = OMA = 90°, i.e. the right angle < PMO = 90° and exists on diameter PO (on arc PO), and since the sum of the other two angles < MPO + MOP exist on the same arc PO = PM + MO, it follows that the sum of angles in a rectangle triangle is 90 + 90 = 180°.
THE TWO ANGLES PROBLEM
ANY TWO ANGLES  \( \alpha = \angle AOB, \beta = \angle A'O'B' \) WITH PERPENDICULAR SIDES ARE EQUAL .

Let  \( \angle AOB = \alpha \) be any given angle and  \( \angle A'O'B' = \beta \) such that  \( AO \perp O'A' \), \( OB \perp O'B' \).

To proof that angle  \( \beta \) is equal to  \( \alpha \).

Proof :

CENTRE  \( O' = O \),  \( \alpha \leq 90^\circ \)
Angle  \(< AOA' = 90^\circ = AOB + BOA' = \alpha + x \) \( (1) \)
Angle  \(< BOB' = 90^\circ = BOA' + A'OB' = x + \beta \) \( (2) \), subtracting (1) , (2) → angle  \( \beta = \alpha \)

CENTRE  \( O' = O \),  \( 90^\circ < \alpha < 180^\circ \)
Angle  \(< AOA' = 90^\circ = AOB' - B'OA' = \alpha - x \) \( (1) \)
Angle  \(< BOB' = 90^\circ = A'OB - A'OB' = \beta - x \) \( (2) \), subtracting (1) , (2) → angle  \( \beta = \alpha \)

CENTRE  \( O' \neq O \).
Draw circle (\( M , MO = MO' \)) with  \( OO' \) as diameter intersecting  \( OA,O'B' \) produced to points  \( A_1,B_1 \).
Since the only perpendicular from point  \( O \) to  \( O'A' \) and from point  \( O' \) to  \( OB \) is on circle (\( M , MO \))
then , points  \( A_1 , B_1 \) are on the circle and angles  \( O'A_1O, O'B_1O \) are equal to  \( 90^\circ \).

The vertically opposite angles  \( a = a_1 + a_2 , b = b_1 + b_2 \) where  \( O'C \perp OO' \).

Since  \( MO = MA_1 \) then angle  \( < MOA_1 = MA_1O = a_1 \).
Since  \( MA_1 = MO' \) then angle  \( < MA_1O' = MO'A_1 = x \)
Since  \( MO' = MB_1 \) then angle  \( < MO'B_1 = MB_1O' = z \)

Angle  \( MO'C = 90^\circ = x + b_1 = z + b_2 \).
Angle  \( O'A_1O = 90^\circ = x + a_1 = x + b_1 \) →  \( a_1 = b_1 \)
Angle  \( O'B_1O = 90^\circ = z + a_2 = z + b_2 \) →  \( a_2 = b_2 \)

By summation  \( a_1 + a_2 = b_1 + b_2 \) or  \( b = a \) (o.e.d.)
i.e. any two angles  \( a , b \) having their sides perpendicular among them are equal .

From upper proof is easy to derive the Parallel axiom , and more easy from the Sum of angles on a right-angled triangle .
POINT M ON A CIRCLE OF DIAMETER AB = 0 → ∞

Let M be any point on circle (O, OM = OA = OB), M1, M2 the middle points of MA, MB and in second figure MM’ ⊥ BA at point B (angle AMM’ = 90°). In third figure MM1 is diameter.

To Show:
1. Angle < AMB = MAB + MBA = a + b = m
2. Triangles MBM1, MBA are always equal and angle < MBM1 = AMB = 90°
3. The Sum of angles on triangle MAB are < AMB + MAB + MBA = 180°.

Proof:

1. Since OA = OM and M1A = M1M and OM1 common, then triangles OM1A, OM1M are equal and angle < OAM = OMA = BAM = a → (a)

Since OM = OB and M2B = M2M and OM2 common, then triangles OM2B, OM2M are equal and angle < OBM = OMB = ABM = b → (b)

By summation (a), (b) BAM + ABM = (OMA + OMB) = AMB = a + b = m ...(c)

i.e. When a Point M lies on the circle of diameter AB, then the sum of the two angles at points A, B is constantly equal to the other angle at M. Concentrated logic of geometry exists at point B, because as on segment AB of a straight line AB, which is the one dimensional Space, springs the law of Equality, the equation AB = OA + OB i.e. The whole is equal to the parts, so the same is valid for angles of all points on the circumference of the circle (O, OM), [as Plane ABM and all angles there exist in the two dimensional Space], and it is m = a + b.

In figure (2), when point M approaches to B, the Side BM’ of angle < ABM tends to the perpendicular on BA and when point M coincides with point B, then angle < ABM = 90° and < OAM = BAM = 0°, therefore angle < AMB = 90° and equation (c) becomes:

BAM + ABM = AMB → 0 + 90° = AMB → AMB = 90°, (i.e. AM ⊥ BM) and the sum of angles is (BAM + ABM) + AMB = 90° + 90° = 180°, or BAM + ABM + AMB = 180°

2. Triangles MBA, MBM1 are equal because they have diameter MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB).

Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from isosceles triangle OM1B, angle < OBM1 = OM1B = a

The angle at point B is equal to MBM1 = MBA + ABM1 = b + a = m = AMB

Rotating diameter MM1 through centre O so that points M, M1 coincides with B, A then angle < MBM1 = MBA + ABM1 = BBA + ABA = 90° + 0 = 90° and equal to AMB

i.e. The required convention for angle MBM1 = AMB = m = a + b = 90°. (o.e.δ)
3. Since the Sum of angles \( a + b = 90^\circ \), and also \( m = 90^\circ \) then \( a + b + m = 90+90 = 180^\circ \). It is needed to show that angle \( m \) is always constant and equal to \( 90^\circ \) for all points on the circle. Since angle at point \( B \) is always equal to \( \text{MBO} + \text{OBM} = b + a = m = \text{AMB} \), by Rotating triangle \( \text{MBM}_1 \) so that points \( M, B \) coincide then \( \text{MBM}_1 = \text{BBA} + \text{ABA} = 90+0 = m \)

Since angle \( < \text{AMB} = a + b = m \) and is always equal to angle \( < \text{MBM}_1 \), of the rotating unaltered triangle \( \text{MBM}_1 \), and since at point \( B \) angle \( < \text{MBM}_1 \) of the rotating triangle \( \text{MBM}_1 \) is \( 90^\circ \), then is always valid, angle \( < \text{AMB} = \text{MBM}_1 = 90^\circ \) (o.e.δ), 22/4/2010.

ANY TWO ANGLES HAVING THEIR SIDES PERPENDICULAR AMONG THEM ARE EQUAL OR SUPLEMENTARY

\[
\begin{align*}
\text{AB} &= \text{Diameter} & \text{M} \to \text{B}, \text{AB} \perp \text{BM}\,^\prime & \text{AM}_1 \perp \text{BM}_1, \text{AM}_2 \perp \text{BM}_2 = \text{AM}_2 \perp \text{BM}
\end{align*}
\]

Let angle \( < \text{M}_1\text{AM}_2 = a \) and angle \( < \text{M}_1\text{BM}_2 = b \), which have side \( \text{AM}_1 \perp \text{BM}_1 \) and side \( \text{AM}_2 \perp \text{BM}_2 \)

To show:
1. Angle \( < \text{M}_1\text{AM}_2 = \text{M}_1\text{BM} = a \)
2. Angle \( < \text{M}_1\text{AM}_2 + \text{M}_1\text{BM}_2 = a + b = 180^\circ \).
3. The Sum of angles in Quadrilateral \( \text{AM}_1\text{BM}_2 \) is \( 360^\circ \).
4. The Sum of angles in Any triangle \( \text{AM}_1\text{M}_2 \) is \( 180^\circ \).

Proof:
1. In figure 3, since \( \text{AM}_1 \perp \text{BM}_1 \) and \( \text{AM}_2 \perp \text{BM}_2 \) or the same \( \text{AM}_2 \perp \text{BM} \), then according to prior proof, \( \text{AB} \) is the diameter of the circle passing through points \( \text{M}_1, \text{M}_2 \), and exists \( a_1 + b_1 = m_1 = 90^\circ \), \( a_2 + b_2 = m_2 = 90^\circ \) and by summation \( (a_1 + b_1) + (a_2 + b_2) = 180^\circ \) or \( (a_1 + a_2) + (b_1 + b_2) = a + b = 180^\circ \), and since also \( x + b = 180^\circ \) therefore angle \( < x = a \)
2. Since the Sum of angles \( \text{M}_1\text{BM}_2 + \text{M}_1\text{BM} = 180^\circ \) then \( a + b = 180^\circ \)
3. The sum of angles in quadrilateral \( \text{AM}_1\text{BM}_2 \) is \( a + b + 90 +90 = 180 + 180 = 360^\circ \)
4. Since any diameter \( \text{AB} \) in Quadrilateral divides this in two triangles, it is very easy to show that diamesus \( \text{M}_1\text{M}_2 \) form triangles \( \text{AM}_1\text{M}_2, \text{BM}_1\text{M}_2 \) equal to \( 180^\circ \) each.

so,
1. Any angle between the diameter \( \text{AB} \) of a circle is right angle (\( 90^\circ \)).
2. Two angles with vertices the points \( A, B \) of a diameter \( \text{AB} \), have perpendicular sides and are equal or supplementary.
3. Equal angles exist on equal arcs, and central angles are twice the inscribed angles.
4. The Sum of angles of any triangle is equal to two right angles.

i.e. two Opposite angles having their sides perpendicular between them, are equal or supplementary between them. This property has been used in proofs of Parallel Postulate

Conclusions, and how Useful is this invention, is left to the reader. marcos
The essential difference between Euclidean and non-Euclidean geometries is the nature of parallel lines.

1. Euclid’s fifth postulate, the parallel postulate, states that, within a two-dimensional plane $ABM$ for a given line $AB$ and a point $M$, which is not on $AB$, there is exactly one line through $M$ that does not intersect $AB$. In Euclidean geometry, in case of two straight lines that are both perpendicular to a third line, the lines remain at a constant distance from each other and are known as parallels.

Now is proved that, a point $M$ on the $N$th Space, of any first dimensional Unit $AB = 0 \rightarrow \infty$, jointly exists, with all Sub-Spaces of higher than $N$ Spaces, and with all Spaces of lower than $N$ Subspaces. This is the Structure of Euclidean geometry.

As in fundamental theorem of Algebra Equations of $N$th degree can be reduced to all $N-a$ or $N+a$ degree, by using the roots of the equations, in the same way Multi–Spaces are formed on $AB$.

Nano-scale-Spaces, unorganig and organig, Cosmic-scale-Spaces are now unified in our world scale. Euclidean Empty Space is Homogenously Continues, but all first dimensional Unit- Spaces Heterogenous and this because all Spaces constitute another Units (the $N$th Space Tensor is the boundaries of $N$ points).

All above referred and many others, are springing from the first acceptance for point, and the approaching of Points. By multiplication is created another one very important logical notion for the laws concerning Continues or Not Continues Transformations in Space and in Time for Mechanics, Physics Chemistry and motions generally. From this logic Yields that a limited and not an unlimited Universe can Spring anywhere. Since Non-existence is found everywhere then Existence is found and is Done everywhere.

If Universe follows Euclidean geometry, then this is not expanded indefinitely at escape velocity, but is moving in Changeable Spaces with all types of motions, < a twin symmetrically axial–centrifugal rotation > into a Steady Space (System $AB \perp AB = 0 \rightarrow AB \rightarrow \infty$), with all types of curvatures. (It is a Moving and Changeable Universe into a Steady Formation).

2. Hyperbolic geometry, by contrast, states that there are infinitely many lines through $M$, not intersecting $AB$.

In Hyperbolic geometry, the two lines “curve away” from each other, increasing in distance as one moves further from the points of intersection with the common perpendicular, which have been called ultra-parallel.

The simplest model for Hyperbolic geometry is the pseudo-sphere of Beltrami-Klein, which is a portion of the appropriate curvature of Hyperbolic Space, and the Klein model, by contrast, calls a segment as line and the disk as Plane.

Mobius strip and Klein bottle (complete one-sided objects of three and four dimensions) transfers the parallel Postulate to a problem of one point $M$ and a Plane, because all curves and other curve lines are not lines (For any point on a straight line exists < the whole is equal to the parts which is an equality > and not the inequality which has to do with Plane, three points only and anywhere.

If Universe follows Hyperbolic geometry then this is expanded indefinitely, which contradicts to the homogenous and isotropic Empty Spaces and also to the laws of conservation of Energy.

3. Elliptic geometry, by contrast, states that, all lines through point $M$, intersect $AB$.

In Elliptic geometry the two lines “curve toward” each other and eventually intersect.

The simplest model for Elliptic geometry is a sphere, where lines are “great circles”???

For any great circle (which is not a straight line ???) and a point $M$ which is not on the circle all circles (not lines ???) through point $M$ will intersect the circle.

If Universe follows Elliptic geometry then this is expanded to a halt and then this will stark to shrink possibly not to explode as is said, but to change the axial-centrifugal motion to the initial Rectilinear.
**Spherical Geometry**, changes a line to a “great circle”, that is, a circle of maximum radius, saying that the sum of the angles of a triangle is no longer $180^\circ$.

Remarks:
1. A line is not a great circle, so anything is built on this logic is false.
2. The fact that the sum of angles on any triangle is $180^\circ$, is springing for the first time, in this article (Rational Figures), proof g. page 8, and after 1/5/2010 → page 11 - 12.

**Hyperbolic Geometry**, is replacing the Parallel Postulate, accepting that, for any line $AB$ and a point $M$ not on $AB$, there are at least two (minimum) distinct lines through $M$, which do not intersect $AB$.(Hyperbolic geometry, Wikipedia, the free encyclopedia).

Remarks:
1. This admission of two or more than two parallel lines, instead of one of Euclid’s, does not prove the truth of the admission. The same to Euclid’s.
2. The proposed Method in this article, based on the prior four axioms only, proofs, (not using any admission) that,
   Through point $M$ on any Plane $ABM$ (three points only), passes only one line of which all points equidistant from $AB$, as point $M$, i.e. the right is to Euclid Geometry.

Question: Which axiom is not satisfied by Hyperbolic or other geometry? 15/4/2010

It has been proved that quadrilateral $MA_1CC’$ is Rectangle (d, p.3) and from equality of triangles $MA_1C$, $MC’C$ then angle $< C’MC = MCA_1$.

Since the sum of angles $< MCA_1 + MCB = 180^\circ$, also, the sum of angles $< C’MC + MCB = 180^\circ$ which answers to Postulate P5, as this has been set (e, p.3).

Hyperbolic geometry, Lobachevsky, non-Euclidean geometry, Wikipedia the free encyclopedia, states that there are TWO or more lines parallel to a given line $AB$ through a given point $M$ not on $AB$.

If this is true, for second angle $C2MC$, exists also the sum of angles $< C2MC + MCB = 180^\circ$, which is Identity ($C2MC = C’MC$), i.e. all (the called parallels) lines coincide with the only one parallel line $MM’$, and so again the right is to Euclid geometry.

*Definitions*, Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature, which is the meter of all logics, and has been found to be the first dimensional Unit $AB = 0 \rightarrow \infty$ (figure 3, page 5) i.e. the reflected Model of the Universe.

Lobachevsky’s and Riemann’s Postulate may seem to be good attempts to prove Euclid’s Fifth Postulate by contradiction, and recently by “compromising the opposites” in the Smarandache geometries. Non of them contradicts any of the other Postulates of what actually are or mean.

From any point $M$ on a straight line $AB$, springs the logic of the equation (the whole $AB$ is equal to the parts $MA$, $MB$), which is rightly followed (intrinsic), in Euclidean geometry only, in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line, disk as planes and others), so all non-Euclidean geometries contradict to the second (D2) definition and to the first (P1) Euclidean Postulate.

Question: Is it possible to show that the sum of angles on any triangle is $180^\circ$? 1/5/2010

Yes by Using Euclidean Spaces and Subspaces.

Since the two dimensional Spaces exists on Space and Subspace then this problem of angles must be on the boundaries of the two Spaces i.e. on the circumference of the circle and on any tangent of the circle and also to that point where Concentrated Logic of geometry exists for all units, as straight lines etc. It was proofed at first that, the triangle with hypotenuse the diameter of the circle is a right angled triangle and then the triangle of the Plane of the three vertices and that of the closed area of the circle (the Subspace), measured on the circumference is $180^\circ$. 

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Some Answers to dear professors,  

**Question 1**: Which axiom is not satisfied by Hyperbolic or other geometry?  

It has been proved that quadrilateral MA1CC’ is Rectangle (d, p.3) and from equality of triangles MA1C, MC’C then angle < C’MC = MCA.  

Since the sum of angles < MCA + MCB = 180°, also, the sum of angles < C’MC + MCB = 180° which answers to Postulate P5, as this has been set (e, p.3).  

Hyperbolic geometry, Lobachevsky, non-Euclidean geometry, Wikipedia the free encyclopedia, states that there are two or more lines parallel to a given line AB through a given point M not on AB. If this is true, for second angle C2MC, exists also the sum of angles < C2MC + MCB = 180°, which is Identity (C2MC = C’MC), i.e., all (the called parallels) lines coincide with the only one parallel line MM’, and so again the right is to Euclid geometry.  

**Definitions, Axioms or Postulates create a geometry, but in order this geometry to be right must follow the logic of Nature, which is the meter of all logics, and has been found to be the first dimensional Unit AB = 0 → ∞ (figure 3, page 5) i.e. the reflected Model of the Universe.**  

Lobachevsky’s and Riemann’s Postulate may seem to be good attempts to prove Euclid’s Fifth Postulate by contradiction, and recently by “compromising the opposites” in the Smarandache geometries. None of them contradicts any of the other Postulates of what actually are or mean.  

From any point M on a straight line AB, springs the logic of the equation (the whole AB is equal to the parts MA, MB), which is rightly followed (intrinsically) in Euclidean geometry only, in contradiction to the others which are based on a confused and muddled false notion (the great circle or segment is line, disk as planes and others), so all non-Euclidean geometries contradict to the second (D2) definition and to the first (P1) Euclidean Postulate.  

**Question 2**: Why Hyperbolic geometry satisfies the same 4 axioms as Euclidean geometry, and the error in my Euclidean derivation of the 5th axiom.  

An analytical trial is done to answer this question.  

**Postulate 1**: states that,  

“Let it have been postulated to draw a straight-line from any point to any point”.  

As this can be done by placing the Ruler on any point A to any point B, then this is not in doubt by any geometry. The world “line” in Euclid geometry is straight line (the whole is equal to the parts) and axioms require that line to be as this is (Black color is Black and White color is White). In ancient Greek “Εὐθεία γραμμή ἐστιν, ἥτις ἐξ ἑξίσου τοῖς ἑαυτῆς σημείωσις κεῖται.” >  

**Postulate 2**: states that,  

“And to produce a finite straight-line”  

Marking points A, B which are a line segment AB, and by using a Ruler then can produce AB in both sides continuously, not in doubt by any geometry.  

**Postulate 3**: states that,  

“And to draw a circle with any centre and radius”  

Placing the sting of a Compass at any point A (centre) and the edge of pencil at position B and (as in definition 15 for the circle) Radiating all equal straight lines AB, is then obtained the figure of the circle (the circumference and the inside), not in doubt by any geometry.  

**Postulate 4**: states that,  

“And all right angles are equal to one another”
In definition D8 is referred as Plane Angle, to be the inclination of two lines in a plane meeting one another, and are not laid down straight-on with respect to one another, i.e. the angle at one part of a straight line.

In definition D9 is stated “And when the lines containing the angle are straight then the angle is called rectilinear” and this because straight lines divide the plane, and as plane by definition is $360\circ$ then the angles on a straight line are equal to $180\circ$.

In definition D10 is stated that a perpendicular straight line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle and this because as the two adjacent angles are equal and since their sum is $180\circ$, then the two right-angles are $90\circ$ each and since this happen to any two perpendicular straight-lines, then all right angles are equal to one another, not in doubt by any geometry.

Postulate 5:

This postulate is referred to the Sum of the two internal angles on the same side of a straight-line falling across two (other) straight lines, being produced to infinity, and be equal to $180\circ$.

Because this postulate, beside all attempts to prove it, was standing for centuries, mathematicians created new geometries to step aside this obstruction.

In my proposed article the followings have been geometrically proved:

1. From any point M to any line AB (the three points consist a Plane) is constructed by using the prior four Postulates, a system of three rectangles MA1CC’, CCB1M’, MA1B1M’.

2. The Sum of angles C’MC and MCB is $< C’MC + MCB = 180\circ$, which satisfies postulate P5 of Euclid geometry, and as this is now proved, then it is an axiom.

3. The extended Structure of Euclidean-geometry to all Spaces (Spaces and Sub-spaces) resettles truth to this geometry, and by the proposed solution which is applicable to any point M, not on line AB, answers to the temporary settled age-old question for this problem.

4. Mathematical interpretation and all relative Philosophical reflections based on the non-Euclid geometry theories, must properly revised and resettled in the truth one.

5. .......

References:

EUCLID’S ELEMENTS IN GREEK
The great text of J. L. Heiberg (1883-1886) and the English translation by Richard Fitzpatrick.
ELEMENTS BOOK 1.

THE PARALLEL POSTULATE IS DEPENDED ON THE OTHER AXIOMS by Markos Georgallides

THE SUCCESSION OF PROOFS FOR THE PARALLEL POSTULATE

AB is a straight line through points A, B and M is a point not on AB.

METHOD

1. Draw the circle (M, MA) be joined meeting line AB in C and let A1, B1 be the midpoint of CA, CB.

2. On mid-perpendicular B1M find point M' such that M'B1 = MA1 and draw the circle (M', M'B = M'C) intersecting the circle (M, MA = MC) at point D.

3. Draw mid-perpendicular of CD at point C'.

4. To show that line MM' is a straight line passing through point C' and it is such that MA1 = M'B1 = C'C = h, i.e. a constant distance h from line AB or, also the sum of angles C'MC + MCB1 = A1CM + MCB1 = 180°.

Proofed Succession:

1. The mid-perpendicular of CD passes through points M, M'. (page 3, step 2)

2. Angle < A1MC' = A1MM' = A1CC', Angle < B1M'C' = B1M'M = B1CC' <A1MC' = A1CC' because their sides are perpendicular among them i.e. MA1⊥CA, MC⊥CC'.


3. The sum of angles A1MM' + B1M'M = 180° because the equal sum of angles A1CC' + B1CC' = 180°, so the sum of angles in quadrilateral MA1B1 M' is equal to 360°.

4. The right-angled triangles MA1B1, M'B1A1 are equal, so diagonal MB1 = M'A1 and since triangles A1MM', B1M'M are equal, then angle A1MM' = B1M'M and since their sum is 180°, therefore angle < A1MM' = MM'B1 = M'B1A1 = B1A1M = 90°

5. Since angle A1CC' = B1CC' = 90°, then quadrilaterals A1CC'M, B1CC'M' are rectangles and for the three rectangles MA1CC', CB1M'C', MA1B1M' exists MA1 = M'B1 = C'C

6. The right-angled triangles MCA1, MCC' are equal, so angle < A1CM = C'MC and since the sum of angles < A1CM + MCB1 = 180° then also C'MC + MCB1 = 180° Which is the second to show, as this problem has been set at first by Euclid.
Let M be any point on circle (O, OM = OA = OB), M1, M2 the middle points of MA, MB and in second figure MM’ ⊥ BA at point B, (angle AMM’ = 90°).

To show that the Sum of angles < AMB, MAB, MBA of any triangle ABM is 180°.

Proof:

1. Since OA = OM and M1A = M1M and OM1 common, then triangles OM1A, OM1M are equal and angle < OAM = OMA = BAM = (a)

Since OM = OB and M2B = M2M and OM2 common, then triangles OM2B, OM2M are equal and angle < OBM = OMB = ABM = (b)

By summation (a) and (b) → BAM + ABM = (OMA + OMB) = AMB .... (c)

i.e. When a Point M lies on the circle of diameter AB, then the sum of the two angles at points A, B is constantly equal to the other angle at M. Concentrated logic of geometry exists at point B, because as on segment AB of a straight line AB, which is the one dimensional Space, springs the law of Equality, the equation AB = OA + OB i.e. The whole is equal to the parts, so the same is valid for angles of all points on the circumference of the circle (O, OM), as Plane ABM and all angles there exist in the two dimensional Space, and it is m = a + b.

2. In figure (2), when point M approaches to B, the Side BM’ of angle < ABM tends to the perpendicular on BA and when point M coincides with point B, then angles < ABM = 90° < OAM = BAM = 0°, AMB = 90° and equation (c) becomes:

BAM + ABM = AMB → 0 + 90° = AMB → AMB = 90° (d), (i.e. AM ⊥ BM)

and the sum of angles is (BAM + ABM) + AMB = 90° + 90° = 180°,
or BAM + ABM + AMB = 180° (o.e.δ)

2a. To show, the Sum of angles a + b = constant = 90° = m. (figure 3)

M is any point on the circle and MM1 is the diameter. Triangles MBA, MBM1 are equal and by rotating diameter MM1 through centre O, the triangles remain equal.

Proof:

a. Triangles MBA, MBM1 are equal because they have MM1 = AB, MB common and angle < OBM = OMB = b (from isosceles triangle OMB) so MA = BM1.

b. Since Triangles MBA, MBM1 are equal therefore angle < MM1B = MAB = a, and from isosceles triangle OM1B, angle < ABM1 = OBM1 = OM1B = a

c. The angle at point B is always equal to MBM1 = MBO + OBM1 = b + a = m = AMB

Rotating triangle MBM1 so that points M, B coincide then MBM1 = ABB + ABA = 90° + 0° = m
Since angle $\angle AMB = a + b = m$ and is equal to angle $\angle MBM_1$, of the rotating unaltered triangle $MBM_1$ and which at point $B$ has angle $m = 90^\circ$, then is valid angle $\angle AMB = MBM_1 = 90^\circ$ i.e. the required convention for angle $\angle AMB = m = a + b = 90^\circ$. (o.e.δ), 22/4/2010.

2b. When point $M$ moves on the circle, Euclidean logic is as follows.

Accepting angle $\angle ABM' = b$ at point $B$, automatically point $M$ is on the straight line $BM'$ and the equation at point $B$ is for $(a = 0, b = 90^\circ, m = 90^\circ) \rightarrow 0 + 90^\circ = m$ and also equal to, $0 + b - b + 90^\circ = m$ or the same $\rightarrow b + (90^\circ - b) = m$ i.e. the required $\angle AMB = MBM_1 = 90^\circ$ i.e. the demand that the two angles $a, b$ satisfy equation (M) is that their sum must be constant and equal to $90^\circ$. (o.e.δ)

3. In figure 3, according to prior proof, triangles $MBA, MBM_1$ are equal. Triangles $AM_1B, AMB$ are equal because $AB$ is common, $MA = BM_1$ and angle $\angle MAB = ABM_1$, so $AM_1 = MB$. Triangles $ABM_1, AMB$ are equal because $AB$ is common $MB = AM_1$ and $AM = BM_1$ therefore angle $\angle BAM_1 = ABM = b$ and so, angle $MAM_1 = a + b = MBM_1$.

Since angle $\angle AMB = AM_1B = 90^\circ$ then $AM \perp BM$ and $AM_1 \perp BM_1$.

Triangles $OAM_1, OBM$ are equal because side $OA = OB$, $OM = OM_1$ and angle $\angle MOB = AOM_1$, therefore segment $M_1A = MB$.

Rotating diameter $MM_1$ through $O$ to a new position $Mx, M_1x$ any new segment $MxB = M_1xA$ the angle $\angle MxBM_1x = MxB + ABM_1x$ and segment $BMx = AM_1x$.

Simultaneously rotating triangle $MxBM_1x$ through $B$ such that $BMx \perp AB$ then angle $\angle MxBM_1x = BBA + ABA = 90^\circ + 0 = 90^\circ$ i.e. in any position $Mx$ of point $M$ angle $\angle AMxB = MBM_1x = 90^\circ$ i.e. two Equal or Supplementary between them Opposite angles have their sides perpendicular between them. (the opposite to that in page 11)

Followings the proofs, Any angle between the diameter of a circle is right angle $(90^\circ)$, central angles are twice the inscribed angles, angles in the same segments are equal to one another and then applying this logic on the circumscribed circle of any triangle $ABM$, then is proofed that the Sum of angles of any triangle is equal to two right angles or $\angle BAM + ABM + AMB = 180^\circ$.

Conclusions, and how Useful is this invention, is left to the reader. marcos

Question 3 : Is any error in the argument that angles with Perpendicular sides are equal? 17/5/10

In Proved Succession (a) page 3, is referred that two angles with perpendicular sides are equal (or supplementary). To avoid any pretext, a clear proof is given to this presupposition in (1) page 11 showing also that,

1 Any angle between the diameter $AB$ of a circle is right angle $(90^\circ)$.
2 Any two angles with vertices the points $A, B$ of a diameter $AB$, have perpendicular sides and are also equal or supplementary.
3 Equal angles exist on equal arcs, and central angles are twice the inscribed angles.
4 The Sum of angles of any triangle is equal to two right angles.

so,

There is not any error in argument of proofs.
The 5th Postulate is Depended on (derived from) the prior four axioms.
To show that angle $\angle AMB = m = 90^\circ$

$BB' \perp BA$ (angle $ABB' = 90^\circ$), $MM'' \perp AB$

F1: It has been proved (page 17) that triangles $AMB, MBM_1$ are equal and angle $\angle AMB = \angle MBM_1 = m$ for all positions of $M$ on the circle.

Since triangles $OMB, OAM_1$ are equal then chord $BM = AM_1$ and arc $BM = AM_1$.

F2: The rotation of diameter $MM_1$ through centre $O$ is equivalent to the new position $M_x$ of point $M$ and simultaneously is the rotation of angle $\angle M_1BM = \angle MBA$ through point $B$, and this because arc $BM = AM_1$, $BM_x = AM_1x$, i.e. when point $M$ moves with $BMM'$ to a new position $M_x$ on the circle, diameter $OMM_1 = OMM_1x$ is rotated through $O$, the points $M, M_1$ are sliding on sides $BMM'$, $BM_1$ because point $M_1$ to the new position $M_1x$ is such that $AM_1x = BM_x$ and angle $\angle M_1BM_1$ is then rotated through $B$. (analytically below)

F3: When diameter $MM_1$ is rotated through $O$, point $M$ lies on arc $MB = AM_1$ and angle $\angle M_1BM_1$ is not altered (this again because $MB = AM_1$) and when point $M$ is at $B$, point $M_1$ is at point $A$, because again arc $BM = AM_1 = 0$, and angle $a = BM_1M = 0$,

angle $\angle M_1BM = M_1BM' = ABM' = 90^\circ = m = a + b$

Conclusion 1.
Since angle $\angle AMB$ is always equal to $\angle MBM_1 = M_1BM_1$ and angle $\angle M_1BM_1 = 90^\circ$ therefore angle $\angle AMB = a + b = m = 90^\circ$.

 Conclusion 2.
Since angle $\angle ABB' = 90^\circ = ABM + MBB' = b + a$, therefore angle $\angle MBB' = a$, i.e. the two angles $\angle BAM, MBB'$ which have $AM \perp BM$ and $AB \perp BB'$ are equal between them.

Conclusion 3.
Any angle $\angle MBB'$ on chord $BM$ and tangent $BB'$ of the circle ($O, OA = OB$), where is ($BB' \perp BA$), is equal to the inscribed one, on chord $BM$.

Conclusion 4.
Drawing the perpendicular $MM''$ on $AB$, then angle $\angle BMM'' = MAB = MBB'$, because they have their sides perpendicular between them, i.e. since the two lines $BB'$, $MM''$ are parallel and are cut by the transversal $MB$ then the alternate interior angles $MBB'$, $BMM''$ are equal.

Conclusion 5.
In Mechanics, the motion of point $M$ is equivalent to, a curved one on the circle, two Rotations through points $O, B$, and one rectilinear in the orthogonal system $M_1BM_1 = MBM_1$. 
Why angle MBM₁ is unaltered when plane MBM₁ is rotated through B to new position MxBM₁x ??

Let Plane ( MBM₁ ) , ( F₄ ) be rotated through B , to a new position B₁BM₁x such that :

1. Line BM → BB₁ intersects circle (O,OB) at point Mₓ and the circle (B,BM = BB₁) at point B₁.
2. Line BM₁ → BM₁x extended intersects circle (O,OB) at the new point M₁x.
3. Angle < M₁BM₁x = MBB₁ = MBMₓ , is angle of rotation.

Since angle < M₁BM₁x = MBMₓ , therefore angle < M₁BM is unaltered by rotation i.e. Angle < M₁BM = M₁xBMₓ and diameter MM₁ is sliding uniformly on their sides.

**Data + Remarks .**

1. Diameter MM₁ is sliding in angle M₁BM which means that points M₁, M lie on the circle (O,OB) and on lines BM₁, BM respectively, and also sliding to the other sides BM₁x, BMₓ of the equal angle < M₁xBMₓ. Any line segment M₁xMₓ = MM₁ is also diameter of the circle.
2. Only point Mₓ is simultaneously on circle (O,OB) and on line BB₁.
3. The circle with point M₁x as centre and radius M₁xMₓ = MM₁ intersect circle (O,OB) at only one unique point Mₓ.
4. Since angle < M₁xBB₁ = M₁BM and since segment M₁xMₓ = MM₁ then chord M₁xMₓ must be also on sides of angles M₁xBB₁, M₁BM, i.e. Point Mₓ must be on line BB₁.
5. Ascertain 2 and 4 contradict because this property belongs to point Mₓ, unless this unique point Mₓ coincides with M₁x and chord MₓM₁x is diameter of circle (O,OB).

Point Mₓ is simultaneously on circle (O,OB), on angle < M₁xBB₁ = M₁xBMₓ and is sliding on line BB₁. We know also that the unique point Mₓ has the same properties as point Mₓ, i.e. point Mₓ must be also on circle (O,OB) and on line BB₁, and the diameter M₁xMₓ is sliding also on sides of the equal angles M₁xBB₁, M₁xBMₓ, M₁BM.

Since point Mₓ is always a unique point on circle (O,OB) and also sliding on sides of angle M₁BM = M₁xBMₓ and since point Mₓ is common to circle (O,OB) and to line BB₁ = BMₓ, therefore, points Mₓ, M₁x coincide and chord MₓM₁x is diameter on the circle (O,OB) i.e. The Rotation of diameter MM₁ through O, to a new position MₓM₁x, is equivalent to the Rotation of Plane (MBM₁) through B and exists angle < MBM₁ = MxBM₁x, so

angle < MBM₁ = MₓBM₁x = AMB = 90° = m = a + b ........... 0.ε.δ

Since angle < MBMₓ = M₁BMₓ is the angle of rotation, and since also arc MMₓ = M₁M₁x (this because triangles OMMₓ, OM₁M₁x are equal) then :

Equal inscribed angles exist on equal arcs.