Combinatorics, observables, and String Theory: part II

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Abstract

We investigate the string configuration that, in the framework of the theoretical scenario introduced in [1], corresponds to the most entropic configuration in the phase space of all the configurations of the universe. This describes a universe with four space-time dimensions, and the physical content is phenomenologically compatible with the experimental observations and measurements. Everything is determined in terms of the age of the universe, with no room for freely-adjustable parameters. We discuss how one obtains the known spectrum of particles and interactions, with massive neutrinos, no Higgs boson, and supersymmetry broken at the Planck scale. Besides the computation of masses and couplings, CKM matrix elements, cosmological constant, expansion parameters of the universe etc..., all resulting, within the degree of the approximation we used, in agreement with the experimental observations, we also discuss how this scenario passes the tests provided by cosmology and the constraints imposed by the physics of the primordial universe.

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Contents

1 Introduction

In the [1] we have discussed a theoretical scenario in which the universe is given by the superposition of all possible configurations which describe the assignment of a certain amount of energy along a vector space, of any possible dimension. The time ordering of the history of the universe is given by the inclusion of the sets containing all the configurations at a certain total energy. All the information about the universe is encoded in a "partition function" which can be expressed as:

$$\mathcal{Z}_{\mathcal{E}} \; = \; \sum_{\psi(E \leq \mathcal{E})} \mathrm{e}^{S(\psi)} \, , \tag{1.1}$$

where $\psi(E)$ indicates a configuration (i.e. a distribution of E energy units along space), and $S(\psi)$ is the entropy, i.e. the logarithm of the volume of occupation of the configuration ψ in the phase space of all the possible configurations. After discussing how 1.1 implies a quantum scenario which also embeds special and general relativity, and therefore quantum gravity, we also identified in String Theory the theory which realizes a representation of this scenario, i.e. the evolution of the superposition of geometries, in terms of propagating fields. The analogous of 1.1 on the continuum is:

$$\mathcal{Z}_V \; = \; \int_V \mathcal{D}\psi \, \mathrm{e}^{S(\psi)} \, , \tag{1.2}$$

where now ψ indicates a string configuration, and $S(\psi)$ its volume in the phase space of all string configurations at finite volume V, measured in the duality-invariant Einstein's frame. The dominant configuration of the universe is the one of highest entropy. As we discussed, the contribution of all the neglected configurations falls under the error accounted for by the Heisenberg Uncertainty, and therefore is already considered by quantization. The spectrum and the interactions of the elementary excitations of the universe and their interactions are therefore basically the ones described by the string configuration of highest entropy.

In this paper we want precisely to investigate the properties of this configuration. In [1] we have discussed how the absolute generality of the scenario described by 1.1, together with the fact of being the properties of a string representation uniquely identified, imply the uniqueness of string theory, namely the fact that all perturbative string realizations are part (dual slices) of a unique underlying theory. This means that we can not only speak of volume in a frame-independent way, but also of the string configuration of highest entropy independently on the particular representation, or perturbative string realization we may find convenient to use in order to represent it. Indeed, as this configuration is characterized by its having the lowest amount of symmetry, a condition which in particular implies strong coupling, it is not possible to realize it explicitly through a perturbative construction. In order to investigate its properties, we will make heavy use of string-string non-perturbative dualities. We will find that, as already expected from the analysis of [1], in this configuration only a four-dimensional subspace is allowed to expand, and indeed, owing to the presence in the spectrum of massless fields, it expands. Along these four coordinates, T-duality is broken. At large volumes a time-like coordinates can therefore be identified with what we ordinary call "time". All the remaining coordinates are twisted and therefore stuck, at the Planck scale. This scenario describes a universe with, on the large scale, the geometry of a three sphere, in which supersymmetry is broken at the Planck scale. The "volume ordering", namely what corresponds to the time ordering of the combinatorial approach, is equivalent to an ordering according to the radius R of the sphere. This implies a non-accelerated expansion of the universe. Nevertheless, it can be shown that to an observer the expansion of the universe appears to be accelerated as a consequence of the time-dependence of red-shifts, in turn due to the time-dependence of masses and energy levels. The dominance of the contribution of the configuration of highest entropy to the mean value of any observable increases the more and more "as time goes by", and the universe expands and cools down. We have in this way a realization, at a non-field theory level, of the idea of spontaneous breaking of symmetry, because the mean value of observables effectively shows a "progress" toward more broken configurations.

The spectrum of the elementary excitations corresponds to the degrees of freedom of all the known elementary particles, and their interactions. However, their identification, as well as a correct computation of masses, couplings, and cosmological parameters such as the so-called cosmological constant, involves a deep change of perspective as compared to the usual string or field-theoretical approach. All this is a consequence of compactness of the whole space: a finite volume space is here not conceived as a formal artifact introduced in order to work with a regularization of the infinity: it is the basic formulation of classical space. This in particular means that there is no invariance under time translations, because any progress in time is a progress in the history of the universe, and therefore a flow toward a configuration with different total energy, etc. There is no invariance under space translations either, because, unless special boundary conditions are imposed, any displacement implies approaching, or going away, from the horizon of space. The basic absence of invariance under space-time translations implies, by construction, a different normalization of string amplitudes, and therefore a different interpretation of the computed mean values: owing to the absence of a normalization factor $1/\mathcal{V}$, where $\mathcal{V}$, the four-volume of space, corresponds to the volume of the group of translations, densities are now lifted to global quantities. The cosmological constant is correctly predicted without fine tuning, despite the lack of low-energy supersymmetry, because the string vacuum energy expectation value, in our case of order one [1], does not correspond to an energy density, but to a quantity that, in order to be transformed into a density, must be rescaled by a Jacobian accounting for the coordinate transformation from the string to the Einstein's frame; this introduces a suppression corresponding to a two-volume, the square of the radius of space-time. The so produced true density is therefore $\Lambda \sim 1/R^2 \sim H^2$, where H is the Hubble constant. The

[1] Indeed it is precisely 1 by choice of normalization of the mean values.

cosmological constant is correctly predicted in its present day value, and it turns out to be not at all a constant, but it evolves with the inverse square of the age of the universe. The masses of the various particles arise as momenta of a space in which no coordinate is infinitely extended. These momenta have typically the size of some root of the inverse of the radius of the classical universe, i.e. the radius of the horizon, or, equivalently, the age of the universe. They originate from a pure string mechanism, they are not introduced via a Higgs mechanism. There is no Higgs field. Masses are produced by shifts along the space-time coordinates, that lift the ground energy of a particle. These shifts have a non-trivial effect because space-time is compact. The values of masses indeed correspond to the experimental masses of the elementary particles.

As discussed in [1], a consequence of the missing space-time translational invariance is also that the energy of the universe in not conserved: the energy density of the universe can be seen to scale, like the cosmological constant, as the inverse square of its age. Indeed, here we can see that the values of the energy, matter and cosmological densities are of the same order of magnitude as a consequence of a symmetry of the string configuration, which is broken in a mild way. A scaling of these quantities like the inverse square of the age of the universe implies that the total energy of the universe scales as its radius.

Inserting the values of the three energy densities in a FRW Ansatz for the universe, we can then solve the equations and obtain the large scale geometry of space-time. As it could have been argued from the fact that the horizon is "stretched" from the expanding light rays, the expansion of the universe turns out to be not accelerated. Nevertheless, to our observations it appears to be accelerated: what we observe is in fact a time-dependent red-shift effect, whose time variation, that could be interpreted as due to an accelerated expansion, is produced by a time-variation of the energy and matter scales.

The chiral nature of the weak interactions too is a consequence of the shifts along the space coordinates, which not only give origin to masses but at the same time also break parity. The separation of the matter world into weakly and strongly coupled is instead a consequence of the breaking of a T-duality along one internal coordinate. From a low-energy effective point of view, this appears as an S-duality. Indeed, the shifts that give rise to masses break not only parity and time reversal, but also explicitly the group of space rotations. This agrees on the other hand with our experience of everyday in the macroscopic world: the presence of objects, i.e. clusters of energy, in the space breaks the invariance under a change in the direction of observation. These symmetries, which appear to be broken at a macroscopic level, are here broken also in the fundamental description: the two levels are therefore sewed together without conceptual separation.

Matter is basically non-perturbative: in Planck units, the coupling of the matter sector is one. From this ground value depart the electro-magnetic, weak and strong coupling; the first two running, as the space-time volume increases, toward lower values; the third one toward higher values. This poses a fundamental problem to the investigation of the matter degrees of freedom, due to the fact that there are particles which feel both strong and weak interactions. An explicit, perturbative representation of particles as elementary states can only be realized through an expansion around a vanishing ground coupling. Since couplings unify only at the Planck scale, i.e. when we can no more speak of "low energy world", as a matter of fact there is no scale at which all the matter degrees of freedom appear all at the same time as perturbative. If we want to see the matter degrees of freedom explicitly represented in a perturbative construction, as they appear in usual field theory models, we must go to a picture in which the internal coordinates are "decompactified". In general, owing to the presence of some amount of T-duality, through decompactification we can have access only to a part of the full theory. A decompactification is non-singular and preserves all the properties of the physical vacuum only if the space under question is flat. Otherwise, as in the cases of interest for us, it is only an approximation, corresponding to considering just the tangent space around a certain point [2]. We may call this a "logarithmic representation" of the physical space, in which a coupling of order one is mapped into its logarithm, therefore of order zero. In a logarithmic representation of space, i.e. on the tangent space, the vanishing of the ground coupling implies also the vanishing of the tuning parameter of the supersymmetry breaking. As a result, the linearized, perturbative representation, in which particles show up as elementary states, appears to be supersymmetric, as is the case of many perturbative approaches to string and field theory. On the other hand, in the real world there is no regime in which both leptons and quarks appear at the same time as elementary, weakly coupled, free, asymptotic states [3].

For the computation of masses and couplings, we use the fact that, in order to be a representation of the combinatorial scenario of 1.1,the entire dynamics of the system is determined by the entropy of the various processes in the phase space of all the configurations. Since couplings and masses determine the interaction and decay probability of particles and fields, masses and couplings must be related to the volume occupied in the phase space by the corresponding matter and field degrees of freedom. The problem of computing these parameters in therefore translated to the one of computing the fraction of phase space these particles and interactions correspond to. This allows us to determine the "bare" mass and coupling values, i.e. the parameters which are usually considered as external inputs in any effective action. Our approach is somehow reversed with respect to the traditional one. Usually, the parameters and terms of the effective action are used in order to compute the full bunch of interactions. From a general point of view, these can be seen as "paths" coming out from, or leading to, a particle, or in general a physical state. Their number and strength can be considered therefore a measure of the entropy of the state: the higher is the mass of a particle, the higher is its interaction/decay probability, because higher is the number of final states it can decay to. Entropy is then computed as a function of the interaction/decay probability, in turn determined by the dynamics. In our approach, things go the other way

[2] This is for instance the case of the so-called non-compact orbifolds.

[3] An artificial linearization of space-time is the cause of another false appearance of the string vacuum, namely the fact that from some respects string theory seems to require for its complete description more than 11 coordinates. Indeed, the 12-th coordinate should be better viewed as a curvature. As is known, we can represent an n-dimensional curved space in n dimensions, with an "intrinsic" curvature, or we can embed it in a $n+1$-dimensional flat space. The degrees of freedom are in any case the same, because in the second case we don't consider the full $n+1$-dimensional space, but an n-dimensional sub-manifold. A perturbative representation of M-theory is something of this kind: when the vacuum corresponds to a curved space, by patching dual representations we have the impression that more than eleven coordinates are required in order to describe the full content, because these representations are necessarily perturbative and therefore built on a flattened, "tangent space".

around: it is the dynamics which, consistently with 1.1, is viewed as being determined by entropy.

Couplings and masses turn out to scale as powers of the inverse of the age of the universe. They naturally unify at the Planck scale. There is no much to be surprised by the fact that, in usual field theory models, supersymmetry seems to improve the scaling behavior of couplings, making possible their unification. If they correspond to a "logarithmic" representation of the physical vacuum, couplings unify because, roughly speaking, they are logarithms of functions which unify. In our framework, a logarithmic scaling shows up if we want to compare the "bare" values we obtain, with the parameters of a low-energy effective action, in which space-time is considered as infinitely extended. In order to make contact with the literature, this step in unavoidable: it happens in fact quite often that data of experimental observations are given as the result of elaborations carried out within a certain type of theoretical scheme. In particular, this is the case of effective couplings and masses run to the typical scale of a physical process. In this case, passing from a large but anyway finite space-time volume to an infinitely extended one results in a "mild", logarithmic correction to the "bare" mass, or coupling. Logarithmic corrections work in this case for small displacements in the "tangent space". The bare parameters are instead derived in the full space, and their running is exponential with respect to the one on the tangent space.

In general, any contact between our computations and the data found in the literature must be established at the level of experimental observations, rather than on effective action parameters, whose derivation always depends on a specific theoretical scheme. Therefore, to be rigorous, better than effective couplings one should directly consider scattering amplitudes and decay ratios; one should explain the emitted frequency spectra rather than trying to match given acceleration parameters of galaxies, and so on. This requires a deep change of perspective and a thorough re-examination of any known result. On the other hand, in all the cases a prejudice-free re-analysis of already known results is carried out, we find that our theoretical framework provides us with a consistent scheme. Although almost any physically observable quantity receives a different explanation than in traditional field theory or cosmology approaches, it is nevertheless consistent with what is experimentally measured. Indeed, precisely the high predictive power of this theoretical scenario, due to the fact that there are no free parameters that can be adjusted in order to fit data, enhances the strength of any matching with experimental results: any discrepancy could in fact rule out the entire construction. Because of this, a large part of our investigation is devoted to re-analyzing the most important data and constraints, coming not only from elementary particles physics but also from astrophysics and cosmology. Our predictions and results are compatible with any experimental datum we have considered, within the degree of approximation introduced in our derivation.

Among the highlights of the predictions, there is the absence of so-called "new-physics" (i.e. new particles or interactions) below the Planck scale. This does not mean that no stringent tests can come from future high energy experiments: the quantum mechanics arising from 1.1 implies departures from the quantum behavior expected in the traditional models of electro-dynamics and weak interactions, especially in relativistic systems characterized by a high geometric complexity. These effects can be viewed as due to quantization of space, as implied in a quantum gravity scenario. When one thinks at the contribution of gravity, one usually has in mind the contribution of *classical* gravity, and thinks at quantization as something that involves only the description of the graviton as a quantum field. Indeed, quantum gravity implies much more: it concerns the quantization of the geometry, and its effects show up in any complex system. For instance, as discussed in Ref. [2], it allows to explain the correlation between complexity of the lattice structure and critical temperature in high-temperature superconductors. Similarly, it predicts at high energy a higher degree of non-locality of wave-functions, something that should show out in a higher degree of correlation among products of a high energy collision [4]. Apart from the absence of a Higgs field and the fact of being the acceleration of the expansion of the universe only apparent, other phenomenological implications of our scenario that depart from what commonly expected are to be found in the so-called "time variation of the fine structure constant α"; and in the non-existence of dark matter. In this scenario, for each of these phenomena the explanation relies in the particular evolution of mass scales and couplings, as functions of the age of the universe.

One of the things that characterize this scenario is its high predictive power, due to the fact that there are no free parameters other than the age of the universe itself. Any measurable quantity is determined as a function of the age of the universe. It is therefore not a trivial fact that, besides its success in producing values of observables (masses, couplings, CP violation parameters etc...) correct within the degree of approximation we used, this scenario also survives to the stringent tests coming from cosmology, which is well known for imposing severe constraints on model building.

1.1 The outline of the work

The work consists for a large part in an update of [3], and is organized as follows. We start in section 2 by investigating the string phase space through orbifold constructions. The configuration of highest entropy, which, as discussed in [1], is also the one of minimal symmetry, is in this subclass of constructions the one with the highest amount of twists and shifts. In particular, we re-derive the result of [1] about the number of dimensions of space-time, here identified with the number of coordinates which remain untwisted, and therefore free to expand. We review also the discussion of [1] about the number of dimensions in which the non-perturbative string theory lives, in the light of non-perturbative string-string dualities between orbifold constructions (section 2.1). Knowing the whole number of dimensions will turn out useful in order to compute the neutron mass in section 4.4.1. After a discussion of the origin of masses and the observable spectrum of the theory, we pass to the breaking of the Lorentz invariance, in particular of the subgroup of space rotations (section 2.3), which explains the observed slight inhomogeneities of the universe when observed in different directions. We comment then the issues related to the magnetic monopoles, in particular the topological ones, which in this scenario are expected to not exist. We conclude the section with a discussion (subsection 2.5) of the meaning of effective action in our framework of compact space with time-dependent volume. In principle it is not obvious that even approx-

[4] It indeed seems that effects of this kind are being detected at LHC.

imately an effective action for our theory can be written at all; it indeed turns out to make sense, even though only as an approximation, because of the breaking of T-duality, which allows to speak of "extended" space-time.

In section 3 we discuss the geometry of the universe. Interestingly, the curvature is given by the sum of all energy densities, namely the cosmological one and those corresponding to matter and radiation. As discussed in [1], the contribution to the curvature of space due to the configuration of highest entropy (which can be viewed as the "classical" one) is of the same order as the sum of the contributions of all the other configurations. According to the discussion of section 7.4 of [1], the part of the curvature which can be seen to correspond to the pure geometry as described by light rays intended in the classical sense amounts to one-third of the actual geometry of the universe. This means that the value of the curvature receives contribution also from non purely "classical" objects. These were interpreted in [1] as quantum corrections, corresponding to matter and radiation, i.e. the degrees of freedom that propagate in the space. Here we finally see, in the light of a string scenario, how this is related to a non completely broken symmetry, which exchanges radiation, matter and gravity. We discuss then the type of expansion the universe undergoes: it results to be non accelerated, in agreement with the properties of the universe as derived through the combinatorial approach discussed in [1].

In section 4 we pass then to the detailed derivation of the couplings, the masses of the elementary particles, and of the massive bosons. All these quantities are given as functions of the age of the universe. We derive then the scaling of the mean mass scale of the universe, a quantity that can be non-perturbatively computed in an exact way: it corresponds in fact to the only eigenvalue of the Hamiltonian at any finite space-time volume. This scale can be seen to basically correspond to the mass of stable matter: if the matter present in the universe was constituted by particles all of the same kind, these would have a mass precisely corresponding to this scale. This scale can be shown to roughly correspond to the neutron mass. With the scaling of the average mass of the universe at hand we can discuss the issue of the apparent acceleration of the universe, and show that in this scenario of non-accelerated expansion the acceleration of red-shifts one observes can be explained as due to the variation with time of the atomic energy levels, and consequently of emitted light. In particular, we comment this point of view in comparison to the usual approach to the acceleration of the expansion of the universe. We discuss then the running of couplings, and the issue of their unification. We conclude the section with some comments about approaches which share something in common to our one, such as those based on the geometric probability.

In section 5 we come to an explicit evaluation of masses and couplings at present time. We discuss the degree of approximation under which these values are obtained, and give a rough estimate of the corrections they would receive if the string vacuum was known with a better accuracy. In particular, in section 5.4 we compute the fine structure constant (indeed its present-day, value because in our case it is not a constant), obtaining a value which falls within an error of $\sim 5 \times 10^{-6}$ away from its most updated experimental value. In section 5.5 we briefly discuss also baryon and meson masses. Also their values agree, within the approximations introduced in the computing procedure, with what experimentally measured (in the case of neutrinos, our computation remains a not yet tested prediction).

The investigation of the mass sector of the theory is completed in section 6, where we consider the mixing angles of weak decays (the Cabibbo-Kobayashi-Maskawa matrix) and the CP violations. Since in our scenario neutrinos are massive, mixing of generations and off-diagonal decays are expected to occur also among leptons.

If the present-time value is useful for a comparison of our predictions with the values of masses and couplings experimentally measured in accelerators or in general in a laboratory, knowing their behavior along the history of the universe allows us to test the predictions of this theoretical framework also in the case of astrophysical and cosmological observations. In section 7 we consider the "Cosmic Microwave Background" radiation, and discuss how the existence of a $\sim 3\,^0$ Kelvin radiation comes out as a prediction in this framework. We discuss then also, in subsection 7.2, the case of dark matter. In our scenario, this is expected to not exist. We comment several cases which are usually considered to provide evidence for its existence, and propose how, within our framework, in each of them the effects attributed to dark matter receive an alternative explanation. In section 8 we discuss then the constraints on the evolution of masses and couplings coming from the observation of ancient regions of the universe, or, as is the case of the Oklo bound, from the history of our planet. We find out that the predicted behavior is compatible with all the constraints. Not only, but in the case of the so-called "time dependence of α", it turns out to correctly predict the magnitude of the observed effect (section 8.1).

2 The non-perturbative solution

The integral 1.2 contains in principle all the informations about our universe. As discussed in [1], section 6.2, the volume V can be put in relation with $\mathcal{T}$, the age of the universe. At any time $\mathcal{T}$ the main contribution to the appearance of the universe is given by the configuration of minimal symmetry, $\psi^{\min}(\mathcal{T}) \equiv \psi^{\min}(V)$, the configuration that dominates in the integral 1.2 because it has at any time the highest entropy in the phase space at fixed volume. The contribution to the mean values coming from *all* the other configurations gives a correction of the order of the Heisenberg's Uncertainty, and therefore, in the whole, becomes comparably smaller and smaller as the universe expands: $\mathcal{T}, V \to \infty$. $\psi^{\min}(\mathcal{T})$ is therefore at any time the configuration that best corresponds to the universe as it is experimentally observed. We stress however that $\psi^{\min}(\mathcal{T})$ is not a solution in a classical sense, but only the (largely) dominant configuration, that dominates the more and more at larger times/temperatures. Our aim is to investigate the physical content of $\psi^{\min}(\mathcal{T})$. To this purpose, we will use a "perturbative" approach. In ordinary quantum field theory one separates the time evolution into a free propagation and an interaction part. The physical configurations are inspected via the conceptual separation of a base of free states, eigenstates of the free Hamiltonian, which are exact solutions of the free theory. As long as the coupling of the interaction is small, the full solution can be considered as a small perturbation of the free propagation, and the perturbative approach makes sense. In our case, we have a truly non-perturbative string system, in which even the space-time is mixed up, and in general will not be factorisable into an extended one, "the" space-time as we experience it, and an internal space. Moreover, we can access to the whole theory only through "slices", the perturbative (string) constructions, to be treated as the patches, the "projections", which allow to shed light into the "patchwork", the whole theory. We will get information about the true vacuum through heavy use of string-string duality, and follow the process of symmetry reduction through the spectrum of possible string constructions in the class of orbifold contructions.

Orbifolds are particular string constructions in which we have, at any energy level, full knowledge about the spectrum of the perturbative states. We are therefore able to write the partition function, the "one loop partition function", which in principle encodes all the information about the construction. With the string orbifold partition function it is possible to perform one-loop computations of scattering amplitudes and threshold corrections, and therefore compare string duals through pure string computations. The orbifold point is however in general a point of enhanced symmetry. As we will discuss, degeneracies are removed by mass differentiations among the states that possess a sub-Planckian mass. The latter turns out to depend on the (average) radius R of the extended space, by definition the only part of the string space of $\psi^{\min}(\mathcal{T})$ whose coordinates result to be non-twisted. Mass corrections cannot be investigated in detail at the orbifold point, and will be treated as a perturbation. These corrections will turn out to be of order $\sim \mathcal{O}\left(1/R^{\beta}\right)$, $0 < \beta < 1$. Masses become equal in any perturbative limit, which is a limit of decompactification of some coordinate. On the other hand, a perturbative investigation of $\psi^{\min}(\mathcal{T})$ makes sense because full theory and perturbative limits differ by the value of sub-Planckian masses, not by the nature of the spectrum: no new particles or fields are generated or lost in taking such limits.

In the search for the orbifold vacuum that corresponds to the highest entropy in the phase space, Z_2 orbifolds will be the most favoured ones, because they mod-out the space by the group with the smallest volume among all the orbifold operations. A product of Z_2 twist/shifts allows therefore to achieve a configuration whith a smaller surviving symmetry group than those obtained through any other product of orbifold operations. Entropy will therefore be the maximal we can obtain with orbifold operations. The most entropic orbifold vacuum will be the one with the highest amount of freely and non-freely acting Z_2 shifts and twists. Unfortunately, this configuration can be constructed explicitly only in a perturbative regime. This corresponds to the decompactification of some coordinate, which serves as coupling of the theory. As a consequence, it will never show explicitly all the properties of the whole theory. The latter can only be indirectly inferred through the comparison of dual constructions. In particular, supersymmetry will appear in a different way, depending on whether the decompactified coordinate does also tune the supersymmetry breaking, or not.

2.1 Investigating orbifolds through string-string duality

Investigating the non-perturbative properties of a string vacuum by comparing dual constructions is neither an easy task, nor a straightforward one. In general, at a generic point in the moduli space the full set of dual constructions, enabling to "cover" the full content, is not known. Some progress on its knowledge has been done in the case of supersymmetric vacua with extended supersymmetry, where it is in general possible to identify a subset of the spectrum made "stable" by the properties of supersymmetry. The case of orbifolds turns out to be particularly suited for the investigation of non-perturbative string-string dualities. In this case it is possible to make a non-trivial comparison of the renormalization of terms that receive contributions only from the so called BPS states, and this not just on the ground of the properties of supersymmetry, but through the computation of true string contributions. Fortunately, Z_2 orbifolds, the case of our interest, are the easiest and therefore more investigated constructions [5]. Indeed, through the analysis of these constructions, it is possible to get an insight into the properties which are typical of string theory in itself: most of the investigations performed at other points in the moduli space must in fact rely on geometrical properties of smooth surfaces, and their singularities. Although for some respects rather powerful, these techniques don't allow to capture the presence of states related to non-geometrical singularities, or even fail in general for the simple reason that, owing to T-duality, the full string space simply cannot be reduced to a geometrical one[6]. Our starting point will be a maximally supersymmetric string vacuum with flat background: in our approach, the curvature of space-time will come out as an output, it is not an external input of the theory. The constraints of two-dimensional conformal field theory impose that Z_2 orbifold twists must act on groups of four coordinates at once. In any string construction, there is room for a maximum of 3 such operations, one of which is however redundant, in that

[5]See for instance Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

[6]For examples, see for instance Ref. [15].

it leads, once combined with the other ones, to the re-introduction in the twisted sectors of the states projected out. Therefore, we can say that only a maximum of two independent Z_2 twists act effectively. However, the amount of supersymmetry surviving to these projections, as well as the amount of initial supersymmetry, is different, depending on whether we start with heterotic, type I, or type II strings. This means that in any construction not all the projections acting on the theory are visible. Indeed, one of them is always non-perturbative. The reason is that, by definition, a perturbative construction is an expansion around the zero value of a parameter, the coupling of the theory, which is itself a coordinate in the whole theory. An orbifold operation acting on this coordinate is forcedly non-perturbative [7].

Our aim now is to derive the structure of the vacuum of lowest symmetry. Dual constructions correspond in general to different perturbative limits, i.e. decompactifications of different coordinates. Essential for our work is that these decompactifications must be possible; for this to make sense, the involved coordinate(s) must not be twisted. Even in the case they are shifted, under decompactification the scenario gets "trivialized" and we loose information about the construction. This problem does not exist for the first steps of symmetry reduction, because it is always possible to decompactify a coordinate not involved in some orbifold operation. Things are different when we reach the maximum of orbifold operations. As we will see, for the configuration of lowest symmetry, a decompactification limit is a trivialization of the theory, under which some physical content is lost: a perturbative treatment of the degrees of freedom does not correspond in this case anymore to a simple "limit" of the theory, but to a phase which can be obtained only through a "logarithmic mapping" of the coordinates of the physical vacuum. This will be a key point of the entire discussion of masses and supersymmetry breaking. In this section we are not concerned with this problem; in the purpose of understanding what is the singularity structure, we proceed, as in Ref. [36], by first "counting" the Z_2 operations in the various decompactification limits, as long as these can be taken. By the way, we remark that it is precisely thanks to this limiting procedure that it is possible to construct supersymmetric string constructions: in a fully compact space-time supersymmetry is always broken.

In the following we will often make use of the language of string compactifications to four dimensions, especially for what matters our reference to the moduli of the string orbifolds. This will turn out to be justified "a posteriori": we will see that indeed the final configuration is the one of a string space with all but four coordinates twisted and therefore "frozen". Only four coordinates remain un-twisted and free to expand, while all the others remain stuck at the string/Planck scale. Massless degrees of freedom move along these and expand the horizon of space-time at the speed of light. Although not infinitely extended, this "large" space is what in our scenario corresponds to the ordinary space-time. The language of orbifold constructions in four dimensions is therefore just an approximation, that works particularly well at large times. Only at a second stage, we will also discuss how and where this picture must be corrected in order to account also for compactness of the space-time coordinates. Although somehow an abuse of language, this approximation allows us to take and use with little changes many things already available in the literature. In particular, for

[7] A first investigation of a non-perturbative orbifold, which produces the heterotic string, has been carried out in [34, 35].

several preliminary results and a rediscussion of the previous literature, the reader is referred to Ref. [15].

Let's see what are in practice the steps of decreasing symmetry we encounter when approaching the most singular configuration. Although at the end it will be irrelevant the order in which we apply freely and non-freely acting orbifold operations, it is convenient to organize the analysis by considering first non-freely acting operations, i.e. pure twists with orbifold fixed points. Starting from the M-theory configuration with 32 supercharges, we come, through orbifold projections, to 16 supercharges and a gauge group of rank 16. Further orbifolding leads then to 8 supercharges ($\mathcal{N}_4 = 2$) and introduces for the first time non-trivial matter states (hypermultiplets). As we have seen in [15] through an analysis of all the three dual string realizations of this vacuum (type II, type I and heterotic), this orbifold possesses three gauge sectors with maximal gauge group of rank 16 in each. The matter states of interest for us are hypermultiplets in bi-fundamental representations: these are in fact those which at the end will describe leptons and quarks (all the others are eventually projected out). As discussed in [15], in the simplest formulation the theory has 256 such degrees of freedom. The less symmetric configuration is however the one in which, owing to the action of further Z_2 shifts, the rank is reduced to 4 in each of the three sectors. These operations, acting as rank-reducing projections, have been extensively discussed in [37, 11, 12, 15]. The presence of massless matter is in this case still such that the gauge beta functions vanish. In this case, the number of bi-charged matter states is also reduced to $4 \times 4 = 16$. These states are indeed the twisted states associated to the fixed points of the projection that reduces the amount of supersymmetry from 16 to 8 supercharges.

Let's consider the situation as seen from the type II side. We indicate the string coordinates as $\{x_0, \ldots, x_9\}$, and consider $\{x_0, x_9\}$ the two longitudinal degrees freedom of the light-cone gauge. The transverse coordinates are $\{x_1, \ldots, x_8\}$. Here all the projections appear as left-right symmetric. The identification of the degrees of freedom, via string-string duality, on the type I and heterotic side depends much on the role we decide to assign to the coordinates, as we will see in a moment. By convention, we choose the first Z_2 to twist $\{x_5, x_6, x_7, x_8\}$:

$$Z_2^{(1)}: \quad (x_5, x_6, x_7, x_8) \to (-x_5, -x_6, -x_7, -x_8)\,, \tag{2.1}$$

and the second Z_2 to twist $\{x_3, x_4, x_5, x_6\}$:

$$Z_2^{(2)}: \quad (x_3, x_4, x_5, x_6) \to (-x_3, -x_4, -x_5, -x_6)\,. \tag{2.2}$$

These two projections induce a third one: $Z_2^{(1,2)} \equiv Z_2^{(1)} \times Z_2^{(2)}$, that twists $\{x_3, x_4, x_7, x_8\}$:

$$Z_2^{(1,2)}: \quad (x_3, x_4, x_7, x_8) \to (-x_3, -x_4, -x_7, -x_8)\,. \tag{2.3}$$

Altogether, they reduce supersymmetry from $\mathcal{N}_4 = 8$ to $\mathcal{N}_4 = 2$, generating 3 twisted sectors. Depending on whether we consider the type IIA or IIB construction, the twisted sectors give rise either to matter states (hyper-multiplets) or to gauge bosons (vector-multiplets). As we discussed in Ref. [15], a comparison with the heterotic and type I duals shows that the underlying theory must be considered as the union of the two realizations: owing to the lack of a representation of vertex operators at once perturbative for all of them, for technical reasons no one of the constructions is able to explicitly show the full content of this vacuum. The matter (and gauge) content in these sectors is then reduced by six Z_2 shifts acting, two by two, by pairing each of the three twists of above with a shift along one of the two coordinates of the set $\{x_1, \ldots, x_8\}$ which are not twisted. Each shift reduces the number of fixed points of a Z_2 twist by one-half; two shifts reduce therefore the matter states of a twisted sector from 16 to 4. Altogether we have then, besides the $\mathcal{N}_4 = 2$ gravity supermultiplet, three twisted sectors giving rise each one to 4 matter multiplets (and a rank 4 gauge group). On the type I side, these three sectors appear as two perturbative D-brane sectors, D9 and D5, while the third is non-perturbative. On the heterotic side, two sectors are non-perturbative. As it can be seen by investigating duality with the type I and heterotic string, the matter states from the twisted sectors are actually bi-charged (see Refs. [38, 39], and [15]), something that cannot be explicitly observed, the charges being entirely non-perturbative from the type II point of view. The moduli $T^{(1)}$, $T^{(2)}$, $T^{(3)}$ of the type II realization, associated respectively to the volume form of each one of the three tori $\{x_3, x_4\}$, $\{x_5, x_6\}$, $\{x_7, x_8\}$, are indeed "coupling moduli", and correspond to the moduli "S", "T", "U" of the theory. On the heterotic side, S is the field whose imaginary part parametrizes the string coupling: $\mathrm{Im}\, S = \mathrm{e}^{-2\phi}$. It is therefore the coupling of the sector that contains the gravity fields. T and U are perturbative moduli, and correspond to the couplings of the two non-perturbative sectors. On the type I side, on the other hand, two of them are non-perturbative, coupling moduli, respectively of the D9 and D5 branes, while only one of them is a perturbative modulus, corresponding to the coupling of a non-perturbative sector [38, 40, 41, 26]. Owing to the artifacts of the linearization of the string space provided by the orbifold construction, gravity appears to be on a different footing on each of these three dual constructions.

2.1.1 The maximal twist

The configuration just discussed constitutes the last stage of orbifold twists at which we can "easily" follow the pattern of projections on all the three types of string construction. It represents also the maximal degree of Z_2 twisting corresponding to a supersymmetric configuration. As we will see, a further projection necessarily breaks supersymmetry. The vacuum appears supersymmetric only in certain dual phases, such as the perturbative heterotic representation. Non-perturbatively, supersymmetry is on the other hand broken. This means that, when further twisted, the theory is basically no more de-compactifiable: perturbative, i.e. decompactification, phases, represent only approximations in which part of the theory content and properties are lost, or hidden. This is what usually happens when one for instance pushes to infinity the size of a coordinate acted on by a Z_2 twist. The situation is the one of a "non-compact orbifold".

The further Z_2 twist we are going to consider is also the last that can be applied to this vacuum, which in this way attains its maximal degree of Z_2 twisting. This operation, and the configuration it leads to, appears rather differently, depending on the type of string approach. Let's see it first from the heterotic point of view. So far we are at the $\mathcal{N}_4 = 2$ level. The next step appears as a further reduction to four supercharges (corresponding to $\mathcal{N}_4 = 1$ supersymmetry). Of the previous projections, $Z_2^{(1)}$ and $Z_2^{(2)}$, only one was realized explicitly on the heterotic string, as a twist of four coordinates, say $\{x_5, x_6, x_7, x_8\}$. The further projection, $Z_2^{(3)}$, acts on another four coordinates, for instance $\{x_3, x_4, x_7, x_8\}$. In this way we generate a configuration in which the previous situation is replicated three times. When considered alone, the new projection would in fact behave like the previous one, and produce two non-perturbative sectors, with coupling parametrized by the moduli of a two-torus, in this case $\{x_5, x_6\}$: $T^{(5-6)}$, $U^{(5-6)}$. The product $Z_2^{(1)} \times Z_2^{(3)}$ leaves instead untwisted the torus $\{x_7, x_8\}$ and generates two non-perturbative sectors with couplings parametrized by the moduli $T^{(7-8)}$, $U^{(7-8)}$. Altogether, apart from the projection of states implied by the reduction of supersymmetry, the structure of the $\mathcal{N} = 2$ vacuum gets triplicated.

The symmetry of the action of the additional projection with respect to the previous ones suggests that the basic structure of the configuration, namely its repartition into three sectors, S, T, U, is preserved when passing to the less supersymmetric configuration. This phenomenon can be observed in the type II dual, that we discuss in detail in Appendix B. From the heterotic point of view, the states of these sectors come replicated ($\{T\} \to \{T^{(3-4)}, T^{(5-6)}, T^{(7-8)}\}$, $\{U\} \to \{U^{(3-4)}, U^{(5-6)}, U^{(7-8)}\}$). On the type II side we observe a triplication also of the "S" sector. However, as we discussed in Ref. [15], we are faced here to an artifact of the orbifold constructions, that by definition are built over a linearization of the string space into planes separated by the orbifold projections. The matter states are indeed charged under three sectors, S^i, T^j, U^k, but we can at most observe a double charge, as it appears on the type I dual side; from an analysis based on string-string duality, we learn that the states are in fact multi-charged for mutually non-perturbative sectors When one of the S^i, T^j, U^k sectors is at the weak coupling, the other two are at the strong coupling, and it doesn't make sense to ask what is this sort of "splitting" of the non-perturbative charge of the states: we simply observe that they have a perturbative index and one running on a strongly coupled part of the theory.

On the type I dual realization of this vacuum, besides a D9 branes sector we have now three D5 branes sectors and a replication of the non-perturbative sector into three sectors, whose couplings are parametrized by $U^{(3-4)}$, $U^{(5-6)}$, $U^{(7-8)}$.

A result of the combined action of these projections is that all the fields S^i, T^j and U^k are now twisted. This means that their vacuum expectation value is not anymore running, but fixed. We will see below, section 2.1.2, that minimization of entropy selects this value to be of order one, thereby implying also the identification of string and Planck scale (section 2.5). Nevertheless, for convenience here we continue with the generic notation S, T, U used so far, because it allows to better follow the functional structure of the configuration we are investigating. Twisting of the "coupling" moduli indeed suggests the non-decompactifiability of this vacuum. This, as discussed, would imply the breaking of supersymmetry. However, this property is not so directly evident: each dual construction is in fact by definition perturbatively constructed around a decompactification limit. The point is to see, with the help of string-string duality, whether this is a real decompactification, or just a singular, non-compact orbifold limit. An important argument in favour of this second situation is that, after the $Z_2^{(3)}$ projection is applied, the so-called "$\mathcal{N} = 2$ gauge beta-functions" are unavoidably non-vanishing. According to the analysis of Ref. [15], this means that there are hidden sectors at the strong coupling [8]. As a consequence, supersymmetry is actually non-perturbatively broken, due to gaugino condensation. Inspection of the type II string dual shows explicitly the instability of the $\mathcal{N}_4 = 1$ supersymmetric vacuum.

In order to construct the type II dual, it is not possible to proceed as with the heterotic and type I string, namely by keeping un-twisted some coordinates. On the type II side the "$\mathcal{N}_4 = 1$" vacuum looks rather differently: the new projection twists all the transverse coordinates, leaving no room for a "space-time". This however does not mean that a space-time does not exist: all non-twisted coordinates, therefore the space-time indices, are non-perturbative. Their volume is precisely related to the size of the coupling around which the perturbative vacuum is expanded. After $Z_2^{(1)}$ and $Z_2^{(2)}$, the only possibility for applying a perturbative Z_2 twist is in fact to act on $\{x_1, x_2,\}$ and on two of the $\{x_3, \dots, x_8\}$ coordinates, already considered by the previous twists. These can be either the pair $\{x_3, x_4\}$ or $\{x_5, x_6\}$, or $\{x_7, x_8\}$. Which pair, is absolutely equivalent. We can chose $Z_2^{(3)}$ such that:

$$Z_2^{(3)}: \quad (x_1, x_2, x_3, x_4) \; \to \; (-x_1, -x_2, -x_3, -x_4)\,. \tag{2.4}$$

The other choices are anyway generated as $Z_2^{(3)} \times Z_2^{(2)}$ and $Z_2^{(3)} \times Z_2^{(2)} \times Z_2^{(1)}$. Assigning a twist to some coordinates is not enough in order to define an orbifold operation: the specification must be completed by an appropriate choice of "torsion coefficients". The analysis of this orbifold turns out to be easier at the fermionic point, where the world-sheet bosons of the conformal theory are realized through pairs of free fermions [42]. We leave to the appendix B a detailed discussion of the construction of this vacuum. There we see how the duality map with $\mathcal{N} = 2 \to \mathcal{N} \to 1$ heterotic theory imposes a choice of "GSO coefficients" that leads to the complete breaking of supersymmetry, and discuss, in appendix C, how the breaking is tuned by moduli which in this vacuum are frozen at the Planck scale. This is also therefore the scale of the supersymmetry breaking [9].

The reason why the breaking of space-time supersymmetry can be observed in a dual in which space-time is entirely non-perturbative relies on the unambiguous identification of the supersymmetry generators. More precisely, what on the type II side it is possible to see is the projection of the supersymmetry currents on the type II perturbative space. Target space supersymmetry is in fact realized in string theory through a set of currents whose representation is built out of the world-sheet degrees of freedom. For instance, in the case of free fermions in four dimensions, we have:

$$G(z) \; = \; \partial_z X^{\mu} \psi_{\mu} \; + \; \sum_i x^i y^i z^i \, , \tag{2.5}$$

and

$$G(\bar{z}) \; = \; \partial_{\bar{z}} X^{\mu} \bar{\psi}_{\mu} \; + \; \sum_i \bar{x}^i \bar{y}^i \bar{z}^i \, , \tag{2.6}$$

[8]We refer the reader to the cited work for a detailed discussion of this issue.

[9]Among the historical reasons for the search of low-energy supersymmetry are the related smallness of the cosmological constant and the stabilization in the renormalization of mass scales produced by supersymmetry. In our framework, the value of the cosmological constant will be justified in a completely different way (section 3). Also the issue of stabilization of scales in this framework must be considered in a different way: masses are no more produced by a field-theory mechanism, and field theory is not the environment in which to investigate their running.

where the index i runs over the internal dimensions. At the $\mathcal{N}_4 = 2$ level it is possible to construct both the representations of the type II dual, namely the one in which space-time is perturbative, and the one in which space-time is non-perturbative., Tracing the representation of the supersymmetry currents in both these pictures allows us to identify them also when the $Z_2^{(3)}$ twist is applied. Although, strictly speaking, there is no simple one-to-one linear mapping between coordinates of dual constructions, the fact that the dual representations of the currents share a projection onto a subset of coordinates common to both, enables us to follow the fate of space-time supersymmetry anyway.

The analysis of the type II dual confirms that the matter states of this vacuum are indeed three replicas of the chiral fermions of the theory before the supersymmetry-breaking, $Z_2^{(3)}$ projection. In the type II construction their space-time spinor index runs non-perturbatively; they appear therefore as scalars. In total, we have three sets of bi-charged states in a $\mathbf{16} \times \mathbf{16}$. In the minimal, semi-freely acting configuration, they get reduced to three sets of $\mathbf{4} \times \mathbf{4}$ by the further Z_2 shifts, acting on the twisted planes. As it was the case of the $\mathcal{N}_4 = 2$ theory, on the type II side their charges are non-perturbative, and they misleadingly appear as $(\mathbf{16}, \mathbf{16}, \mathbf{16})$, reduced to $(\mathbf{4}, \mathbf{4}, \mathbf{4})$. The impression is that we have three families of three-charged states. However, this is only an artifact of the orbifold construction. From the heterotic point of view, namely, the vacuum in which gauge charges are visible, two sectors of each family are non-perturbative and, as previously mentioned, the structure of their contribution to threshold corrections is an indirect signal that they are at the strong coupling (see Ref. [15]). The situation is the following: *either* we explicitly see all the gauge sectors, on the type II side, but we don't see the gauge charges, *or* where we can explicitly construct currents and see gauge charges (the heterotic realization), we see the gauge sector, and the currents, corresponding to just one index born by the matter states: the other are non-perturbative and strongly coupled.

The type II realization appears to be a different "linearization", or linear representation, of the string space, in which the non-perturbative curvature has been "flattened" through an embedding in a higher number of (flat) coordinates, which goes together with a redundancy of states due to an artificial replication of some degrees of freedom. On the type II string, twisted states can only be represented as uncharged, free states. Their charges are in any case non-perturbative, and we cannot observe a "non-abelian gauge confinement". These gauge sectors appear as partially perturbative on the type I side. However, the type I vacuum, like the heterotic one, corresponds to an unstable phase of the theory: it appears as supersymmetric although it is not. Moreover, inspection of the gauge beta-functions reveals that they are positive. Therefore, although appearing as free states, the currents on the D-branes run to the strong coupling and the apparent gauge symmetries are broken by confinement.

Let's **summarize** the situation. The initial theory underwent three twists and now is essentially the following orbifold:

$$Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)} \, . \tag{2.7}$$

In terms of supercharges, the supersymmetry breaking pattern is:

$$32 \xrightarrow{Z_2^{(1)}} 16 \xrightarrow{Z_2^{(2)}} 8 \xrightarrow{Z_2^{(3)}} 0 \ \ (4 \text{ only perturbatively}) \,. \tag{2.8}$$

The "twisted sector" of the first projection gives rise to a non-trivial, rank 16 gauge group; the twisted sector of the second leads to the "creation" of one matter family, while after the third projection we have a replication by 3 of this family. The rank of each sector is then reduced by Z_2 shifts of the type discussed in Ref. [11, 8, 12], two per each complex plane. As a result, each **16** is reduced to **4**. On the type II side one can explicitly see, besides the shifts, both the total breaking of supersymmetry and the doubling of sectors under which the matter states are charged. The product of these operations leads precisely to the spreading into sectors that at the end of the day separate into weakly and strongly coupled, allowing us to interpret the matter states as quarks [10]. On the type I side, the states appear in an unstable phase, as free supersymmetric states of a confining gauge theory, while on the heterotic side they appear on the twisted sectors, and their gauge charges are partly non-perturbative, partly perturbative. The perturbative part is realized on the currents. Like the type I realization, also the heterotic vacuum appears to be an unstable phase, before flowing to confinement; they are indeed non-perturbatively singular, non-compact orbifolds. This reflects on the fact that, as also discussed in Ref. [15], both on the heterotic and type I side, perturbative and non-perturbative gauge sectors have opposite sign of the beta-function. This signals that, as the visible phase is confining, the hidden one is non-confining.

2.1.2 Origin of four dimensional space-time

The product (2.7) represents the maximal number of independent twists the theory can accommodate: a further twist would in fact superpose to the previous ones, and restore in some twisted sector the projected states. Therefore, further projections are allowed, but no further twists of coordinates. These twists allow us to distinguish between "space-time" and "internal" coordinates. While the first ones (the non-twisted) are free to expand, the twisted ones are "frozen". The reason is that the graviton, and as we will see the photon, live in the non-twisted coordinates. Precisely the fact that graviton and photon propagate along these coordinates, and therefore "stretch", expand the horizon, allows us to perceive these as our "space-time". We get therefore "a posteriori" the justification of our choice to analyze sectors and moduli from the point of view of a compactification to four dimensions.

The radius at which the internal coordinates are twisted is fixed by the requirement of minimization of symmetry to be of the order of the string scale. In order to understand this, let's consider an ordinary, bosonic lattice, that for simplicity we restrict to just one coordinate. The partition function consists of a sum of two terms: the un-twisted/projected and the twisted/projected part. Roughly, the un-twisted part reads:

$$\mathcal{Z} \sim \int \left\{ 1 + \sum_{m,n \neq 0,0} \mathrm{e}^{-\tau[(m/R)^2 + (nR)^2]} \right\} \times |\eta|^{-1} \,. \tag{2.9}$$

[10]As we will discuss, the leptons show up as singlets inside quark multiplets.

The η factor encodes the contribution of the oscillators that build up a tower of states over the ground momentum and winding. The term of interest for us, the one that varies, is the factor within brackets. Even in the case a shift acts on this coordinate (as is in our case), what remains of the (broken) T-duality is enough to state that the string scale $R_0 \sim \mathcal{O}(1)$ is a point of minimal symmetry of the theory. This may seem odd, because it is somehow a self-dual point, therefore related to an "enhanced symmetry". However, even though there is a Z_2 enhancement of symmetry (at least if T-duality is not broken, something which is not really the actual case), the whole spectrum has, modulo Z_2, the highest differentiation. As the radius increases (resp. decreases beyond the extremal point), the probability of the states with non-zero winding (resp. momentum) number in fact decreases the more and more rapidly, while the probability of the momentum (resp. winding) states approaches the limit value:

$$P_m \sim \frac{\int \mathrm{e}^{-\tau\, m^2/R^2}}{\mathcal{Z}} \quad \stackrel{R \to \infty}{\longrightarrow} \quad \frac{\int 1}{\mathcal{Z}} \,. \tag{2.10}$$

Therefore, in the limit $R \to \infty$, for any fixed momentum number m_R, the tower of momentum states with momentum number $m < m_R$ "collapses" to nearly the same probability: many of the formerly well separated steps shrink to a "continuum" of states of nearly identical, maximal probability. Similarly goes for $R \to 0$ and the winding instead of momentum states. Therefore, as the radius departs from the extremal value $R = R_0$, either by increasing or by decreasing, the distribution of probabilities "collapses" toward two kinds of possibilities: half of the states tend to acquire the same, maximal probability, while the other half tend to disappear. From a well differentiated configuration, in which any energy level possessed a non-trivial probability, we move toward a situation of higher symmetry. These two decompactification limits lead either to a restoration of the broken symmetry, or to a configuration of purely projected, i.e. a non-freely acting orbifold, but without the additional states coming from the twisted sectors.

2.1.3 In how many dimensions does non-perturbative String Theory live?

We have seen that, with the maximal twisting, supersymmetry is broken. The string space is therefore necessarily curved. For some respects, this may seem strange. As we also discuss in Appendix B and C, even with the maximal twist, not all the string coordinates are twisted. At most, we have twisted eight of them. Why should then not be possible to decompactify one of the non-twisted ones, and obtain anyway a flat space? The fact that the space gets curved at the maximal twisting, even though this does not involve all the coordinates, is a property of the orbifold constructions; they are a kind of singular spaces, with a geometry which is flat everywhere apart from some singular points. The curvature is somehow all "concentrated" on these points. Although from a global point of view orbifold spaces are curved, locally they are almost everywhere flat. By the way, this is the reason why one can have the impression of being able to decompactify them and build a consistent supergravity theory at the decompactification limit [11]. Supergravity is a part of field theory, it is related

[11] This is also the case of Refs. [34, 35]: the Hořava-Witten analysis indeed obtains the Heterotic string in ten dimensions as a non-compact orbifold (see Appendix C).

to differential geometry concepts, it is a local theory. The signal that something illegal has been done comes from the investigation of the *pure stringy*, non-perturbative properties of the vacuum under consideration. The fact that with the $Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)}$ twisting the string orbifold space is curved can be clearly seen by considering the type II point of view. There, all the transverse coordinates are twisted. Not the coupling, which remains untwisted. Nevertheless, we are in the presence of a perturbative series in which, at any order in the expansion around the non-twisted coordinate(s), we have a fully twisted, curved space. This sums up therefore to a curved space; the perturbative contribution to the curvature cannot in fact be cancelled by a possible non-perturbative term: the terms should be cancelled order by order.

Besides the above mentioned twists/shifts, the only way left out to further minimize symmetry is to apply further shifts along the non-twisted coordinates. How many are they? From the type II point of view, there are no further, un-twisted coordinates. But we know that they are there, "hidden" as longitudinal coordinates eaten in the light-cone gauge and in the coupling of the theory. Some of these coordinates appear on the heterotic/type I side as *two* transverse coordinates. If we count the total number of twisted coordinates by collecting the information coming from intersecting dual constructions, and the coordinates which are "hidden" in a certain construction and are explicitly realized in a dual construction, we get the impression that the underlying theory possesses 12 coordinates. For instance, on the heterotic side we have a four-dimensional space-time plus six internal, twisted coordinates, and a coupling. On the type II side we see eight twisted coordinates. We would therefore conclude that the two additional twisted coordinates correspond to the coupling of the heterotic dual. On the other hand, no supersymmetric 12-dimensional vacuum seems to exist, at least not in a flat space: the maximal dimension with these properties is 11. This seems therefore to be the number of dimensions in which non-perturbative string theory is natively defined.

Let's have a better look at the properties of supersymmetry. As is known, the supersymmetry algebra closes on the momentum operator. When applied to the vacuum, we have:

$$\left\{ Q, \bar{Q} \right\} \approx 2\, M \,. \tag{2.11}$$

From a dimensional point of view, a mass can be viewed as the inverse of a length, so that we can also write:

$$< \left\{ Q, \bar{Q} \right\} > \cong \frac{1}{R} \,. \tag{2.12}$$

The supersymmetry algebra suggests that the mass on the right hand side of 2.11, in all respects an order parameter for the supersymmetry breaking, could be interpreted as the inverse of the length of a coordinate of the theory. This coordinate refers to an extra internal dimension, or, perhaps more appropriately, to a curvature, i.e. a function collecting the contribution of several coordinates, perturbative as well as non-perturbative. Precisely the fact that, in the breaking of $\mathcal{N}_4 = 2$ supersymmetry to $\mathcal{N}_4 = 1$, the dilaton and the other "coupling" fields get twisted, is a signal that a non-vanishing curvature of the string space has been generated. As we discussed in section 2.1.1, this means that, even in the case of infinite volume, we are in a situation of non-compact orbifold. Simply assigning a non-vanishing value to the coupling does not imply the generation of a non-vanishing curvature of the string space: the corresponding field must also be twisted. In the orbifold language, this is implemented by the fact that, whenever the coupling field is "explicitated" by going to a dual construction, the corresponding perturbative geometric field appears as a volume of a two-dimensional space. This phenomenon can be observed for reduced supersymmetry (for maximal supersymmetry, there is just the type II string construction). Consider for instance the eleventh coordinate of M-theory, that should correspond to the dilaton of the heterotic string. In the type II orbifold constructions (K3 orbifold compactifications), the heterotic coupling corresponds to a two-torus volume. Considering that this two-dimensional space corresponds, from the heterotic point of view, to "extra-coordinates", one would say that, in order to realize all these degrees of freedom, the full underlying theory should be (at least) twelve-dimensional. However, this is only an artifact of the linearization implied by the orbifold construction, and it means that the simple compactification on a circle is not enough, we need also an additional "curvature coordinate" in order to parametrize a truly curved space.

From the type II dual we learn that supersymmetry is not restored by a simple decompactification: the string space is twisted [12]. Flatness of the string space is broken by a "twist" of coordinates that fixes them to the Planck scale. As a consequence, the supersymmetric partners of the low-energy states are boosted above the Planck scale. In a situation of supersymmetry restoration, they should come down to the same mass as the visible world, and space should become "flat". However, this is only possible when the twist is "unfrozen" and we can take a decompactification limit, such as for instance the M-theory limit. Otherwise, at the decompactification limit the space becomes only locally flat (non-compact orbifold). In order to get a true flat space we must take out the "point at infinity" (the "twist"), which closes the geometry to a curved space.

Let's collect the informations so far obtained:

1. As soon as the string space is sufficiently twisted, supersymmetry is broken.
2. Equations 2.11 and 2.12 suggest in this case a non-vanishing curvature of space.
3. In the class of orbifolds, the phenomenon of curving the string space can only be partially and indirectly seen, through the comparison of dual constructions.
4. These constructions are built on a (perturbatively) flat, supersymmetric background: they provide therefore "linearizations" of the string space.
5. The maximal dimension of a supersymmetric theory on a flat background is 11.

[12] In some type II/heterotic duality identifications, the heterotic coupling is said to correspond to un-twisted coordinates of the type II string. This however does not change the terms of the problem: in the artifacts of the flattening implied by the orbifold constructions, part of the curvature may be "displaced", or referred, to some or some other coordinates. This "rigid" distribution of the twists, basically dictated by the need of recovering a description in terms of supergravity fields referring to the same space-time dimensionality for both the dual constructions, may induce to misleadingly conclusions. The intrinsic twisted nature of the space has to be considered by looking at the string space in its whole (for more details and discussion, see for instance Ref. [15]).

All this suggests that, when supersymmetry is broken, we are in the presence of an eleven-dimensional *curved* background. Any, forcedly perturbative, explicit orbifold realization requires for its construction a linearization of the background. Since a 11-dimensional curved space can be embedded in a 12-dimensional flat space, we have the impression of an underlying 12-dimensional theory. However, this is only an artifact; in fact, we never see all these 12 flat coordinates at once: we infer their existence only by putting together all the pieces we can explicitly see. But this turns out to be misleading: the linearization is an artifact.

The 12 dimensional background is only fictitious, we need it only in order to describe the theory in terms of flat coordinates. At the perturbative string level, of these coordinates we see only a maximum of 10.

These conclusions match with those of section 6.1.1 of Ref. [1]. The counting of twisted and un-twisted coordinates has to be considered from this point of view. Trading the two "space-time" transverse coordinates on the type II side for the coupling coordinates of "M-theory", as we did in section 2.1.2, doesn't mean that we really have two such coordinates: the only thing we know is that, taking the union of all the coordinates of the various dual constructions, in which this curved space appears to be linearized, one has to the impression to have up to 12 dimensions. Indeed, going to the type II picture is a trick enabling to explicitly see the instability of the $\mathcal{N}_4 = 1$ vacuum, by switching on the operation on the "hidden", non-perturbative part of the theory. As a matter of fact, we are however in the presence of a maximum of seven "twisted" coordinates, i.e. coordinates along which the degrees of freedom don't propagate, and four un-twisted ones, along which the degrees of freedom can propagate. By comparison of dual string vacua, we can see that there is room to accommodate two more "perturbative" Z_2 shifts: through the heterotic and/or type I realization, we can explicitly see only two transverse non-twisted coordinates, plus two longitudinal ones, along which no shift can act. It remains then one "internal", truly non-perturbative coordinate, to which no shift has yet been applied. This can only be indirectly investigated: if we try to explicitly realize it, it will appear as a set of two coordinates, giving the fake impression to have room for two independent shifts.

Let's now count the number of degrees of freedom of the matter states. We have three families, that for the moment are absolutely identical: each one contains $16 = 4 \times 4$ chiral fermions. These degrees of freedom are suitable to arrange in two doublets of two different $SU(2)$ subgroups of the symmetry group. Each doublet is therefore like the "up" and "down" of an $SU(2)$ doublet of the weak interactions, but this time with a multiplicity index 4. As we will see, this 4 will break into 3+1, the 3 corresponding to the three quark flavours, and the 1 (the singlet) to the lepton. The number of degrees of freedom is therefore the right one to fit into three families of leptons and quarks. However, so far all these fields are massless and charged under an $SU(2)$ symmetry. We will see how precisely shifts along the space-time coordinates lead to the breaking of parity of the weak interactions and to a non-vanishing mass for these particles.

2.1.4 *The origin of masses for the matter states*

Shifts applied to the "internal" coordinates reduce the symmetry group through mass lifts that, owing to the fact that the coordinates are also twisted, remain for ever fixed; in this specific case, at the Planck scale. Also the shifts acting on the space-time coordinates reduce further the rank of the symmetry group. However, the breaking in this case is obtained through mass shifts that depend on the length of the untwisted coordinates, i.e. on the size of space-time. Therefore, the matter states "projected out" by these shifts are not thrown out from the spectrum of the low energy theory: they acquire a "weak" differentiation in their masses; the mass difference is inversely related to the scale of space-time.

We have seen that, before any shift in the space-time coordinates is applied, each twisted sector gives rise to four chiral matter fermions transforming in the **4** of a unitary gauge group. The first Z_2-shift in the space-time breaks this symmetry, reducing the rank through a "level doubling" projection. On the heterotic string, this operation acts by further doubling the level of the gauge group realized on the currents [13]. The consequences of this operation are that: 1) half of the gauge group becomes massive; 2) half of the matter states become also massive. The initial symmetry is therefore broken to only one rank-2 unitary group, under which only half of matter transforms. The remaining matter degrees of freedom become massive. Since this phenomenon takes place at a scale related to the inverse of the space-time size, therefore lower than the Planck scale, field theory constitutes a good framework allowing to understand the fate of these degrees of freedom, leading to the creation of massive states. In field theory massive matter is made up of four degrees of freedom, corresponding to two chiral massless fields. In order to build up massive fields, the lifted matter degrees of freedom must combine with those that a priori were left massless by the shift. Namely, we have that the **4**, corresponding to $U(4)$, is broken by this shift to $\mathbf{2} + \cancel{\mathbf{2}}$:

$$U(4) \to U(2)_{(\mathrm{L})} \otimes \cancel{U}(\cancel{2})_{(\mathrm{R})} , \tag{2.13}$$

where the second symmetry factor is the broken one, with the corresponding bosons lifted to a non-vanishing mass. The two matter degrees of freedom charged under this group acquire a mass below the Planck scale, and combine with the two charged under $U(2)_{(\mathrm{L})}$. Therefore, of the initial fourfold degeneracy of massless matter degrees of freedom, we make up light massive matter, of which only the left-handed part feels an $U(2)$ symmetry, namely what survives of the initial symmetry. Indeed, the surviving group is the non-anomalous, traceless $SU(2)_{(\mathrm{L})}$ subgroup. As we will see, this group can be identified with the group of weak interactions. The chirality of weak interactions comes out therefore as a consequence of a shift in space-time. This had to be expected: the breaking of parity is in fact somehow like a free orbifold projection on the space-time. Together with the generation of non-vanishing particle masses and the breaking of parity, this shift also breaks the rotation symmetry of space-time, by separating the role of the two space-time transverse coordinates. We will come back to this issue in section 2.3.

[13] Notice that, once the four-dimensional space-time is included in the orbifold operations, owing to the rank reduction produced by the further shift made in this way possible, the gauge group realized on the currents becomes non-confining, inverting thereby the situation discussed at the end of section 2.1.1.

The spectrum does not contain the degrees of freedom of a possible Higgs boson. On the other hand, here there is no need of such a field, because masses are generated with a pure stringy mechanism, and are basically related to the compactness of the whole space. As remarked in [1], the Higgs boson of ordinary field theory can in some way be thought as the parametrization of a boundary term through a field propagating in the bulk of space.

It is legitimate to ask what is the mass scale of the gauge bosons of the "missing" $SU(2)$, namely whether there is a scale at which we should expect to observe an enhancement of symmetry. The answer is: there is no such a scale. The reason is that the scale of these bosons is simply T-dual, with respect to the Planck scale, to that of the masses of particles. Let's consider this shift as seen from the heterotic side. On the heterotic vacuum, matter states originate from the twisted sector, while the gauge bosons (the visible gauge group, the one involved in this operation) originate from the currents, in the untwisted sector of the theory. Similarly, on the type I side, gauge bosons and the charged states we are considering originate from D-branes sectors derived respectively from the untwisted, and the twisted orbifold sectors of the starting type II theory [14]. It is therefore clear that a shift on the string lattice lifts the masses of gauge bosons and those of matter states in a T-dual way. Since the scale of particle masses is below the Planck scale, the mass of these bosons is above the Planck scale; at such a scale, we are not anymore allowed to speak of "gauge bosons" or, in general, fields, in the way we normally intend them.

The shift just considered is the last "level-doubling, rank-reducing" projection allowed by this perturbative conformal approach. We are left however with two more coordinates which can accommodate a shift. One is a further coordinate of the extended space-time, the other is one of the twisted coordinates. From the heterotic point of view, this is an internal non-perturbative coordinate; just for simplicity, we can identify it with the 11-th coordinate of M-theory. A shift along the extended coordinate is somehow related to the breaking of the last perturbative symmetry we are left with, the $SU(2)_{(\mathrm{L})}$ symmetry. A shift along the 11-th coordinate breaks instead the underlying S-duality of the theory. This symmetry exchanges the role of strong and weak coupling.

With our approximation of Z_2 orbifolds it is however not possible to investigate these effects in a complete way: with the first shift, we have reached the boundary of the capabilities of the approximation we are using. Z_2 orbifolds are a good base on which to expand the string vacua of interest for us, but they constitute anyway an approximation. This results particularly clear if one considers that, in the orbifold description, the string target space does not account for the whole detail of the extended space, namely for its actual content, the full, real geometry of the universe, as implied by 1.1. This space appears as "factorized out". Indeed, in the perturbative orbifold construction the whole target space appears factorized into orbifold planes, and also for the internal space we get the fake impression of a

[14] The type II vacua are on the other hand not appropriate for the investigation of this phenomenon, because the gauge charges are non-perturbative. In any case, although in the form of just the Cartan subgroup of their symmetry group, gauge bosons and matter states arise from mirror constructions, related each other by the type II dual of the heterotic T-duality under consideration [15].

symmetry between these planes. As a consequence, the three matter families appear on the same footing. We have on the other hand an indirect indication that, non-perturbatively, this symmetry is broken, and the extended space gets somehow "embedded" into the internal space.

2.1.5 Breaking the symmetry between the three matter families

By comparing dual string constructions, it is possible to indirectly see how the shifts acting on the various coordinates of the string space break the symmetry between orbifold planes. This holds both in the case of the symmetry between the three orbifold twists that give rise to the three matter families, and for the symmetry between these twists and the twist "along the eleventh coordinate". In this second case, the breaking of the symmetry will mean not only the breaking of the strong-weak coupling symmetry (S-duality), and the related equivalence of weakly and strongly coupled matter, but also the breaking of the $S-T-U$ symmetry exchanging weakly with strongly coupled matter, and either of the two with radiation (the physical meaning of this symmetry will be discussed in section 3).

Let us consider the action of shifts at the "$\mathcal{N}=2$" level, the step at which the three sectors, that we will shortly indicate as the "S", "T" and "U" sector, originate. The interpretation of them as the sectors corresponding to the three matter families, or to the $S-T-U$ non-perturbative symmetry, depends on the interpretation one gives to the specific dual perturbative constructions, namely on whether one decides to explicitate the "eleventh coordinate" trading it with one of the perturbative coordinates of the string target space, or not. These sectors appear on a different footing on dual constructions: what in a heterotic construction appears as a perturbative shift on the momenta of a "T-U" lattice which affects the corrections to some coupling of the same sector (the "S" sector, by convention), on a type II dual construction shows out as an operation that lifts the mass of states which were non-perturbative from the heterotic point of view. From the heterotic point of view, the T and U sectors are related by a perturbative T-duality. In this realization one can explicitly see that the shifts break this symmetry. In particular, the action of the shifts on the internal coordinates results in a different (T-dual) rescaling of the volume of the string space. This implies a rescaling of the coupling, which as a consequence affects the measure of masses in units of the Planck mass [15]: as a consequence of the shift, that rescales differently the coupling of the T and of the U sector, in each of these sectors the dependence of sub-Planckian masses on the radius of the extended space enters multiplied by a different factor. As we will discuss in section 4, the orbifold construction corresponds to a logarithmic realization, i.e. on the tangent space, of the full, non-perturbative string configuration. Instead of the true coordinates of the physical space, one works with their logarithm: $R \to \log R \equiv r$. As

[15] The coupling is related to the ratio of Planck and string scale (see for instance expression 2.21 or 2.22 for the dependence at the leading order). In the simple case of a series of freely-acting orbifold projections realized through lattice shifts on a two-torus, the loop corrections to the coupling amount to a sum of logarithms of Jacobi θ functions, which distinguish between the dependence on the modulus T and U. In more complicated cases, when also a dependence on the coordinates of the extended space is introduced, the θ functions are substituted by more complicated expressions. It remains however the differentiation in the contribution to the volume factor, i.e. in the scaling of the coupling.

a consequence, the rescaling of a dependence in the logarithmic realization on $1/r$ by some numerical factor, $1/r \to k/r$ reflects in a power-law dependence on the real radius of space: $1/R \to (1/R)^{1/k}$. As we will see, the ratios of masses between different families are indeed of the order of some power of the radius R: $m_i/m_j \approx (1/R)^{\alpha_{ij}}$.

Through a comparison of dual constructions we can see where the differentiation of sectors comes from, but we cannot follow the whole pattern of symmetry breaking, because we can only see the fate of pairwise two sectors out of three: one of the three sectors is always excluded from the investigation, because it involves the coupling over which one of the dual realizations is perturbatively constructed. All this leads to expect that the configuration of highest entropy lies a bit "displaced" from the orbifold point. However, we expect the corrections to the orbifold approximation to be related to the scale of extended space-time. More precisely, owing to the breaking of T-duality produced by the shift along the coordinates of the extended space, we expect these corrections to be of the order of some root of the inverse of the radius, or age, of the universe. The mean value of the observables on the configuration of minimal symmetry is expected to be of the type:

$$< \mathcal{O} >_{\psi^{\min}} \;\; \sim \;\; < \mathcal{O} >_{\psi^{\min}}\Big|_{Z_2} + \Delta < \mathcal{O} >_{\psi^{\min}} , \tag{2.14}$$

with

$$\Delta < \mathcal{O} >_{\psi^{\min}} \;\; \approx \;\; \mathcal{O}\left(1/\mathcal{T}^p\right) , \qquad p > 0 . \tag{2.15}$$

The dependence on this scale as a root and not simply to the first power depends on the action of the orbifold shifts, something we will discuss more in detail.

2.2 Origin of the $SU(3)$ of QCD and low-energy spectrum

At the point we arrived, the low energy world appears made out of light fermionic matter (*no scalar* fields are present!), massless and massive gauge bosons. Matter states are charged with respect to bi-fundamental representations. More precisely, they transform in the $((\mathbf{2} \oplus \not{\mathbf{2}}), \mathbf{4})$, replicated in three families: $((\mathbf{2} \oplus \not{\mathbf{2}}), \mathbf{4}')$, $((\mathbf{2} \oplus \not{\mathbf{2}}), \mathbf{4}'')$, $((\mathbf{2} \oplus \not{\mathbf{2}}), \mathbf{4}''')$. As we discussed, a perturbative shift lifts the mass of the gauge bosons of the second $\mathbf{2}$ above the Planck scale and gives at the same time a light mass to the matter states. The $\mathbf{4}$ (i.e. $\mathbf{4}'$, $\mathbf{4}''$ and $\mathbf{4}'''$) are however at the strong coupling, and therefore broken symmetries. Indeed, outside of the orbifold point, we should better speak in terms of a larger symmetry, into which the matter degrees of freedom transform as $\mathbf{12} \supset \mathbf{4}' \oplus \mathbf{4}'' \oplus \mathbf{4}'''$, which is broken and at the strong coupling. The (perturbative) orbifold point constitutes a simplification, in which the $\mathbf{12}$ has been rigidly broken by the orbifold twists; the real configuration is a perturbation of this simplified situation, in which the bosons of the broken symmetry, which were acting among orbifold planes, acquire a non-vanishing mass, of the order of some power of the age of the universe. On the other hand, minimization of symmetry tells us that indeed the $\mathbf{12}$, besides being broken into $\mathbf{4}' \oplus \mathbf{4}'' \oplus \mathbf{4}'''$, are also further broken to the minimal confining subgroup. Had we to represent this in terms of field theory, this would imply as unique possibility the breaking of $\mathbf{4}$ into $\mathbf{1} \oplus \mathbf{3}$, corresponding to $SU(4) \to U(1) \times SU(3)$. However, from the string point of view, strictly speaking the $SU(3)$ symmetry does not exist: the vacuum appears to be already at the strong coupling, and the only asymptotic states are $SU(3)$ singlets. More than talking about gauge symmetries, what we can do is to *count* the matter states/degrees of freedom, and *interpret* their multiplicities as due to symmetry transformation properties, as they would appear, in a field theory description, "from above", i.e. before flowing to the strong coupling. This is an artificial situation, that does not exist in the actual string realization. The string vacuum indeed describes the world as we observe it, namely, with quarks at the strong coupling. From: i) the counting of the matter degrees of freedom, ii) knowing that this sector is at the strong coupling as soon as we identify the "$(\mathbf{2} \oplus \not{\mathbf{2}})$" as the sector containing the group of weak interactions, and iii) the requirement of consistency with a field theory interpretation of the string states below the Planck scale [16], we derive that the only possibility is that this representation is broken to $U(1) \times SU(3)$. A further breaking would in fact lead to a negative beta-function, in contradiction with point ii). The operation that breaks the **4** into **3** and **1** corresponds to the "shift" along the 11-th coordinate we mentioned in section 2.1.4. This operation, that cannot be represented at the orbifold point, doesn't work as a "level doubling", but rather as a "Wilson line". Notice that, coming from the breaking of an $SU(4)$ symmetry, the $U(1)$ factor is traceless. This means that it acts by transforming with opposite phase states charged under $SU(3)$ and uncharged ones:

$$
\begin{aligned}
U(1)_\beta \, \varphi &= \mathrm{e}^{\mathrm{i}\beta} \, \varphi \, , \\
U(1)_\beta \, \varphi_a &= \mathrm{e}^{-\mathrm{i}\,\beta/3} \, \varphi_a \, , \qquad a \in \mathbf{3} \, \mathrm{of} \, SU(3) \, .
\end{aligned}
\tag{2.16}
$$

At this point, we are left with one more extended space-time direction where to accommodate a shift, and therefore further break the symmetry.

At the point of minimal symmetry, the gauge groups are broken to $U(1)$'s. This is true for the groups which are at the weak coupling, namely those corresponding to the $(\mathbf{2} \oplus \not{\mathbf{2}})$. It doesn't hold for the $SU(3)$ "colour" symmetry, which is broken and at the strong coupling. Let's therefore concentrate on the $(\mathbf{2} \oplus \not{\mathbf{2}})$. This derives from an initial $U(4)$, then broken to $U(2) \otimes U(2)$ and "patched" to a single, massless $U(2)$, with the second $U(2)$ lifted at over-Planckian mass. Of this massless $U(2)$, acting on half of the chiral matter states, indeed only an $SU(2)$ subgroup survives, the traceless part. This can be identified with the group of the weak interactions, $SU(2)_{\mathrm{w.i.}}$, which acts only on the left moving part of the matter states. A slight displacement away from the orbifold point, driven by moduli of the size of the inverse of the coordinates of the extended space-time ($\mathcal{O}(1/\mathcal{T}^p)$, $p < 1$), breaks this group to $U(1)$. This displacement does not break parity: it commutes with the operation that lifted the second $U(2)$ (indeed, one can think to perform this displacement first, and then the level-doubling shift on the $U(1) \times U(1)$ group).

Consider now the last shift operation, either at the $U(1)$ or at the extended $SU(2)_{\mathrm{w.i.}}$ gauge symmetry point. This shift cannot act as a level-doubling projection, as it can be seen by looking from a heterotic point of view, in which the group from which the $SU(2)_{\mathrm{w.i.}}$ of the

[16] For us, field theory is not something realized below the string scale. It is rather an approximation (that locally works), obtained by artificially considering the space-time coordinates as infinitely extended. As a consequence, the "field theory" scale is in our set up the scale of the compact, but non-twisted, string coordinates. In order to keep contact with the usual approach and technology, what we have to do is to send to infinity the size of space-time, and consider the <u>massless</u> spectrum of four dimensional string vacua.

weak interactions originates is realized on the currents. A further rank reduction through a level-doubling orbifold projection is forbidden by the embedding of the spin connection into the gauge group. The effect of this shift is that of lifting the mass of all the gauge bosons. This action cannot be followed in a heterotic construction, where the gauge fields arise from the currents, and cannot be "shifted". What happens can be observed on the type II side, where the gauge fields originate in the twisted sectors. From the type II point of view, it is clear that such a shift corresponds to making the orbifold projection to act freely. For a shift realized "on the momenta" of space-time, i.e. a "field theory shift", as the one we are considering, the mass of the bosons will be below the Planck mass:

$$m_{W^i} \to \approx \frac{1}{2} \times \frac{1}{R}, \tag{2.17}$$

where R is the radius of the shifted coordinate. We will come back to discuss the mass of the $SU(2)_{\rm w.i.}$ bosons after having gained some insight into the relation between perturbative mass expressions and the perturbatively resummed ones, in sections 4 and 5. Differently from the usual case, in which the matter states are lifted in a T-dual way to the bosons, here the mass of the matter states receives a shift of the same order as the gauge bosons. This phenomenon can be traced by looking at the type II dual realization. As discussed also in Ref. [15], on the type II side there are two mirror constructions: in one it is the (Cartan subgroup of the) gauge group which is explicitly realized on the twisted sectors; in the mirror we see instead the matter states. The two constructions are related by a T-duality in the internal coordinates, therefore not involving a transformation of the space-time coordinates in which this last shift acts. A shift that lifts gauge boson masses by acting on the momenta of one such coordinate remains a shift in the momenta also as viewed from the mirror construction, in which it is seen to lift the mass of matter states. What we expect is therefore that the mass difference between the up and the down of a broken $SU(2)_{\rm w.i.}$ doublet is of the order of the broken boson masses. Namely, we expect the mass difference of the heaviest doublet, the one that sets the scale of the breaking of this symmetry in the matter sector, to correspond to the scale of the breaking of this symmetry in the gauge sector (m^i_W mass). We will come back for a more detailed discussion in sections 4 and 5. At the point of minimal entropy, the mass lift 2.17 is "combined" with a further mass shift produced by the breaking of the $SU(2)_{\rm w.i.}$ symmetry to $U(1)$. This is a "second order" deformation, driven by moduli as in 2.14, 2.15. The $U(1)$ boson will have therefore a mass in first approximation of the same order of the one of the W^+ and W^- bosons, plus a mass difference produced by a parity-preserving, and therefore left-right symmetric, displacement.

Leptons and quarks originate from the splitting of the $\mathbf{4}$ of $SU(4)$ (replicated in three families, $(\mathbf{4}', \mathbf{4}'', \mathbf{4}''')$), as $\mathbf{3} \oplus \mathbf{1}$ of $SU(3)$. All in all, we have therefore two $U(1)$ groups, or, better, if we also consider the Cartan of the lifted $SU(2)$ "Right", that we know to have acquired an over-Planckian mass, three $U(1)$s. Let's indicate their generators as $T_3^{\rm L}$, $T_3^{\rm R}$ and $\tilde{Y}$. The charge assignments along each of the three $(\mathbf{4} \otimes \mathbf{4}^i)$ are then:

$$\left(Q(T_3^{\rm L}) \oplus Q(T_3^{\rm R})\right) \otimes \left(Q(\tilde{Y})\right) \; = \; \left(\frac{1}{2}, -\frac{1}{2} \oplus \frac{1}{2}, -\frac{1}{2}\right) \otimes (\beta, -\beta/3, -\beta/3, -\beta/3) \; . \tag{2.18}$$

$U(1)_{\tilde{Y}}$, the group 2.16, can be identified with some kind of hypercharge group. This is not exactly the hypercharge of the Standard Model: the charge assignments are here left-right symmetric, and $SU(2)$ invariant, because the two $SU(2)$, $SU(2)_{(\mathrm{L})}$ and $SU(2)_{(\mathrm{R})}$, of which the "Left" gives rise to the weak interactions group, are here factorized out. Since the hypercharge and the $SU(2)$ groups arise from mutually non-perturbative sectors, the hypercharge eigenstates are here singlets of the $SU(2)$, left *and* right, symmetries. This means that the phase refers to the sum of the up and down of each family:

$$\beta \;\propto\; \tilde{Y}_e + \tilde{Y}_\nu \,, \tag{2.19}$$

$$\beta/3 \;\propto\; \tilde{Y}_{\mathrm{up}} + \tilde{Y}_{\mathrm{down}} \,, \tag{2.20}$$

this for any family and colour. By using the standard normalization of Lie group generators, we set $|\beta| = 1$, and, by pure convention, $\beta = -1$. This group, the only surviving gauge group, whith a truly massless boson, can be identified with the electromagnetic group, and its charges with the "electric" charge assignment of the elementary particles of this vacuum. The problem is to see how the electric charge is distributed among the $SU(2)$ doublets, namely, according to which fraction the decompositions 2.19 and 2.20 are made. We know that, as a matter of fact, in nature this is not done "democratically": the electron has charge -1, and the neutrino is uncharged. The same charge difference applies also to colour triplets of quarks. This can be interpreted as the result of having taken a linear combination, more precisely the sum, of the $\tilde{Y}$, T_3^{L} and T_3^{R} charges. In our scenario, this distribution of the electric charge among the $SU(2)$ multiplets turns out to be required by minimization of the amount of symmetry of the configuration. Among all the possible arrangements of this kind of hypercharge, the one corresponding to the most entropic string configuration is the one for which one particle becomes uncharged for one of the interactions, the electromagnetic one. In this case, highest is the differentiation among the spectrum of particles, i.e. the symmetry of the configuration is the lowest possible. Of course, in the whole phase space also other choices are present, but they correspond to less entropic configurations, which therefore give a minor contribution to the mean value of observables. The less interacting particle must also be the lightest one. As we will discuss in section 4.1, the mass of a particle is related to its weight in the phase space. The latter is in turn related to the number of paths leading to its configuration. The number of paths is related to the strength of the interacting power of the particle: the more are the interactions the particle possesses, the more entropic is the physical configuration it corresponds to, because larger is the particle's "spread out" in the phase space. The neutral particle we obtain must be identified with the neutrino.

The inclusion of the right-moving quantum numbers is not an "ad hoc" assignment, chosen in order to get the correct charge assignments: these are instead the consequence of the fact that, in this scenario, $\mathcal{T} \to 1$ is a limit of left-right symmetry restoration, where the dual, over-Planckian masses, come down to match the sub-Planckian ones. This is basically the meaning of having broken parity in a soft-way, by a shift along the extended coordinates. Therefore, no one of the charge assignments in 2.18 can explicitly break this symmetry [17].

[17] The case of the strong-weak coupling separation, related to the breaking of an S-duality, is different, because it involves a shift along coordinates which are also twisted, and therefore there is no restoration at the limit. The breaking is neither soft nor "spontaneous".

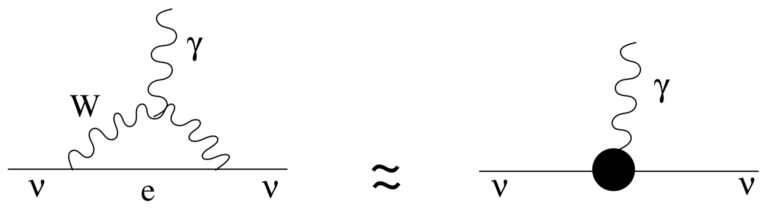


Figure 1: An effective neutrino electric charge is generated through higher order corrections.

Here is a point of difference with respect to the ordinary approach to the building of the electromagnetic group: in the usual approach, the hypercharge assignments explicitly break the $SU(2)$ and the parity symmetries.

Once the charge of the neutrino has been fixed, all other charges result correctly determined. Owing to the shift of the "hypercharge" with the $SU(2)$ charges, the electromagnetic current doesn't couple only with the "long range" force, the "photon", i.e. the massless field associated to the $U(1)_{\tilde{Y}}$ symmetry (indeed the sum of the up and down electric charges), but also with the massive neutral fields originating from the broken $SU(2)$s. The "Right" one is very massive, over the Planck scale, and therefore the corresponding interaction channel can be neglected: from a field theory point of view, "it does not exist" [18]. In section 5.6 we will discuss the masses of the broken $SU(2)$ and the "Left" $U(1)$ boson. The coupling of the electromagnetic current will be derived and discussed in sections 4.3.3 and 4.4.4. The $SU(2)_{\text{w.i.}}$ coupling and the coupling of the neutral current with the under-Planckian massive neutral boson, the "Z" boson, will be discussed in sections 4.3.2 and 5.6 respectively.

It may seem disappointing that relations that we are used to consider associated to an exact symmetry, or concerning the exact vanishing of a charge, in this case the electric charge of the neutrino, appear in this scenario "softened", a consequence of maximization of entropy: there are no "conservation laws" strictly forbidding other solutions, and protecting these charges and masses. However, this is precisely what we should expect in a quantum scenario, and is accounted for in 1.1 and 1.2. In a way similar to what the well known path integral does, here *all* possible configurations contribute to build up the universe as we observe it, although not all with equal weight. Also in traditional quantum field theory the electric charge of the neutrino vanishes only as long as we consider the neutrino as a free, asymptotic state. Interactions generate indeed an effective non vanishing electric charge for this particle, in the sense that, as illustrated in figure 1, at higher orders, a neutrino effectively interacts also with the photon.

[18]This is a major point of difference with respect to the field theoretical "left-right" extensions of the Standard Model.

2.3 The breaking of Lorentz/rotation invariance

We have seen that in our framework the origin of masses is related to the breaking of parity and time-reversal, and that the same operation implies also the breaking of rotation invariance. This is not a surprise, because 1.2 is expected to describe the universe "on shell", i.e. as it is, the space-time *with* its energy and matter content at any scale, and the breaking of these symmetries at the macroscopical level is part of the common experience of everyday. Indeed, the entire analysis of the string orbifold vacua has been performed in the light-cone gauge, where the Lorentz symmetry is explicitly broken. However, the breaking due to a particular choice of gauge is not a real breaking: it is just a convenient representation, through a particular "slice", of an invariant construction. On the other hand, in our framework, what happens is that the Lorentz symmetry is really broken. The breaking is realized at two levels: 1) the breaking of the Lorentz boosts; 2) the breaking of the subgroup of space rotations. The Lorentz boosts are not a symmetry in a compact space-time, in which transformations in space and time correspond to an evolution of the universe. Nevertheless, rotations in principle could remain a symmetry of space: they are not in contradiction with its compactness. However, here we discover that maximization of entropy not only implies the breaking of parities, but also the breaking of the symmetry of space under rotations. And this precisely at the same time as masses are produced. This does not come unexpected: the very fact of placing a matter excitation somewhere in space breaks this invariance, selecting a preferred direction, in a way similar to the one a Higgs field breaks a gauge symmetry by selecting a position of minimum among the entire orbit of the symmetry group. In the average, owing to the presence of a lot of particles, existing as single sources of curvature or grouped into larger objects almost homogeneously distributed in the space, we can still say that the universe is on a large scale invariant under rotation. Nevertheless, strictly speaking this is not a "pure" invariance. From a field theory point of view we would say that it is "softly broken". It is not a surprise that precisely the mechanism that in our framework substitutes the Higgs mechanism in giving rise to masses is also responsible for the breaking of the space rotation symmetry, through shifts acting on the transverse space coordinates: the one associated to the breaking of parity, and the one associed to the breaking of $SU(2)_{\mathrm{w.i.}}$. The amount of breaking of space rotations produced in our framework is of the same order of the particle masses. There is no fracture between a fundamental, microscopical physics possessing certain symmetries, and a macroscopical world in which for some reasons these laws are violated. In our framework the microscopical and macroscopical descriptions of the world are sewed together: time-reversal, space-parity, reversibility, and symmetry under rotations are broken at a very fundamental level.

2.4 The fate of the magnetic monopoles

Under the conditions of the scenario we are discussing, namely of a universe "enclosed" within a finite, compact space, also the issue of the existence of magnetic monopoles changes dramatically. Magnetic monopoles can be of two kinds: the "classical" ones, namely those associated to a non-vanishing "bulk" magnetic charge that parallels the electric charge in a symmetric version of the Maxwell's equations, and the topological ones. In our scenario there are no "classical" monopoles: their existence would be possible only in the absence of an electro-magnetic vector potential, what we have called the "photon" A_μ; their existence has therefore been ruled out as soon as we have discussed the existence and the masslessness of this field.

The first idea about the existence of magnetic monopoles in the classical sense (i.e. non-topological) originated by a request of symmetry: were not for the absence of magnetic charges, the Maxwell equations would be completely symmetric in the electric and magnetic field. However, the symmetry of these equations, preserved in empty space, is precisely spoiled by the presence of matter states that are also electrically charged. In our scenario, the description of the universe is "on-shell" and the presence of matter comes out as "built-in": it cannot be disentangled from the existence of space itself. As the most often realized configuration of the universe is the one with the lowest amount of symmetry, it is not so surprising that, for the same reason for which charged matter states are generated, in correspondence with a local breaking of the invariance of space under $SO(3)$ rotations, also the symmetry of the Maxwell's equations is broken.

In this scenario there are no topological monopoles either. As all vector fields are twisted (i.e. massive at the Planck scale or above it) with the only exception of the photon A_μ, propagating in the four-dimensional space time, and as this space-time dimensionality is electro-magnetically self-dual, the only possible topological monopoles would be those of the four-dimensional space coupled to the same photon field A_μ, namely, configurations à la t'Hooft and Polyakov or similar [19]. However, any such topological configuration is characterised by its being living in an infinitely-extended space: only in this way it is in fact possible to make compatible the existence of a p-form working as a "potential" $A_{(p)}$, defined as an analytic function in every point of the space, with the presence of a non-trivial magnetic flux. As is well known, the magnetic flux through a surface can be computed as a loop integral of the vector potential. In the case of a surface enclosing a finite volume, the total flux is the sum of the loop integral circulated in both the opposite directions, so that it always trivially vanishes. However, things are different if the field has a non-trivial behaviour at infinity. At infinity we need just the circulation in one sense, because there is no "outside" from which field lines can "re-enter" in the space: if there is a non-vanishing circulation, there is a non-vanishing magnetic flux, and therefore also a non-vanishing magnetic charge. This however also means that, provided it exists, such a magnetic monopole is a highly non-localised object, with a magnetic field/vector potential such that the magnetic flux vanishes through any compact finite closed surface [20]. As a consequence, also the magnetic charge density vanishes point-wise at any place in the "bulk". Therefore, in our setup, where space is compact, these monopoles cannot exist. Moreover, in our case we don't have a Higgs mechanism either, and, since the surface at infinity does not belong to any configuration of space-time, there is no smooth limit with a true restoration of the conditions at infinity allowing the existence of non-trivial topologies and homotopy groups. Light states with

[19] for a review and references, see for instance [43, 44].

[20] Notice that the situation around the zero-dimensional point is equivalent to the one around the surface at infinity: if on one side the Dirac string can be considered as somehow the "dual" picture of the surface at infinity of the t'Hooft and Polyakov construction, in our scenario both infinity and the dimensionless point are excluded. Differential geometry and gauge theory are here only approximations.

topological magnetic charges do not exist at all, not even approximately as the time becomes very large [21].

2.5 *Effective action*

The various string constructions have to be considered as slices, at different points in the moduli space, of the same theory. Dual constructions corresponding to slices of overlapping regions of the moduli space. The comparison of dual string constructions makes only sense once the expressions of the physical quantities are converted into common units, i.e. when, instead of being expressed in terms of the proper length of each string construction, they are expressed in terms of the Planck length or mass scale. Whenever one can write an effective action for the string light modes, this conversion corresponds to the passage from the string frame to the duality-invariant, so called Einstein's frame, also referred to as the "supergravity frame". In the case of compact space-time, the existence of an effective action is however not obvious. Traditionally, i.e. in an infinitely extended space time, the limit of infinite volume selects as elementary excitations of the propagating modes one of the two T-dual worlds, or towers of states whose quantum numbers correspond either to windings or to momenta. The other ones (therefore either the momenta or the windings) are "trivialized" to a constant contribution. In the case of compact space, at any volume both T-dual degrees of freedom are instead non-trivially present, and the selection of a configuration close to the case of infinite volume is only possible when T-duality along the space-time coordinates is broken. The breaking of T-duality leads in fact to the choice of a "direction", or scale, for a field theory representation: either below or above the Planck scale, two situations no more equivalent. As we have seen, in the configuration of minimal symmetry T-duality in the space-time coordinates is broken. It is then possible to talk about "extended" space-time, along which degrees of freedom, representable as fields, can propagate. We are therefore allowed to talk about an effective action, in which the light modes of the theory, namely those whose mass is below the Planck scale, move in a space-time frame of coordinates larger than the Planck length, while heavier modes are integrated out and their existence manifests itself only through their contribution to the parameters of the effective theory. Although the case of infinitely extended space-time is never realized, at large space-time volumes it can constitute a useful approximation. This justifies the use of the language of traditional string compactifications, and an analysis in terms of orbifold dualities and comparison of dual constructions as we did in section 2.1, namely in terms of the "sectors" S, T and U of the theory, compared through the reduction of the effective actions to the common, Einstein's frame. At the $\mathcal{N}_4 = 2$ level, the last step at which we could still explicitly follow the pattern of dual constructions, for the heterotic string the relation between the string and the Planck scale is given by:

$$\mathrm{M}_{\mathrm{P}}^2 \equiv \mathrm{M}_{\mathrm{Het}}^2 \, \mathrm{Im}\, S^{-1} , \tag{2.21}$$

[21] The situation is similar to the case of the volume of the group of translations and its identification with the regularized volume of space in the usual normalization of operators and amplitudes, completely absent in our scenario, something that leads to a different interpretation of string amplitudes as global quantities instead of densities, cfr. Ref [1].

where $S = \chi + \mathrm{i}e^{-2\phi}$, ϕ is the dilaton field. On the type I side there are two such fields, called S and S', that parametrize the coupling of different D-branes sectors of the theory. Each sector has therefore its own "string length", to be rescaled to the Planck length. By inspecting string-string duality with the type II string, we have on the other hand learned that the vacuum of interest for us possesses at the $\mathcal{N}_4 = 2$ level three such "coupling fields", corresponding to three sectors of the theory (see also Ref. [15] for a more detailed discussion). This structure, namely the triplication of the string vacuum, is preserved when going to lower (super)symmetric, less entropic configurations, up to the one of minimal symmetry. We can therefore talk of three "waves", the "S", "T" and "U" "wave" of the theory. Correspondingly, there are three "string scales" to be related to the Planck scale. In terms of the notation "S, T, U", these relations read:

$$\mathrm{M}_{\mathrm{P}}^2 \; = \; \mathrm{M}_{\mathrm{S}}^2 \, \mathrm{Im}\, S^{-1} \;\; = \;\; \mathrm{M}_{\mathrm{T}}^2 \, \mathrm{Im}\, T^{-1} \;\; = \;\; \mathrm{M}_{\mathrm{U}}^2 \, \mathrm{Im}\, U^{-1} \, . \tag{2.22}$$

As we have seen in section 2.1.2, the internal space is frozen at the self-dual radius, $\mathcal{O}(1)$ in string units. This implies that string and Planck scales are equivalent, as anticipated. From this equivalence, we conclude that the masses of the states lifted by the shifts acting along the internal coordinates are of the order of the Planck mass.

Owing to the implied breaking of time reversal, the breaking of T-duality leads to the choice of an arrow for the time evolution: space-time volume expansion is not equivalent to space-time contraction. Indeed, in this scenario the effect of the boundary of a finite volume space cannot be neglected: it reflects in a dependence of the parameters of the effective action on the age of the universe. To be more precise, instead of one effective action, we should better speak in this case of a series of effective actions, $S_{\mathrm{eff}} \to S_{\mathrm{eff}}(\mathcal{T})$, for each one of which one can formally write a Lagrangian density for propagating fields bearing a dependence on a "bulk" time coordinate t, but this remains only an approximation.

3 The geometry of the universe

According to the results of section 7.2 of Ref. [1], the energy density scales as [22]:

$$\rho(E) \sim \frac{1}{\mathcal{T}^2}, \tag{3.1}$$

whereas the entropy scales as:

$$S \propto \mathcal{T}^2. \tag{3.2}$$

The total energy at a certain time $\mathcal{T}$ of the history of the universe, given by the integral of the energy density over the space volume of the universe at time $\mathcal{T}$, scales then as:

$$\mathrm{E}(\mathcal{T}) \sim \int_{\mathcal{T}} d^3 \frac{1}{\mathcal{T}^2} \approx \mathcal{T}. \tag{3.3}$$

We will now discuss in detail the evaluation of the mass and energy content of the universe, and confirm that the metric of the universe is the one of a sphere.

3.1 The energy density and the cosmological constant

Let us start by computing in detail the energy density of the universe. When investigating the minimization of symmetry, in section 2.1, we have seen that $\mathcal{N}_4 = 2$ was the last step at which the couplings of the theory appeared as moduli of the string space. There, the theory consists of three "sectors", corresponding to couplings parametrized by the moduli "S", "T" and "U" [23]. When the space is further twisted, these moduli are frozen. Nevertheless, as we already pointed out in section 2.1.1, the less supersymmetric theory inherits the structure from $\mathcal{N}_4 = 2$. As we have seen, at the $\mathcal{N}_4 = 2$ level the heterotic realization, being constructed around a small/vanishing expectation value of the coupling parametrized by S ($\mathrm{Im}\, S$), corresponds to a perturbative realization of the "S-sector". This is the "slice" of the theory which in natural way describes the coupling with gravity. We will call it the "S-picture". When going to vacuum of lowest symmetry, by "S-picture" we have to intend a non-perturbative slice, whose perturbative limit would correspond to the heterotic construction.

By comparison of dual constructions at the extended supersymmetry level, it is possible to see the symmetry of the theory under exchange of the three sectors, symmetry which is then inherited by the $\mathcal{N}_4 = 0$ vacuum. In each sector, the energy of the universe is given by the mean value of the identity operator [24]; the total amount is given by the sum of the contributions of the three sectors. In the configuration of minimal entropy, the $S \leftrightarrow T \leftrightarrow U$ symmetry is broken. As discussed in section 2.1.5, the breaking is tuned by the non-twisted

[22] We recall that in our theoretical framework, owing to the absence of symmetry under translations, string amplitudes are normalized in such a way that densities scale as the inverse of the Jacobian of the transformation between string world-sheet and two target space coordinates (see Ref. [1], section 7.2).

[23] As we already pointed out, we use the language of compactifications to four dimensions, whereas strictly speaking in our framework all string coordinates are compactified. Nevertheless, as we are interested in situations in which the extended space is very large as compared to the string scale, this abuse of language is not completely inappropriate, and allows to simplify the discussion.

[24] See Ref. [1], section 7.2.

coordinates, and, owing to the breaking of T-duality, the corrections to mean values are of the order of some root of the inverse of the age of the universe, as in 2.14, 2.15. As a consequence, we expect that also the contributions to the energy of the universe differ by some power (indeed some root) of the inverse of its age:

$$\frac{E_i}{E_j} \approx 1 + \mathcal{O}\left(1/\mathcal{T}^{p_{ij}}\right) , \quad p_{ij} > 0 , \tag{3.4}$$

where E_i, E_j stay for E_S, E_T, E_U. These three contributions to the energy of the universe differ in their interpretation. A closer look at the situation through comparison of the heterotic and type I pictures (see also Ref. [15]) reveals that the S, T and U pictures are related by S- and T-dualities, that exchange weakly and strongly coupled sectors (i.e. confining matter and non-confining particles, together with their gauge bosons, including the photon) and the "bulk", i.e. the gravity sector. Therefore, the mean value of the identity operator, when inserted in the gravity sector, computes the vacuum energy of the "empty space-time" in the "M-theory", or "String Theory" frame. In the other sectors, the mean value of the identity corresponds to the energy of matter (the confining part) and of "relativistic particles" respectively. The basic equivalence of these three contributions to the energy of the universe is therefore a consequence of the symmetry between the "S-". "T-" and "U- waves" of the dominant configuration; symmetry which is eventually broken by the very minimization of symmetry, but the breaking is "soft" as compared to the scale of the twisted space.

We want to pass now from energies to energy densities. As discussed in [1], this passage introduces a dependence on the Jacobian of the transformation of two space-time coordinates from the string/Planck scale to the actual scale of space-time. Let's start by considering the contribution to the energy density in the "S-wave". This corresponds to the standard heterotic picture. According to 3.1, it is given by:

$$\begin{aligned} E_S|_{\text{Einstein frame}} &= \ < \mathbf{1} > |_{\text{Einstein frame}} = \\ &= \ < \mathbf{1} >_S |_{\text{string frame}} \times \frac{\kappa}{\mathcal{T}^2} = E_S|_{\text{string frame}} \times \frac{\kappa}{\mathcal{T}^2} , \end{aligned} \tag{3.5}$$

where $\kappa/\mathcal{T}^2$ is the Jacobian of the transformation from the string frame to the Einstein's frame, and κ a normalization constant. According to 3.4, $E_S|_{\text{string frame}}$ depends on time at the second power, and, owing to the fact that:

$$< \mathbf{1} >_S |_{\text{string frame}} + < \mathbf{1} >_T |_{\text{string frame}} + < \mathbf{1} >_S |_{\text{string frame}} = 3 , \tag{3.6}$$

we have also that:

$$E_S|_{\text{string frame}} + E_T|_{\text{string frame}} + E_S|_{\text{string frame}} = 3 . \tag{3.7}$$

The quantity 3.5 corresponds to the so called cosmological constant. In order to fix the normalization κ we must take into account that in passing from the string picture to the effective action term, besides the conversion from target space to world sheet of the two longitudinal coordinates, responsible for the two-volume factor, there is also a further space-time contraction due to the Z_2 orbifold projection. On the four (two transverse and two longitudinal) space-time coordinates act altogether two Z_2 shifts, which lead to a factor 4 in the conversion of a four volume. In our case, we have a two-volume, and the factor is 2. We obtain:

$$\Lambda(\mathcal{T}) \equiv E_S|_{\text{Einstein frame}} = \frac{2}{\mathcal{T}^2} \times \left[1 + \mathcal{O}\left(\frac{1}{\mathcal{T}^{q_S}} \right) \right] , \tag{3.8}$$

where the quantity within brackets accounts for the second order correction mentioned in 3.4, which can be positive or negative, i.e. contribute to slightly increase the value of the cosmological constant as compared to the other energies, or decrease it. In any case, the quantity 3.8 is here not a constant, but, through the age of the universe, turns out to depend on time. For the sake of simplicity, we will anyway keep the usual terminology, and refer to it as to the "cosmological constant". According to 3.8, at present time the value of this parameter is:

$$\Lambda(\mathcal{T} = 10^{61}\mathrm{M}_{\mathrm{P}}^{-1}) \approx 10^{-122}\,\mathrm{M}_{\mathrm{P}}^2 , \tag{3.9}$$

as it effectively seems to be suggested by the experimental observations [45, 46, 47].

Let's now introduce a "cosmological density", ρ_Λ, defined through:

$$8\pi\, G_{\mathrm{N}}\, \rho_\Lambda \equiv \Lambda = \frac{2}{\mathcal{T}^2} \times \left[1 + \mathcal{O}\left(\frac{1}{\mathcal{T}^{q_S}} \right) \right] . \tag{3.10}$$

In a similar way, we introduce two other densities, related to the energy density of the T- and U-"waves", ρ_m and ρ_r:

$$8\pi\, G_{\mathrm{N}}\, \rho_m \equiv E_T|_{\text{Einstein frame}} = \frac{2}{\mathcal{T}^2} \times \left[1 + \mathcal{O}\left(\frac{1}{\mathcal{T}^{q_T}} \right) \right] , \tag{3.11}$$

and

$$8\pi\, G_{\mathrm{N}}\, \rho_r \equiv E_U|_{\text{Einstein frame}} = \frac{2}{\mathcal{T}^2} \times \left[1 + \mathcal{O}\left(\frac{1}{\mathcal{T}^{q_U}} \right) \right] . \tag{3.12}$$

The correction terms in the brackets are of different signs, in order to sum up to a constant, in agreement with 3.7. Each one of the above equations can be written as:

$$\begin{aligned} \rho_i &= 2 \times \frac{< \mathbf{1} >_i |_{\text{string frame}}}{3} \times \frac{R}{2G_{\mathrm{N}} \left[\frac{4}{3}\pi R^3 \right]} \\ &= 2 \times \frac{< \mathbf{1} >_i |_{\text{string frame}}}{3} \times \frac{M_{\text{Schw.}}}{V} , \end{aligned} \tag{3.13}$$

where $M_{\text{Schw.}} \equiv R/2G_{\mathrm{N}}$ is the "Schwarzschild mass", and V is the volume of the universe up to the horizon, $R = \mathcal{T}$. If we sum up the three densities, taking into account 3.6 we obtain:

$$\rho \equiv \rho_\Lambda + \rho_m + \rho_r = \frac{2M_{\text{Schw.}}}{V} . \tag{3.14}$$

This agrees with the interpretation of the universe as a black hole: the mass/energy content corresponds to the Schwarzschild mass of a black hole of radius $R = \mathcal{T}$ (to be more precise, the energy content is twice as much as the one of a black hole, because quantum space-time is an orbifold, and the volume is effectively reduced by a factor 2 with respect to the one of an ordinary smooth space). It may seem absurd that a space with the geometry of a sphere, therefore a geometrical object without boundary, behaves for what matters energy, entropy, and the traveling of information, like a black hole, i.e. like an object with boundary, a ball. Indeed, the coexistence of these aspects is possible because the universe is not simply a classical object. As discussed in [1], the strange geometry of the universe implies that geometric points as entities without dimensions are substituted by "cells" of Planck-length size. In such a scenario of quantum gravity / quantum geometry, close to the horizon "points" are completely non-local (see Ref. [48], and the discussion in section 5.5 of Ref. [1]), and they are topologically equivalent to two-spheres. At the origin of any such geometric puzzle is basically the fact that any attempt to view the geometry of the universe as something "really" existing, beyond the measurement that can make of it an observer, is not legitimate. The universe is not a sphere nor a ball: simply, the local geometry around the observer is the one of a three-sphere, and light rays are interpreted by him as coming from the boundary of a three-ball (see the discussion in Ref. [1]).

The fact that 1.1 receives contributions from any type of configurations implies that the relation 3.14, the relation between radius and total energy, is modified by vacua of non-maximal entropy. In this sense, we can say, something not at all surprising, that the universe is a "quantum Black Hole". The Schwarzschild relation, a classical relation, is violated by "quantum gravity" fluctuations. Indeed, if we consider the whole universe as a fluctuation out of the vacuum, from the Heisenberg's Uncertainty Relations we get that its energy must satisfy a relation of the type:

$$\Delta M \geq \frac{1}{2\mathcal{T}}, \tag{3.15}$$

where the equality is saturated by the "classical", Schwarzschild mass.

There is nothing to worry about considering ourselves at the border, or, depending on the point of view, at the center, of a black hole, our universe: although we are intuitively led to consider black holes as small, extremely dense objects, indeed the mass of a black hole scales linearly with its radius, while the mass density scales with the (inverse) volume, i.e. the (inverse) cubic power of the radius. Therefore, the larger the black hole, the lower is its density. For a radius equal to the age of the universe, matter inside is as rarefied as we see it to be in our universe.

Owing to the interpretation of the universe as a black hole, we can define a temperature of the universe as the total energy divided by the entropy. The temperature turns out therefore to be proportional to the inverse of the age of the universe:

$$T \stackrel{\text{def}}{=} k^{-1}\frac{\mathrm{E}(\mathcal{T})}{\mathcal{S}(\mathcal{T})} = (4\pi k)^{-1}\frac{1}{\mathcal{T}}, \tag{3.16}$$

where $k \approx 8,62 \times 10^{-5}\mathrm{eV}K^{-1}$ is the Boltzmann constant, and the normalization has been fixed according to the usual black hole thermodynamics [49, 50] (see also section 6.4 of [1]). The present value of the age of the universe, converted in mass units, is $\mathcal{T} \sim 10^{61}\mathrm{M}_{\mathrm{P}}^{-1}$. At present time, the temperature of the universe is therefore:

$$T \approx 1,1 \times 10^{-29}\,{}^{0}K\,. \tag{3.17}$$

This temperature, for any practical purpose indistinguishable from the absolute zero [25], has not to be confused with that of the CMB radiation ($\sim 3^{0}K$), that we will discuss in detail in section 7.1.

3.2 The solution of the FRW equations

In the previous paragraph we have seen that the energy density and the scaling of entropy agree with the interpretation of the universe as a black hole. As the universe evolves, the energy density and the curvature of space-time decrease toward a flat limit, and the dominant configuration tends to a "classical" description. At large $\mathcal{T}$ it is therefore reasonable to suppose that this configuration admits a description in terms of Robertson-Walker metric, i.e. a classical metric of the type:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(\,d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{3.18}$$

where for us $t \equiv \mathcal{T}$, and $r \leq 1$. The metric should correspond to a closed universe, $k=1$. Under the assumption of perfect fluid for the energy-momentum tensor, the Einstein's equations lead to:

$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{k}{R^2} + \left\{\frac{8\pi\, G_{\mathrm{N}}\,\rho}{3} + \frac{\Lambda}{3}\right\}, \tag{3.19}$$

where we have collected within brackets the contribution of the stress-energy tensor and of the cosmological term. According to our previous discussion, the stress-energy contribution consists of two terms ($\rho = \rho_m + \rho_r$), each one of the same order of the Λ-term. Inserting the values given in 3.10, 3.11, 3.12 with the "Ansatz" $R = \mathcal{T}$, and summing up, we obtain:

$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{(k=1)}{R^2} + \left\{\frac{2}{R^2}\right\} = \frac{1}{R^2}\,. \tag{3.20}$$

The equation is solved by $R = t$, consistently with our Ansatz. This confirms that the dominant configuration corresponds to a spherical Robertson-Walker metric, describing a universe bounded by a horizon expanding at the speed of light. If instead of the densities we introduce the usual quantities Ω_m, Ω_r, Ω_v, i.e. the densities rescaled in units of the Hubble parameter $H \equiv (\dot{R}/R)$, the Hubble equation 3.19 becomes trivially:

$$0 = \frac{k\,(=1)}{R^2} = H^2\,(\Omega_m + \Omega_r + \Omega_v - 1)\,, \tag{3.21}$$

[25] The temperature was around one Kelvin at a time in Planck units 10^{29} times earlier than today, i.e. at $\mathcal{T} \sim 10^{33}\mathrm{M}_{\mathrm{P}}^{-1}$. Since 1 yr $\sim 10^{51}\mathrm{M}_{\mathrm{P}}^{-1}$, this temperature corresponds to the age $\mathcal{T}_0 \sim 10^{-19}$yr $\sim (3\times)10^{-12}$sec.

where we dropped the label "$_0$", which usually indicates the current, present-day value, from the Hubble parameter: this equation is now valid at any time, and is trivially solved by $\Omega_m + \Omega_r + \Omega_v = 2$ and $\dot{R} = 1$.

Besides the Hubble equation, one usually derives further equations of motion, by imposing energy-momentum conservation to the Roberts-Walker solution of the Einstein's equations. In the present case however no further equation can be derived: energy-momentum conservation remains valid only as a "local" law. At the cosmological scale, energy is not conserved: $E_{tot} \propto R = \mathcal{T}$.

⋆⋆⋆

The comparison of our results with astronomical data, as we did in eq. 3.9, contains a possible weak point. Experimental data are given as a result of a process of interpretation of certain measurements, for instance through a series of interpolations of parameters. All this is consistently done within a well defined theoretical framework. Usually, one takes a "conservative" attitude and lets the interpolations run in a class of models. However, this is always done within a finite class of models. In principle, we are not allowed to compare theoretical predictions with numbers obtained through the elaboration of measurements in a different theoretical framework: in general, this doesn't make any sense. However, in the present case this comparison is not meaningless, and this not on the base of theoretical grounds: the reason is that, for what concerns the time dependence of cosmic parameters and energy densities, the solution we are proposing does not behave, at present time, much differently from the "classical" cosmological models usually considered in the theoretical extrapolations from the experimental measurements. The rate of variation of energy density is in fact: $\dot{\rho} \sim \partial(1/R^2)/\partial\mathcal{T} = 1/\mathcal{T}^3 = 1/R^3$. The values of the three kinds of densities can therefore be approximated by a constant within a wide range of time. For instance, as long as the accuracy of measurements does not go beyond the order of magnitude, these densities can be assumed to be constant within a range of several billions of years. For the purpose of testing the statements and conclusions of the present analysis, the use of the known experimental data about the cosmological constant, derived within the framework of a Robertson-Walker universe with constant densities, is therefore justified.

⋆⋆⋆

A universe evolving according to eq. 3.20 is not accelerated: $\dot{R} = 1$ and $\ddot{R} = 0$. Owing to the existence of an effective Robertson-Walker description, the red-shift can be computed as usual. We have:

$$1 + z = \frac{\nu_1}{\nu_2} = \frac{R_2}{R_1} = \frac{\mathcal{T}_2}{\mathcal{T}_1}, \tag{3.22}$$

where ν_1 is the frequency of the emitted light, ν_2 the frequency which is observed, and R_1, R_2 are respectively the scale factor for the emitter and the observer. $R = \mathcal{T}$ is precisely the statement that the expansion is not accelerated. Expression 3.22 however accounts for just the "bare" red-shift, namely the part due to the expansion of the universe: it does not account for the corrections coming from the time dependence of masses, that we will

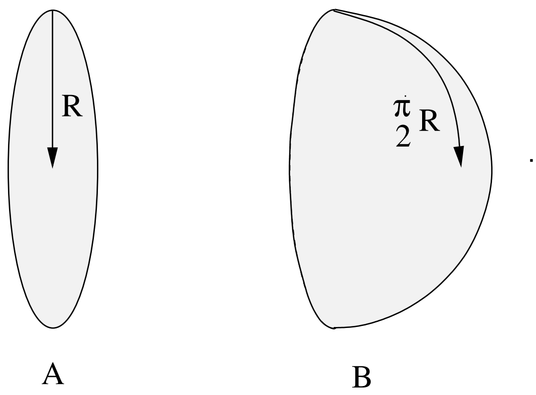


Figure 2: A schematic representation of the universe in reduced dimension. A light ray seen by the observer as coming from the horizon, represented as the boundary of a flat space, the disc of picture A, follows in reality a curved path, a geodesic on the hemisphere depicted in B. The real distance is longer: R in (A) and $\frac{\pi}{2}R$ in (B).

discuss in section 4.4.2. Usually, this effect is not taken into account, because in the standard scenarios masses are assumed to be constant. As we will discuss, in our scenario they depend instead on the age of the universe. A change in the values of masses reflects in a change of the atomic energy levels, and therefore in a change of the emitted frequencies. In order to properly include this effect in the computation of the red-shift, we must therefore discuss the time dependence of masses. We will see that, once the observed frequencies in expression 3.22 are corrected to include also the change in the scale of energies, the scaling of the emitted to observed frequency ratio is not anymore proportional to the ratio of the corresponding ages of the universe. Since the conclusions about the rate of expansion are precisely derived by comparing red-shift data of objects located at a certain space-time distance from each other, this explains why the expansion *appears to be accelerated*.

3.2.1 Speed of light versus speed of expansion

As we discussed in section 7 of [1], to an observer the universe appears in the average as a ball bounded by a two-sphere, with radius $R = c\mathcal{T}$, where $\mathcal{T}$ is the age of the universe. However, the geometry of space-time is curved, and corresponds to the one of a three-sphere, this too of radius proportional to $R = c\mathcal{T}$. Light paths correspond therefore to geodesics in this curved space. A light beam that appears to the observer to be long as much as $R = c$ times the age of the universe $\mathcal{T}$, i.e. coming from the origin of time, corresponding to the horizon of the ball, in reality follows a longer, curved path, because light travels through a curved space (see figure 2 for an illustration of the situation in lower dimension). The "true" speed of light is therefore higher than the speed of expansion, or equivalently the "scale factor" R of the universe is shorter, and expands at a lower speed, than real distances. However, this is of no physical relevance: as long as the two speeds are proportional to each other, with a constant proportionality factor, it does in fact make no sense to ask what is the "real" speed of expansion, the "real" age of the universe, in a world in which everything looks like if we were living in a flat space bounded by a horizon at distance $R = c\mathcal{T}$. The only signal of the existence of a curvature comes from indirect experiments, in which cosmological data are interpreted within a theoretical framework. Namely, for what matters parameters such as the energy density of the universe, the matter content, the speed of expansion of the horizon, and consequently the age of the universe, everything works consistently with the identification of the speed of light with the speed of expansion, the total energy with the integral of the energy density over the (hyper)disk, i.e. the ball enclosed by the horizon, and the universe itself with a black hole. For what matters the cosmological evolution too, densities and equations of motion are normalized for a space with boundary, the "ball" enclosed by the horizon. The two scales, either of times or of lengths, are proportional, and any computation of red-shifts and similar parameters is insensitive to this detail. Things are in this interpretation rather different from the usual cosmological scenario, of a universe described by a Roberts-Walker metric which undergoes an accelerated expansion. In section 4.4.2 we will make a comparison of the two points of view.

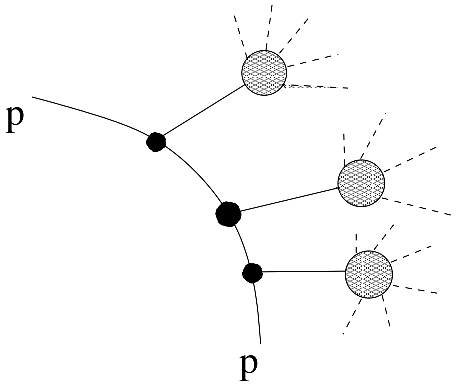
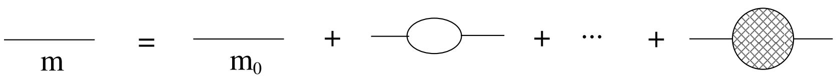


Figure 3: The more and stronger are the interactions of the particle p, the higher its entropy.

4 Masses and couplings

4.1 the mass of a particle

We consider now the masses of the elementary particles. In our approach, a particle is a certain localized amount of energy that rests or moves in space (indeed, the difference between a massive particle and a massless one relies precisely in the fact that the massive particle can stay at rest, whereas the massless one can not). When this scenario is translated into the string representation, masses arise as ground momenta associated to the states of the string spectrum. Since through 1.2 the string scenario is a representation of the combinatorial one, even in the string space a mass is related to the weight of a certain state in the phase space. In the ordinary perturbative approach to field theory (no matter whether it is string-inspired, string-derived, or not) masses, after they have been introduced via some mechanism (Higgs mechanism), are attributes which in general receive corrections at various perturbative orders. The corrections appear as the sum of a series of insertions in the free propagator:



(4.1)

Mass and volume in the phase space are related by the fact that the more are the decay channels of a particle, the larger is its entropy, and also the correction to the mass, because higher is the number of virtual lines (Feynman diagrams) contributing to the mass renormalization (all this is illustrated in figure 3 of page 44). Heavier particles possess a huger decay phase space: quarks are heavier than leptons, and among leptons neutrinos are the lightest particles. Inside each family of particles, the heavier (for instance the top as compared to the bottom of an $SU(2)$ doublet) has the larger absolute value of the electroweak charge. In each family, the lightest particle is the one which has less interactions, or less charge (and therefore a lower interaction probability). For instance, $|Q_\nu| < |Q_e|$, $|Q_b = -1/3| < |Q_t = +2/3|$, and quarks, that feel also the $SU(3)$ interactions, are heavier than leptons [26]. Along this line, we can view the lightest particle as the end-point of a chain of projections that reduce the symmetries of the internal space. In string theory, this corresponds to the fact that heavier momenta are given by inverse of mean radii obtained by taking the geometric average over a higher number of internal coordinates. Masses are the lowest momenta of the string space, corresponding to inverses of radii. Owing to the existence of internal dimensions, not all the radii correspond to the radius of the extended space, R, and therefore there are masses which are higher than $1/R$. In general, we have:

$$m_{(p)}^{-1} = \langle R \rangle = \sqrt[p]{R \times r_1 \times r_2 \cdots \times r_{p-1}}\,. \tag{4.2}$$

Since all internal radii are of Planck size, that is, in our conventions, $r_i = 1$, this relation becomes:

$$m_{(p)}^{-1} \;=\; \sqrt[p]{R \times 1 \times 1 \cdots \times 1}\,. \tag{4.3}$$

In terms of string phase space, this expression says that masses are in relation with the "weight", the volume of an internal space: $\langle R \rangle_p = \sqrt[p]{R}$ and $\langle R \rangle_q = \sqrt[q]{R}$ stay in ratio related to the number of "1"s, i.e. of internal dimensions, they involve. Larger masses correspond to larger internal space, i.e. radii of spaces with a larger number of internal dimensions. Heavier particles are therefore those which "occupy" a larger space; they correspond to a larger internal symmetry than lighter particles. Lighter particles correspond to sub-volumes, sub-spaces of those of the heavier particles: the phase space of lighter particles is contained in the phase space of heavier ones. To figure out this point, think for instance at the case of a heavy particle that decays into lighter ones: the physics of these latter is "contained", in the sense that it is produced, derived, by the physics of the heavier one. In terms of combinatorials of distribution of energies, this simply means that the ways of distributing an amount of energy E along space contain the ways we can distribute an amount $E' < E$.

Although in the most entropic string configuration the string space is not simply factorized into a product of radii as in the above expression of masses, 4.3, from 4.3 one can see that the volumes corresponding to the various masses are not simply those of the internal space, but they realize a kind of "embedding" of the extended space into the internal one, as indicated by their dependence on R. Mass ratios turn out to be of the type $m_{(p)}/m_{(q)} = \sqrt[q-p]{R}$, with p and q however not simply integer, but rational numbers.

4.2 *The couplings*

In this scenario, also couplings are related to weights in the phase space. Indeed, in the combinatorial formulation of the physical world, that, we recall, is the fundamental one,

[26]The first quark family makes an apparent exception: the down quark is heavier, although less charged, than the up quark. This issue will be discussed in detail in section 5.1.2.

a configuration is representated by a group of symmetry, and its weight corresponds to the volume of the symmetry group. Also ratios of masses correspond therefore to ratios of symmetries, i.e. to cosets. When we pass from a particle to another, lighter one, we reduce the symmetry of the subset of the configuration of the universe (i.e. the subgroup of the whole symmetry group representing the configuration of the universe) which represents the particle under consideration. We say we break a certain group to a subgroup. The inverse of the volume of the coset is what we call "coupling". Indeed, in the case we start from a particle and follow a decay process leading to some products (particles and fields with a certain energy), the ratio of the volume of the initial particle to the volume of its decay products gives the coupling of the interaction. Interactions are therefore in a natural way related to groups and symmetries. We will see that this point of view includes both the symmetries which, in the representation in terms of fields living on the continuum, correspond to ordinary gauge symmetries, and the symmetries that in such a representation are going to be "hardly broken", i.e. appear as rigid symmetries. This second type does not give rise to interactions propagating through gauge bosons, but describe for instance the leading contribution to "transitions" such as the passage from a family of quarks or leptons to another one. In this interpretation, they are viewed as an appropriate type of rotations in the phase space. In both cases, these processes, and their relative couplings, follow the rules of composite probabilities typical of the weights in the phase space. A composite transition/decay process: $A \to B \to C$, corresponds to a rotation with an element of the group $\mathbf{G}_{(AC)}$, given by a product $\mathbf{G}_{(AC)} = \mathbf{G}_{(AB)} \times \mathbf{G}_{(BC)}$. This transition corresponds therefore to: 1) first a rotation with an element of the group $\mathbf{G}_{(AB)}$ and then: 2) a rotation with an element of $\mathbf{G}_{(BC)}$. Therefore, we expect the probability of the decay $A \to C$ to be the product of the decay probability of $A \to B$ and of $B \to C$:

$$P(A \to C) = P(A \to B) \times P(B \to C) . \tag{4.4}$$

The effective coupling determines the probability amplitude:

$$P(A \to B) \sim \alpha_{(AB)} , \tag{4.5}$$

where, as usual, $\alpha_{(AB)} \equiv g^2_{(AB)}/4\pi$. The effective coupling for the transition from A to C is given by the product of the effective couplings of the single steps:

$$\alpha_{(AC)} \propto \alpha_{(AB)} \times \alpha_{(BC)} . \tag{4.6}$$

The square coupling of the group $\mathbf{G} = \mathbf{G_1} \otimes \mathbf{G_2} \otimes \ldots \otimes \mathbf{G_n}$ is therefore:

$$\alpha_{\mathbf{G}} = \alpha_{\mathbf{G_1}} \times \alpha_{\mathbf{G_2}} \times \cdots \times \alpha_{\mathbf{G_n}} . \tag{4.7}$$

In the usual renormalization group approach one works in the algebra $\mathcal{G}$ of the group $\mathbf{G}$. If $\mathbf{G} = \mathbf{G_1} \otimes \mathbf{G_2} \otimes \ldots \otimes \mathbf{G_n}$, the algebra decomposes as $\mathcal{G} = \mathcal{G}_\mathbf{1} \oplus \mathcal{G}_\mathbf{2} \oplus \ldots \oplus \mathcal{G}_\mathbf{n}$, and the inverse of the effective coupling seems to renormalize additively. For instance, the one-loop beta-function of $SU(N)$ with gauge bosons in the adjoint and (massless) matter states in the fundamental representation is one half of the beta-function of $SU(2N)$ with an analogous content of matter and gauge states. From the point of view of our approach, what seems to behave additively is just the logarithmic derivative of the inverse of the coupling.

From a technical point of view, subplanckian masses are introduced by a shift along the coordinates of the extended space, required in order to reach the configuration of lowest symmetry. This shift lifts to a non-zero mass all matter states. Their mass is the lowest energy excitation of these states, and, corresponding to the lowest momentum of space, is of the order of the inverse of its radius. As discussed, perturbatively the Z_2 orbifold appears as a point of enhanced symmetry of the theory, and it doesn't allow to see the fact that such an operation doesn't produce an equal mass for all the matter sectors. On the other hand, as we discuss in section 2.1.5, the symmetry between the three matter families (u,d,e, ν_e), (c,b,μ,ν_μ) and (t,b,τ,ν_τ) is non-perturbatively broken. The fact that both masses and couplings are related to volumes of symmetries implies that masses are related to couplings. The hierarchy of masses, as implied by the hierarchy of the corresponding volumes in the string target space, reflects into a hierarchy of the interactions experienced by the particles. We will now see in detail how the ratios of masses are in relation to the ratios of the volumes of these groups, in turn related to the ratios of the strengths of the corresponding couplings.

Let us first see how masses are in general generated by shifts. From the inspection of the generic action of orbifold shifts that produce masses we will also gain a further indication that the perturbative construction corresponds to a logarithmic representation of space. Let us consider the simple example of the orbifold vacua analysed in Ref. [15]. One starts with a construction with matter in the $N \oplus N$ of $SU(N) \otimes SU(N)$. With a level-doubling, rank-reducing Z_2 shift one derives a configuration with matter in the N of $SU(N)$, whereas the other N has been lifted up, and acquired a mass, in the case of a shift acting on the momenta of a circle contained in an internal torus. The mass of the lifted states is given by:

$$m \; = \; \sim \frac{1}{R_1} \, , \tag{4.8}$$

where R_1 is the radius of the circle. Let us now suppose to apply a second shift, acting on another circle, that we assume to be of radius R_2, in a way to lift all matter states. We have therefore two kinds of massive states: those which already acquired a mass with the first shift, that now have mass:

$$m_{1+2} \; = \; \sim \left[\frac{1}{R_1} + \frac{1}{R_2} \right] \, , \tag{4.9}$$

and those which after the first shift were left massless, and now have mass:

$$m_2 \; = \; \sim \frac{1}{R_2} \, . \tag{4.10}$$

Let us suppose that $R_1 = R_2$; m_{1+2} results in this case to be exactly twice m_2. Generalizing, we can say that the mass is proportional to the gauge beta-function:

$$m \; \propto \; \frac{\beta_0}{R} \, , \tag{4.11}$$

where β_0 is the gauge beta-function coefficient (the beta-function is $\beta_0 \log \mu$, μ an appropriate scale). This can also be expressed as:

$$\Delta\left(\frac{1}{m}\right) \approx \frac{\Delta m}{m^2} \sim \Delta\beta_0 \times R\,, \tag{4.12}$$

to be compared with a similar expression one can write for the couplings:

$$\Delta\left(\frac{1}{\alpha}\right) \sim \Delta\beta_0 \times \log\mu\,, \tag{4.13}$$

that we derive from the one-loop renormalization of the inverse of a gauge coupling: $\frac{4\pi}{g^2} \sim \frac{4\pi}{g_0^2} + \beta_0 \ln\mu$. According to our previous discussion about the couplings, the strength $\alpha(G)$ is by definition proportional to the volume of the group, $||G||$ (not to be confused with the volume of the Lie algebra $||\mathfrak{g}||$), and we can write:

$$\frac{\alpha(G_i)}{\alpha(G_j)} = \frac{||G_i||}{||G_j||}\,. \tag{4.14}$$

On the other hand, also masses are related to volumes of symmetries, so that we can write a similar expression:

$$\frac{m_k}{m_\ell} = \frac{||G_k||}{||G_\ell||}\,. \tag{4.15}$$

The comparison of these two expression with 4.12 and 4.13 suggests that:

$$\frac{m_k}{m_\ell} = \frac{\alpha(G_k)}{\alpha(G_\ell)}\,, \tag{4.16}$$

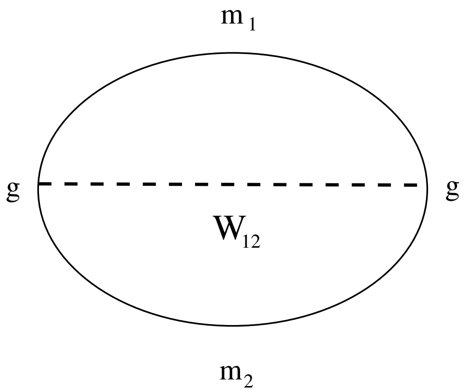
a relation which implies the identification of the radius R entering in the perturbative expression of masses with $\log\mu$, as it is to be expected if *in the perturbative construction the space coordinates are a logarithmic representation of the physical space* [27] . Expression 4.16 can also be written as:

$$\frac{m_i}{m_j} = \alpha(G_{ij}) = ||G_{ij}||\,, \tag{4.17}$$

where G_{ij} too is a symmetry group. In terms of familiar Feynman diagrams, this can be undersood as follows. Let's consider the following diagram for the vacuum renormalization

[27] More precisely, the comparison suggests the identification up to a proportionality constant κ on the tangent space, i.e. $R \sim \mu^\kappa$.

in the case of a two-particle theory with a broken symmetry relating particle 1 and particle 2:



(4.18)

This has to be thought not as a field theory diagram, but as a diagram for our effective non-perturbative theory. There is no question about non-renormalizability of a theory without a Higgs particle: here masses have to be treated as effective parameters, more or less like what one does with the electric field in a semiclassical approximation of a quantum relativistic scattering by an external field. The theory is finite, and we know from above that this diagram, intended as the non-perturbative resummation of the terms corresponding to this transition process, should correct the vacuum amplitude *multiplicatively*, by a factor 1 [28]: this means in fact that there is no renormalization of the vacuum, when for g, m_1, m_2, $m_{W_{12}}$ we take the "on-shell", non-perturbatively resummed physical parameters. By computing the contribution of this diagram, we find:

$$1 \approx g^2 \times \int \frac{d^4p_1 d^4p_2}{(\not{p}_1 + m_1)(\not{p}_2 + m_2)\left((p + p_1 - p_2)^2 - M_W^2\right)} \sim g^2 \int^{<p>} \frac{d^8p}{p^4}\,, \tag{4.19}$$

where the integration has to be performed up to a cut-off $< p >$. This corresponds to the typical mass scale of the (sub)space in which the process takes place. In practice, we can consider that the subsystem consisting of the particles 1, 2 and the boson W_{12} "lives" in a space with "temperature" given by the average of the temperatures of the three states. Equivalently, we can think that the typical length of the process is the (multiplicative) average of the typical lengths associated to the three states. In any case,

$$< p > \sim \left(m_1 m_2 M_W^2\right)^{1/4}. \tag{4.20}$$

Therefore, the integral pops out a factor $[< p >]^4 = m_1 m_2 M_W^2$, where the W_{12} mass is T-dual to the lightest particle mass:

$$M_W^2 = \frac{1}{m_2^2}\,, \tag{4.21}$$

where by convention we have chosen m_2 to be the lower mass, and T-duality is performed with respect to the Planck mass. From this we derive that:

$$g^2 m_1 m_2 M_W^2 = g^2 \frac{m_1}{m_2} \approx 1 \tag{4.22}$$

[28]We recall that for us the additive representation corresponds to a logarithmic representation of the physical string vacuum.

The ratio of the two masses is therefore given by the inverse square coupling of the broken symmetry. We stress that these arguments make only sense in our scenario, not in a generic field theory. In this example, the boson mass, T-dual to the mass of the particle, is higher than the Planck scale, and there is no boson line really coming out from the line of the particle transition: the latter looks like a rigid symmetry. When the transition is "off-diagonal" with respect to the traditional classification of particles into families, we interpret this phenomenon as a "family mixing", whose effect is collected in the off-diagonal entries of the CKM mass matrix.

Expressions 4.16 and 4.17, together with the identification of the logarithm of the scale of perturbative renormalization of the couplings, $\log \mu$, with the scale R of the extended space, tell us that not only masses, but also the couplings scale as powers of the radius, or equivalently the age, of the universe. Indeed, the idea of renormalization group is based on the possibility of letting the parameters run as functions of a scale, in such a way that the sum of higher order terms can be re-written so as to reproduce the functional expression of the first non-trivial correction. This means that there is always an appropriate scale $\tilde{\mu}$ and an effective "beta-function" $\tilde{\beta}$ for which $\tilde{\beta} \ln \tilde{\mu}$ constitutes a good approximation to the expression of the coupling, at least for its perturbative part. Since the logarithmic picture is for us precisely the representation in which the coupling of the theory vanishes, this could be taken somehow as the definition of the running of a coupling in this picture. And indeed, a series like the usual perturbative expansion of the corrections to a coupling:

$$\partial \ln \alpha / \partial \ln \mu \approx \sum_n \beta_n \alpha^n \,, \tag{4.23}$$

somehow looks like the expansion around a small value of the coupling of an exponential expression, of the type:

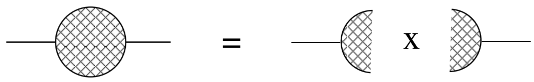
$$\alpha(\mu) \sim \exp \left[\beta \ln \mu \right] \,, \tag{4.24}$$

over $\ln \alpha$:

$$\ln \alpha \sim \beta \ln \mu \,. \tag{4.25}$$

In our approach, 4.24 expresses the full, non-perturbative behaviour of a coupling, whereas a perturbative series 4.23 constitutes only an approximation. Since any field-theoretical approach is in itself only an approximate representation of the full physical content, encoded in 1.1, in this framework it does not make sense to look for a rigorously proof that the first perturbative corrections to a gauge coupling are the first terms of the expansion of 4.24.

Since the perturbative slices of the string configuration we want to consider correspond to logarithmic realizations (i.e. on the tangent space), in order to evaluate the volumes in the phase space associated to the masses it is convenient to think in terms of entropy. The mass is the ground energy of a system, and corresponds to a "ground entropy": we may think that an object with mass possesses a "ground entropy". In the case of a black hole, entropy can be seen as somehow related to a counting of the elementary-state paths going from outside into the horizon of the black hole [51, 52, 53, 54]. In a similar way, here the relation between mass corrections and entropy, established in 4.1 and 4.27, suggests that the "ground entropy" of a particle can be seen as a way of counting the paths leading to the particle. The "bubbles" of the terms in 4.1 represent in fact paths that come out of the particle and end up back into the particle itself. The equivalence with the usual interpretation of the entropy of a black hole is more evident if we ideally view these terms as made up of two mirror cuts:



(4.26)

Each half looks like a decay process, or the other way around as the process of formation of a black hole. The mass renormalization can be seen as a physical process in which the final state is the same particle as the initial state: no lines are propagating out of the bubble. It corresponds therefore to an adiabatic process. The first law of thermodynamics tells us that in this case,

$$dQ \, = \, 0 \, = \, dE \, - \, TdS \, . \tag{4.27}$$

If the temperature were constant, we would conclude that the variation of energy along this process corresponds to the variation of entropy. However, the temperature of the system is related to the energy, and varies with energy. Consider the case of the universe itself. We have seen that its mass, i.e. its rest energy, scales with time, $\mathrm{M}_{\mathrm{Univ.}} \sim \mathcal{T}$, while the temperature scales as the inverse of its age: $T \sim \mathcal{T}^{-1}$. Finally, entropy scales as $S \sim \mathcal{T}^2$. In this case, $dS \sim d\mathcal{T}^2 \sim (2)\mathcal{T}$ and the relation 4.27 reads:

$$dE \, \equiv \, d\mathrm{M}_{\mathrm{Univ.}} \ \sim \ \frac{1}{\mathcal{T}} \times dS \, d\mathcal{T} \ \approx \ d\mathcal{T} \, .$$

Let's now consider the case of a "universe" consisting of just one particle. In other words, let's restrict these considerations to the system illustrated in fig. 4.1. In this case, the "universe" behaves in itself like a quantum black hole. It possesses a "temperature", which is related to the inverse of its "age", and is therefore proportional to its minimal energy gap, given by the lower bound of the Heisenberg inequality: $\Delta E \sim 1/\mathcal{T}$. This energy gap is the mass of the particle itself: $m \sim \Delta E$, and therefore the mass of the particle is proportional to the temperature: $m \sim T$. In this case, equation 4.27 reads:

$$\frac{dm}{T} \sim \frac{dm}{m} \, = \, dS \, . \tag{4.28}$$

Once integrated, it gives:

$$\ln m \, = \, S \, + \, \mathrm{const.} \, , \tag{4.29}$$

which is again the relation $m \sim \exp S = W$, stating that the mass is proportional to the weight in the phase space.

In order to evaluate entropy, we proceed as follows. If we think the string space as fibered over the "space-time" base, we can say that for any coordinate of the fiber there is a phase space of space-time size. Since the size of the internal coordinates, those of the fiber, is one in Planck units, the volume of the phase space will be:

$$\mathcal{V}_P \, = \, \mu^{\beta} \, , \tag{4.30}$$

where μ is the length (the volume) of space-time, and β a coefficient. For instance, if we start with a phase space of volume μ^{β_0} and act on the states with a Z_2 projection that halves the internal space, the un-projected states will have a phase space at disposal for their decays given by $\mu^{\beta_0/2}$. The probability density is proportional to the inverse of the volume 4.30:

$$P(x) \, = \, \frac{1}{\mathcal{V}_P} \, , \tag{4.31}$$

and entropy, given by definition as $S = -\sum P \ln P$, is then:

$$S \, = \, - \int_{\mathcal{V}_P} dx \, \frac{1}{\mathcal{V}_P} \, \ln \frac{1}{\mathcal{V}_P} \, = \, \beta \ln \mu \, , \tag{4.32}$$

where the result follows immediately from the fact that $\mathcal{V}_P$ is a constant in the domain of integration. After a projection like the one of above, entropy will be reduced by one half: $S \, \to \, (\beta/2) \ln \mu$. The mass renormalization reads:

$$\ln m \, = \, \beta_0 \ln \mu \, + \, \beta \ln \mu \, + \, \text{const.} \, , \tag{4.33}$$

that we can also write as:

$$\ln m \, = \, \ln m_0 \, + \, \beta \ln \mu \, + \, \text{const.} \, . \tag{4.34}$$

This expression suggests that the term $\beta \ln \mu$ can be calculated in a logarithmic representation, and the coefficient β, corresponding to the "exponent" of the volume of the subspace of the phase-space of the particle, can be obtained by computing the amount of projections acting on the sector under consideration. We have separated a term β_0, because we must allow for a minimal value of entropy. At finite space-time volume, the minimal mass gap is not arbitrarily small. This sets also the minimal entropy allowed for the process, and results in a "minimal subtraction" in the phase-space volume. After exponentiation, we obtain:

$$m \, = \, (\text{Const.}) \, \times \, m_0 \, \exp(\beta \ln \mu) \, = \, (\text{Const.}) \, \times \, m_0 \, \mu^{\beta} \, . \tag{4.35}$$

The "bare" mass m_0 is given by the inverse square root of the age of the universe: $m_0 = 1/\mathcal{T}^{1/2}$. This value, common to all particles, is produced by the shift acting along the space-time coordinates, that we described in section 2.1.4. In the logarithmic picture, where, owing to the linearization of space, shifts along the coordinates contribute additively, it appears as an additive term:

$$\ln m_0 \, = \, \frac{1}{2} \ln \frac{1}{\mathcal{T}} \, \equiv \, \beta_0 \ln \mu \, , \tag{4.36}$$

as in 4.34. In the two pictures the mass renormalization reads therefore respectively:

$$
\begin{array}{llcl}
(\text{log picture}) & \tilde{m}\,[=\ \ln m] & = & \dfrac{1}{2}\ln\dfrac{1}{\mathcal{T}} \; + \beta \ln \mu \\[2ex]
 & & \equiv & \tilde{m}_0 \; + \; \beta \ln \mu \\[2ex]
 & \xrightarrow{\exp} & & \\[2ex]
(\text{real picture}) & m & = & \mathcal{T}^{-\frac{1}{2}} \times \mu^{\beta} \, .
\end{array}
\tag{4.37}
$$

In these expressions, μ is the age of the universe raised to some power: $\mu = \mathcal{T}^p$. It is clear that any change in p can be reabsorbed by a change in β and β_0. We set by convention $\mu \equiv \mathcal{T}$, so that $\beta_0 = -1/2$. By inserting these values in 4.35, we obtain that masses scale as:

$$
m = (\text{Const}) \times \mathcal{T}^{\beta - 1/2} \, , \tag{4.38}
$$

With the identification of μ^{κ} with the age of the universe, $\mathcal{T}$, the ratios 4.15–4.17 can be written as:

$$
\frac{m_i}{m_j} = \alpha(G_{ij}) = \mathcal{T}^{\beta_i - \beta_j} \, . \tag{4.39}
$$

Notice that, although the mass of a particle decreases with time, the "ground entropy" increases. This is due to the fact that entropy is related to the statistics of the physical system, i.e. to the structure of the phase space. As it can be seen from the Uncertainty Relations ($\Delta E \Delta t \geq 1/2$), in this space the volume of the minimal cell goes like $\sim 1/\mathcal{T}$. It decreases therefore much faster than any mass scale, which is bounded by the square root scale: $m \geq 1/\mathcal{T}^{1/2}$ for any mass m. The "ground" phase space volume of a particle is:

$$
\mathcal{V}_0 = \frac{m}{\Delta E} \rightsquigarrow \frac{\mathcal{T}}{\mathcal{T}^{\frac{1}{2}}} \sim \mathcal{T}^{1/2} \, . \tag{4.40}
$$

As time goes by, the phase space of a particle increases, and therefore also its entropy. In section 4.4.6 we will comment on the relation of this approach to the one based on the "geometric probability" techniques.

In order to obtain the masses, we must first obtain the "beta-functions" β_i, β_j. According to our discussion, we cannot compute them using the rules of ordinary field theory: here we are interested in the full, non-perturbative beta functions. We can proceed as follows: we can determine the ratios of these beta-functions if we know the ratios of the phase-space volumes. For instance, if a projection reduces by one half the phase space, the beta-function will be one half of the initial one. On the other hand, the phase space volumes can be determined if we know the spectrum of interactions of the various particles (i.e., the pattern of figure 3), but the important point is that, in this framework, this in principle is equivalent to knowing the chain of projections, ≡ symmetry reductions, giving origin to the sector a certain massive state belongs to.

In order to recover the correct dependence on the radius of the extended space, which as we have seen does not show out correctly in perturbative orbifold constructions, we will have to combine several considerations in subsequent steps, that somehow constitute a sequence of perturbative approximations, in which the fine details of the structure of the physical configuration of highest entropy are the better and better investigated. This way of proceeding makes sense, because these corrections, depending on inverse powers (roots) of the age of the universe, at present time are sufficiently small.

Once obtained the ratios of beta-functions, in order to get all their values we must fix one of them. To this purpose, we must consider that the mass of the state with maximal, unbroken symmetry, does not change with time, it is a constant. Maximal symmetry, and therefore also supersymmetry, implies in fact that masses either vanish (as also the cosmological constant does), or they do not renormalize out of the initial value at the Planck scale: among the preserved symmetries, there is in fact also time reversal, so that masses do not run. In this case, we have:

$$m_{\text{max. symm.}} \; = \; \frac{1}{2}\,\mathcal{T}^{\beta_{\max}-\frac{1}{2}} \; = \; \text{Constant}\,, \tag{4.41}$$

where we have set the normalization of the mass as a function of the inverse of a proper time to be 1/2, as according to the Heisenberg's Uncertainty Principle. The condition 4.41 implies $\beta_{\max} = \frac{1}{2}$. With this normalization, at the Planck time the maximal mass is the one of a black hole, in agreement with our discussion of section 3, namely it is given by the relation: $2M \; = \; R$, with the identification $R = \mathcal{T}$. Therefore, at the Planck time the minimal mass excitation is forced by the Uncertainty Principle to be 1/2. On the other hand, being the universe a Planck size black hole, this is also the maximal allowed mass excitation. If we allow one coefficient to be greater than 1, there exists a certain size of the universe, *larger* than the Planck length, at which the mass is larger than the black hole's Schwarzschild mass. If we run back in time, starting from the present age, where this state appears among those of the sub-Planckian spectrum, this excitation drops out from the spectrum, by definition formed by the degrees of freedom which are below the black hole threshold, *before* reaching the Planck scale. Therefore, it cannot correspond to a perturbation over the massless spectrum corresponding to some momentum (= inverse of radius) of the string space, of the kind of those giving rise to the effective, "low energy" theory. From 4.39 we see that the overall normalization is the same for all the states which are $SU(3)$, $SU(2)_{\text{w.i.}}$ and $U(1)_{\text{e.m.}}$ singlets:

$$m_i \; = \; \kappa_i \mathcal{T}^{\beta_i}\,, \qquad \kappa_i = \kappa \;\; \forall\, i\,. \tag{4.42}$$

Therefore all these masses are expressed as:

$$m \; = \; \frac{1}{2}\,\mathcal{T}^{\beta-\frac{1}{2}}\;, \qquad \beta \in \{0, \frac{1}{2}\}\,. \tag{4.43}$$

4.3.1 Elementary masses

Let's indicate with p the "beta" function coefficient, i.e. the exponent of the phase-space volume, $\mathcal{V} = \mu^{\beta}$, of the maximally symmetric configuration, and with q the one of the end-point, minimal symmetry (minimal entropy) configuration. The couplings of the various subgroups in which the initial symmetry group gets divided are then given by:

$$\alpha_i^{-1} \, = \, \mu^{\frac{n_i - n_{i+1}}{p}} \, , \tag{4.44}$$

where the ratios $\frac{n_i}{p}$, with $n_i : p = n_0 \geq n_{\rm i} \geq q$, are the volume exponents of the various steps. n_0 corresponds to the last step (the one which selects the lightest neutrino). An accurate evaluation of masses is then equivalent to an exact determination of these coefficients. Our "perturbative" approach starts with a first degree of approximation, consisting of a "rough" determination of the volume of the phase space of each elementary particle, as seen "at the Planck scale". This allows us to map to a logarithmic picture, where all couplings are perturbative, because they are of order one in the original picture. Further corrections of the weak coupling scale are to be expected at the present age of the universe. This leads us out of the domain of a logarithmic picture; the problem can in principle be treated as an ordinary perturbative correction, whose complete evaluation must take into account the details of every decay channel. In any case, these corrections should be of second order, with a relative magnitude proportional to the inverse ratio of the phase volume of the particle under consideration and the one of its decay products. Namely, we expect:

$$m \, = \, m_0 \, + \, \delta m \, , \tag{4.45}$$

where

$$\frac{\delta m}{m} \, \sim \, \mathcal{O} \left[m_{\rm final} / m_{\rm initial} (\equiv m) \right] \, . \tag{4.46}$$

We will consider these corrections in sections 5.2–5.5.2.

Let's consider the first step of the analysis, the one in which we can map to the logarithmic picture. If the hierarchy of the three families were ordered in such a way that all the particles of the higher family were heavier than those of the neighbouring lighter family, we could immediately conclude that the phase space is divided by the particle's families into three parts, with volumes staying in ratios given by 3:2:1. Namely, in passing from one family to the other one, the phase space volume $\mathcal{V}$ should undergo a contraction $\mathcal{V} \to \mathcal{V}^{\frac{2}{3}} \to \mathcal{V}^{\frac{1}{3}}$. However, this sequence is valid only as long as *all* the particles of the "heavier" family are indeed all heavier than the particles of the following family. Otherwise, the pattern changes. In order to understand the implications of this condition, we must consider that relating the mass to the corresponding volume in the phase space roughly means that heavier particles are also the more interacting ones. On the other hand, saying that all the particles of the heaviest family are heavier than those of the lighter families means in particular that even the tau-neutrino, ν_τ, is heavier than the charm and strange quarks, something not expected for a particle definitely less interacting. Indeed, we expect the three neutrinos to be the lightest among all particles. This means that the phase space is *first* separated into the block of neutral and charged particles, and *then* each block is contracted according to the 3:2:1 rule. Apart from the separation of the $SU(4)$ symmetry into $\mathbf{1} \oplus \mathbf{3}$ of leptons and quarks, all the other separations are built on $SU(2)$ steps, as a consequence of the Z_2 structure of the projections. We expect therefore the $SU(2)$ coupling to play a key role in the mass ratios. In principle, an $SU(2)$ coupling factor separates also the up and down, both lepton and quark, inside each family. However, the coupling, and the group, of interest for us is not the left-moving $SU(2)$ of the weak interactions, which at the string scale remains unbroken: mass separations are in relation with the breaking of the $SU(2)_{(L)} \leftrightarrow SU(2)_{(R)}$ symmetry. Requiring that all neutrinos are lighter than any charged particle implies a "flip" in the action of the up/down separation of the $SU(2)$ doublets, such that the minimal block, the $SU(2)$-coupling, instead of acting "diagonally", by separating the tau lepton from its neutrino, acts off-diagonally, being interposed between the ν_τ and the lightest charged particle, the electron.

Let's start by considering the lightest steps: the mass ratios of neutrinos. They should be separated by "minimal blocks" consisting of $SU(2)$ coupling factors:

$$\frac{m_{\nu_\tau}}{m_{\nu_\mu}} \sim \frac{m_{\nu_\mu}}{m_{\nu_e}} \sim x\,, \qquad \frac{m_{\nu_\tau}}{m_{\nu_e}} \sim x^2\,, \qquad x = \alpha^{-1}_{SU(2)}\,. \tag{4.47}$$

According to the hypothesis of the 3:2:1 separation of the neutrino subspace of the phase space, a $x = \alpha^{-1}_{SU(2)}$ factor should also separate the mass of the lightest neutrino, $\nu_{\rm e}$, from the pure "vacuum" of the mass sector, namely the square-root energy scale corresponding to the basic shift: $\mathcal{T}^{-1/2}/2$, the scale which, in the language of 4.41, corresponds to $\beta = 0$. A further $\alpha^{-1}_{SU(2)}$ should then separate the heaviest neutrino from the lightest charged particle, the electron:

$$\frac{m_e}{m_{\nu_\tau}} \sim \alpha^{-1}_{SU(2)}\,. \tag{4.48}$$

As we discussed, from now on we should expect a 3:2:1 relation between the phase space sub-volumes of the three families in the logarithmic picture:

$$\ln \mathcal{V}(t,b,\tau) \;:\; \ln \mathcal{V}(c,s,\mu) \;:\; \ln \mathcal{V}(u,d,e) \quad \approx \quad 3:2:1\,. \tag{4.49}$$

In order to determine one of these volumes, what we need now is to know the down-quark-to-lepton mass ratio. From the down to the up quark the separation should be again an $SU(2)$ coupling factor. There are however subtleties. First of all, we must notice that it is $\mathcal{V}(t,b,\tau)$ that contains these factors at their "ground" level. The reason is that, as we have seen in section 2.2, the shift which, acting along the space-time coordinates, realizes a further "$SU(2)$ step" in the reduction of symmetry, breaking the group of the weak interactions, also introduces a mass differentiation in the matter states of the same order of the scale of the breaking of the gauge group. The full width of the phase space separation corresponds therefore to the scale of the breaking, i.e. the heaviest scale at which a mass separation between matter states related to this broken symmetry appears. We will derive $\mathcal{V}(c,s,\mu)$ and $\mathcal{V}(u,d,e)$ as fractional powers:

$$\mathcal{V}(c,s,\mu) \sim [\mathcal{V}(t,b,\tau)]^{\frac{2}{3}}\;; \quad \mathcal{V}(u,d,e) \sim [\mathcal{V}(t,b,\tau)]^{\frac{1}{3}}\,, \tag{4.50}$$

A second subtlety is that, in the case of quarks, we expect the $\alpha_{SU(2)}$ factor between the top and bottom quark to separate not the masses of the single quarks, but $SU(3)$ triplets, i.e. singlets of the confining symmetry. In other words, the equivalence of leptons and quarks is established at the level of asymptotic states, which are singlets for the confining theory. We expect therefore a factor 1/3 correcting the mass ratio between the top and bottom quark. This normalization is due to the requirement that, once run back to the Planck scale, only $SU(3)$ singlets (in this case quark triplets) go to the same limit mass value as the leptons [29]. The top-to-bottom mass ratio should therefore be:

$$\frac{m_t}{m_b} \sim \frac{1}{3} \alpha^{-1}_{SU(2)} \,. \tag{4.51}$$

The mass separation between quarks and leptons is the consequence of the breaking of the **4** of each family into $\mathbf{3} \oplus \mathbf{1}$. This separates the phase space in two parts of unequal volumes. In first approximation, this separation corresponds to disentangling "one quarter" of $SU(4)$, and therefore we expect the "up" of the **1** part to lie a $\sqrt{\alpha_{SU(2)}}$ factor below the "down" of the **3** part. This is the separation factor between bottom quark and τ lepton:

$$\frac{m_b}{m_\tau} \approx \frac{1}{3} \frac{1}{\sqrt{\alpha_{SU(2)}}} \,. \tag{4.52}$$

Again a 1/3 factor is needed in order to account for the passage from $SU(3)$ singlets to free quarks. Altogether, the top-tau mass ratio is:

$$\frac{m_t}{m_\tau} = \frac{m_t}{m_b} \times \frac{m_b}{m_\tau} \sim \frac{1}{3} \alpha^{-1}_{SU(2)} \times \frac{1}{3} \frac{1}{\sqrt{\alpha_{SU(2)}}} \,. \tag{4.53}$$

According to 4.50, the analogous separation for the first family should read:

$$\frac{m_u}{m_e} \sim \frac{1}{9} \left(9 \frac{m_t}{m_\tau} \right)^{\frac{1}{3}} \,, \tag{4.54}$$

where we have first removed the $\frac{1}{3} \times \frac{1}{3}$ factor from the m_t/m_τ ratio, and then reintroduced it after having taken the third root. This was required by the fact that these normalization factors, accounting for the passage from free quarks to $SU(3)$ singlets, don't enter in the contraction of phase sub-spaces. Putting all the informations together, we conclude that the phase-space sub-volume of the charged particles of the first family, $\mathcal{V}(u, d, e)$, should be given by:

$$\mathcal{V}(u, d, e) = 9 \frac{m_u}{m_{\nu_\tau}} \sim \alpha^{-1/3}_{SU(2)} \left(\sqrt{\alpha_{SU(2)}} \right)^{-1/3} \times \alpha^{-1}_{SU(2)} \,. \tag{4.55}$$

The second and third power of this volume give finally $\mathcal{V}(c, s, \mu)$ and $\mathcal{V}(t, b, \tau)$. To summarize, the mass ratios are:

$$\frac{m_\mu}{m_{\nu_\tau}} \sim \alpha^{-2}_{SU(2)} \,; \tag{4.56}$$

[29] We will discuss below how further normalization factors are needed in order to take into account the fact that at the Planck scale also the electromagnetic and weak interactions are "strong", so that masses must be normalized with respect to singlets of these interactions too.

$$\frac{m_s}{m_{\nu_\tau}} \sim \frac{1}{3}\left(\sqrt{\alpha_{SU(2)}}\right)^{-2/3}\ \alpha_{SU(2)}^{-2}\,; \tag{4.57}$$

$$\frac{m_c}{m_{\nu_\tau}} \sim \frac{1}{9}\,\alpha_{SU(2)}^{-2/3}\ \left(\sqrt{\alpha_{SU(2)}}\right)^{-2/3}\ \alpha_{SU(2)}^{-2}\,; \tag{4.58}$$

$$\frac{m_\tau}{m_{\nu_\tau}} \sim \alpha_{SU(2)}^{-3}\,; \tag{4.59}$$

$$\frac{m_b}{m_{\nu_\tau}} \sim \frac{1}{3}\left(\sqrt{\alpha_{SU(2)}}\right)^{-1}\ \alpha_{SU(2)}^{-3}\,; \tag{4.60}$$

$$\frac{m_t}{m_{\nu_\tau}} \sim \frac{1}{9}\,\alpha_{SU(2)}^{-1}\ \left(\sqrt{\alpha_{SU(2)}}\right)^{-1}\ \alpha_{SU(2)}^{-3}\,. \tag{4.61}$$

These relations are completed by:

$$m_{\nu_\tau} \sim \frac{1}{2}\,\mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \left[\alpha_{SU(2)}^{-1}\right]^3\,. \tag{4.62}$$

$$m_{\nu_\mu} \sim \frac{1}{2}\,\mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \left[\alpha_{SU(2)}^{-1}\right]^2\,. \tag{4.63}$$

$$m_{\nu_e} \sim \frac{1}{2}\,\mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \alpha_{SU(2)}^{-1}\,. \tag{4.64}$$

What we expect to be able to normalize is not the single electron's and neutrino mass, but the $SU(2)_{\text{w.i.}}$-neutral combination (e, ν_e). Furthermore, what we should be able to normalize is not the pure electron's mass but that of the electrically neutral "compound" $(e, \bar{e})$. In the first case, we can say that $m_{(e,\nu_e)} \sim m_e + m_{\nu_e} \sim m_e$. In the second case, however, similarly to what happens with $SU(3)$ and the quarks, we expect $m_{(e,\bar{e})} \sim 2m_e$. Of course, analogous considerations apply to all families, and to quarks as well, because they are all charged also under $SU(2)_{\text{w.i.}}$ and $U(1)_{\text{e.m.}}$. If an almost logarithmic sequence of masses in passing from ups and downs of $SU(2)$ doublets allows in general to neglect the correction due to the lighter particle of the pair, what we cannot neglect is the factor 2 due to the fact that, as it was for the case of the non-perturbative mean scale, section 4.4.1, we are calculating the mass of a particle-antiparticle pair. As a consequence, we expect that with the formulae obtained in this section what we get is twice the mass of any state.

The values we obtain in this way are just the "bare" values of the mass ratios, the first step in the approximation, which must be improved by "actual time" corrections, in order to account for finer details of the phase spaces. We didn't consider yet the quark masses of the first family. As it appears from our discussion, the up quark seems to be heavier than the down quark, as it is reasonable to expect by analogy with the other families. However, this is wrong, as is also clearly indicated by the experimental observations. In the next sections we will pass to the explicit evaluation of all the mass values. We will there discuss also the corrections to the bare expressions, required by an improved description of the details of the string configuration. This is particularly necessary in order to discuss the masses of the second family, strongly affected by the "stable" mass scale of the universe, the mean scale discussed in section 4.4, and the quarks of the first family. As we will see in section 5.1.2, what happens in this case is that consistency of the vacuum implies an exchange in the role of the up and down quark. The mass relations are shown in the table 4.

4.3.2 The $SU(2) \equiv SU(2)_{\Delta m}$ coupling

In order to compute masses, what remains to know is the beta-function of the broken $SU(2)$ group which constitutes the basic ingredient of mass ratios. In principle, this is not the $SU(2)_{\text{w.i.}}$ of weak interactions, which acts only on the "left-moving" part of the particles. We indicate it as "$SU(2)_{\Delta m}$", to distinguish it from the group of the weak interactions. In order to determine the $SU(2)_{\Delta m}$ beta-function, we cannot proceed as in the traditional approach, through a (perturbative) analysis of the spectrum and the corrections to the gauge coupling. Any perturbative computation, therefore performed in a logarithmic representation, does not simply account for the "logarithm of" the true beta-function: in any explicit string realization, part of the spectrum is perturbative and weakly coupled, but there is also a part which is "hidden", either because entirely non-perturbative, or because at least part of its interactions are non-perturbative. Therefore, it makes no sense to count the states of the spectrum, compute the beta-function as is usual in field theory, and then trade this quantity for the logarithm of the real beta-function. In order to get this quantity, we proceed in another way. For the computation of the beta-function a passage to the logarithmic picture is dangerous: we don't know what are all the states, perturbative and non-perturbative, of the spectrum, and we don't know how do they exactly appear in the logarithmic picture [30]. The analysis of the spectrum and the interactions carried out in section 2 was based on a comparison of several, dual "logarithmic pictures", or perturbative constructions, no one of them accounting for the full content of the theory. The gauge charges appeared in a rather different way in dual constructions. As long as it is a matter of counting the degrees of freedom, and listing their transformation properties, gathering together information obtained by "patching" dual pictures proved to be sufficient. However, in order to derive the strength of the couplings, the question is: how do we "patch beta-functions"? On the other hand, we have seen that, with a good degree of approximation, we can analyze the pattern of projections leading to the minimal entropy vacuum through the counting of Z_2 projections in orbifold representations. Through this procedure, non-perturbative sectors are automatically taken into account, being generated and/or projected out by operations that we can easily trace in some picture. In order to be sure to not forget, in the counting of the beta function, some states or misunderstand their role, we will therefore derive the volume occupied by the broken $SU(2)_{\Delta m}$ by counting the volume reductions produced by the various projections we have applied in order to reach the configuration of minimal symmetry.

The analysis of section 2 tells us that, within the conformal theory, we have at disposal 7 internal and 2 extended transverse coordinates for the projections. In total, we can apply 7

[30]Indeed, we will discuss in section 4.4.5 how in this picture the spectrum of the configuration of highest entropy appears to be supersymmetric.

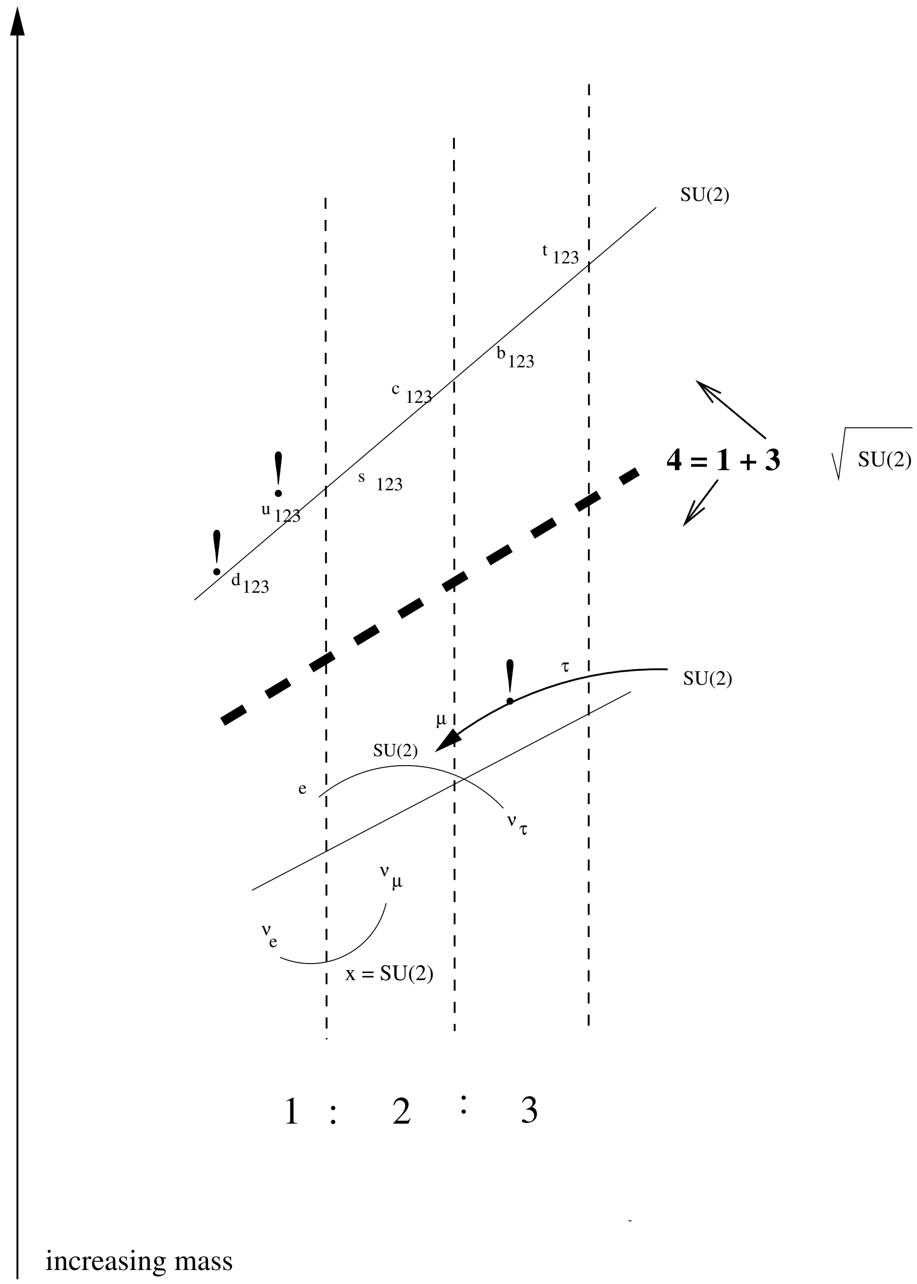


Figure 4: The diagram of elementary masses. Notice that up and down quarks are flipped. For leptons, the $SU(2) = SU(2)_{\Delta m}$ coupling factor separates the heaviest neutrino, ν_τ, from the lightest charged particle, the electron. This is due to the rearrangement of the $\mathbf{1}$ in the $\mathbf{4} = \mathbf{1} \oplus \mathbf{3}$, so that the three neutral particles are also the lightest ones, as required from entropy considerations.

+ 2 = 9 projections [31]. We must however consider that, for what concerns our problem, two of them don't reduce the volume of the phase space: with a first projection, supersymmetry is reduced from $\mathcal{N}_4 = 8$ to $\mathcal{N}_4 = 4$. With a second projection, supersymmetry is further reduced to $\mathcal{N}_4 = 2$, and new sectors of the spectrum are generated. However, in our specific case also at the $\mathcal{N}_4 = 2$ level the gauge beta functions vanish. This is not a general property of any $\mathcal{N}_4 = 2$ vacuum, but it is precisely what happens in the case of a configuration obtained with non-freely acting projections, even when coupled with freely acting, rank-reducing shifts, as is our case [32]. Indeed, $\mathcal{N}_4 = 2$ is the step at which matter is generated in its full content, with the maximal amount of twisted sectors. For what concerns the gauge interactions, it is therefore the point of largest symmetry group. It is only through a further projection that, owing to the reduction to $\mathcal{N}_4 = 0$ [33], the gauge beta-function do not vanish anymore. Starting from this point, any reduction of the spectrum of matter states results in a corresponding reduction of the volume of the symmetry group. By counting the projections with the exclusion of the first two, we obtain that the last step, the one corresponding to the breaking to the minimal symmetry, corresponds, in the logarithmic picture, to a reduction of the volume of the phase space, with respect to the $\mathcal{N}_4 = 2$ configuration, by a factor $2 \times 7 = 14$. Through these projections the full initial symmetry group has *effectively* been reduced into a product of 14 equivalent factors, which correspond to $SU(2)$ enhancements of $U(1)$ symmetries. In other words, we can consider that the full symmetry of the phase space is a group G such that $G \supset U(2)^{\otimes 7}$. Each $U(2)$ factor rotates a subspace of dimension 2. Namely, on the tangent space (logarithmic representation) the "fundamental" representation is a direct sum of **2**:

$$\underbrace{\mathbf{2} \oplus \mathbf{2} \oplus \ldots \oplus \mathbf{2}}_{7 \text{ times}}, \tag{4.65}$$

or, after the last step, in which also the $U(2)$ symmetry is broken and we remain with $U(1)$, a sum of 14 $U(1)$ representations:

$$\underbrace{\mathbf{1} \oplus \mathbf{1} \oplus \ldots \oplus \mathbf{1}}_{14 \text{ times}}. \tag{4.66}$$

The "beta-function coefficient" (or better "exponent") of $SU(2)$ is then $\frac{1}{14}$ of the full exponent, which is fixed as follows. For $\mathcal{N}_4 = 2$ the beta function must vanish: no renormalization at all. The range of values is therefore $[0 - 1/2]$. According to expression 4.43, the beta-function exponent of $SU(2)_{(\Delta m)}$ is:

$$\beta_{SU(2)} = \frac{1}{14} \times \frac{1}{2} = \frac{1}{28}. \tag{4.67}$$

The coupling of $SU(2)_{(\Delta m)}$ is therefore:

$$\alpha_{SU(2)} = \mathcal{T}^{-\frac{1}{28}}. \tag{4.68}$$

[31] Said differently: we have room for shifts along 9 coordinates.

[32] See the discussion of section 2.1 and Ref. [15].

[33] We have seen that, although in some representation this configuration may appear as perturbatively supersymmetric with $\mathcal{N}_4 = 1$, supersymmetry is indeed broken.

Using the value of the age of the universe given in appendix A, we obtain that, at the present day, $\alpha^{-1}_{SU(2)} \sim 147$. If more precisely we use the age of the universe suggested by the agreement with neutron's mass, eq. 4.92 (i.e. $\sim 5,038816199 \times 10^{60} \mathrm{M}_{\mathrm{P}}^{-1}$, see Appendix A), we obtain:

$$\alpha^{-1}_{SU(2)} \sim 147,2 \ (147,211014) \,. \tag{4.69}$$

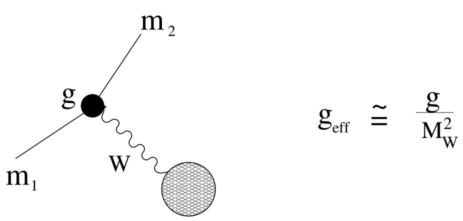
Being obtained through a counting of projections in the Z_2 orbifold approximation, 4.68 probably constitutes only an approximation of the real value of the beta function. However, we expect the relative correction to $\beta^{-1} = 28$ to be small, of the order of the relative magnitude of an inverse root of the age of the universe, as compared to this integer value:

$$\beta^{-1} \approx 28 + \mathcal{O}\left(\alpha^{-1} \mathcal{T}^{-1/p_\beta}\right) \,, \tag{4.70}$$

which should reflect in a similar correction also for the coupling α ($\alpha^{-1} \to \alpha^{-1}(1+\mathcal{O}(1/\mathcal{T}^{1/p_\beta}))$.

4.3.3 The $U(1)_\gamma$ coupling

In order to obtain the coupling of $U(1)_\gamma$, the electromagnetic group, we don't need to determine the absolute fraction of a group factor within the full symmetry group: we can determine the ratio of the $U(1)_\gamma$ and $SU(2)$ phase spaces, or equivalently the ratio of the two exponents, by counting the charged matter states, and subtracting the number of gauge bosons. We can justify this if we consider that the latter contribute somehow "in opposite way" to the matter-to-matter scattering probability amplitudes. Consider a diagram corresponding to a matter-to-matter transition:



(4.71)

For what concerns the initial and final matter states, we have that the larger the mass ratio between initial and final state, the larger is the decay amplitude. The boson mass appears instead at the denominator in the expression of the effective coupling, and suppresses the process.

A better way to see this is to consider that, as we will discuss in section 4.4.5, in the logarithmic picture the most entropic vacuum appears as effectively supersymmetric, with $\mathcal{N}_4 = 2$ extended supersymmetry. As seen from the logarithmic picture, the beta-function exponent is a $\mathcal{N}_4 = 2$ beta function coefficient. In this case $b = T(R) - C(G)$. An equal number of matter states and gauge bosons, transforming in the same representation, corresponds to an effective $\mathcal{N}_4 = 4$ restoration, a situation of non-renormalization, with vanishing beta-function exponent [34]. The phase space coefficient of $U(1)_\gamma$ is proportional to: 3(families) $\times$ 2($SU(2)$doublets) $\times$ $(\mathbf{1}+\mathbf{3})$(leptons + quarks) $\times$ 2(left + right chirality) [= 48] $-$ 1(gauge boson) = 47. Notice that, in the counting, we have considered that *all* the matter states are charged under $U(1)_\gamma$. Three states, the three neutrinos, are however uncharged. However, the electromagnetic charge is simply "shifted" from the central value $(\frac{1}{2}, -\frac{1}{2})$, but the traceless condition is preserved. As a result, the charge is only "rearranged" among the states: some states result more charged, some less. In total, the strength of the renormalization is the same as with a traceless $U(1)$ with a charge equally distributed among all the states. This is true in first approximation: from a field theory point of view, this would be strictly true if all the masses of the matter states were the same, i.e. vanishing. Otherwise, the diagrams corresponding to the contribution of different particles have different amplitudes. From the point of view of this work, the corrections to this first order approximation are kept to a "minimal" degree by the requirement of entropy maximization, which implies a minimization of the total strength of the electro-magnetic interaction.

The beta-function coefficient of $SU(2)$ is proportional to 48 (the same effective number of states as for $U(1)_\gamma$) minus 3 (the number of gauge bosons), i.e. 45, where the coefficient of proportionality is the same as for $U(1)_\gamma$. The ratio of the two coefficients is therefore:

$$\frac{\beta_{U(1)}}{\beta_{SU(2)}} = \frac{47}{45} \,. \tag{4.72}$$

Using 4.68 and 4.72, and the scale $\mu = \mathcal{T} \sim 5,038816199 \times 10^{60} \mathrm{M}_{\mathrm{P}}^{-1}$, the present age of the universe 4.92, adjusted on the neutron mass, we get:

$$\alpha_\gamma^{-1} \sim 183,777867 \,. \tag{4.73}$$

The has to be considered as a "bare" value of the coupling, not an effective coupling in the field theory sense. We will discuss in sections 4.4.4 and 5.4 how this value should be "run back" to obtain the effective coupling to be compared with the value experimentally measured at a certain scale.

4.3.4 The $SU(2)_{\mathrm{w.i.}}$ coupling

Determing the coupling of the $SU(2)$ of the weak interactions is even more problematic than determining α_γ. The point is that for us this symmetry is not spontaneously broken in the classical sense, and we cannot compute the beta-function coefficient in an effective theory with unbroken gauge symmetry. In the usual field theory approach, the $SU(2)$ acting on

[34] In the philosophy of this analysis, when the number of bosons matches the number of matter states, this also means that the representation of the symmetry group is the same: a differentiation in the symmetries of gauge and matter states would correspond to a less symmetric/more entropic phase produced by some operation, even in the case this is not explicitly related to broken gauge states. This is for instance the case of the separations into different planes introduced by orbifold projections, where these mechanisms appear in a "frozen phase".

just one of the two helicities transforms only half of the matter degrees of freedom, and therefore, if we neglect the contribution of the gauge bosons, its beta-function coefficient turns out to be one half of that of a "full" gauge group, namely, with a vectorial coupling to the matter currents. This is however true as long as the matter states are massless (on the other hand, once they acquire a mass, the gauge symmetry is broken). Massive states consist of both left and right degrees of freedom. From the point of view of the volume occupied in the phase space, although interacting with just their left-handed part, massive matter degrees of freedom count as much as left + right chiral states. The volume occupied by $SU(2)_{\text{w.i.}}$ is therefore something "intermediate" between the situation of pure chiral gauge symmetry acting on massless states, therefore on half the space of the degrees of freedom, and a full vectorial interaction. We don't know a rigorous way of counting the volume of this interaction in the phase space. If we want just to give a rough estimate, we can approximately consider that our coupling lies somehow "in between" the two situations: since in first approximation the matter states acquire a mass through a shift that reduces by half the logarithmic volume of the space (resulting therefore in a square-root scaling law), we can expect that the logarithmic volume occupied by $SU(2)_{\text{w.i.}}$ is the mean value between the one of the vectorial interaction (acting therefore on the same number of degrees of freedom as the massive matter states), and the one of the pure chiral interaction, viewed as acting on massless states:

$$\beta_{SU(2)_{\text{w.i.}}} \approx \frac{1}{2}\left(1+\frac{1}{2}\right) \times \frac{1}{28} \,. \tag{4.74}$$

The present-day value of the inverse of the $SU(2)_{\text{w.i.}}$ coupling should therefore be:

$$\alpha_w^{-1} \approx \mathcal{T}_0^{-\left(\beta_{SU(2)_{\text{w.i.}}}\right)} \sim 42,26 \,, \tag{4.75}$$

where we have used the estimate of the age of the universe 4.92. The value 4.75 is roughly a factor 4,4 smaller than the inverse electromagnetic coupling, given in 4.73. Also this number has to be considered a "bare" value, to be corrected in the way we will discuss in section 4.4.4.

4.3.5 The strong coupling

In our framework, the $SU(3)$ colour symmetry is always broken, and in principle there is no phase in which the strong interactions can be treated as gauge field interactions at the same time as the electromagnetic ones. In particular, there is no (under-Planckian) phase in which the strongly coupled sector comes down to a "weak" coupling, which merges with the other couplings of the theory to build up a unified model with a unique coupling, taking up the running up to the Planck scale. For us, the strongly coupled sector is strongly coupled at any sub-Planckian, i.e. field theory, scale. The coupling α_s will always be larger than one:

$$\alpha_s \sim \mathcal{T}^{-\beta_s} \,, \qquad \beta_s < 0 \,. \tag{4.76}$$

Indeed, the representation in terms of an $SU(3)$ gauge symmetry is something that belongs more to an effective field theory realization than to the non-perturbative string configuration we are considering. Namely, in our case we just know that, as soon as the space is sufficiently curved (i.e. symmetry sufficiently reduced), we have the splitting into a weakly and a strongly coupled sector, mutually non-perturbative with respect to each other.

In order to derive the exponent β_s, we must proceed as in section 4.3.2, by computing the amount of symmetry reduction, this time however in the "S-dual" representation. As we discussed in section 2, when seen from the point of view of the full space, this duality is indeed a T-duality . This is basically the reason why the coupling increases as the temperature of the universe decreases (or equivalently its volume increases). We expect therefore that, when seen from the point of view of a dual picture, the coupling arises in a vacuum which underwent the same amount of symmetry reduction as in the case of the dual $SU(2)$ case of section 4.3.2. However, the space-time coordinates feel a "contraction" which is T-dual to the one experienced in the picture of the electro-weak interactions. Therefore, when referred to the time scale of the "electroweak picture", the "beta-function" exponent, the coefficient β_s, should be 1/4 of its analogous given in 4.67. Of course, as seen from the electroweak picture, the sign is also the opposite (an inversion in the exponential picture reflects in a change of sign of the logarithm). We expect therefore:

$$\beta_s = -\frac{1}{4} \times \frac{1}{28} . \tag{4.77}$$

In other words, the strong coupling in itself should run as:

$$\alpha_s \sim (\mathcal{T}_{\text{dual}})^{\frac{1}{28}} , \tag{4.78}$$

but the time scale $\mathcal{T}_{\text{dual}}$ is related to $\mathcal{T}$ by an inversion *times* a rescaling. As the value 4.68 can be seen as the "on-shell" value at the matter scale $1/\sqrt{\mathcal{T}}$, logarithmically rescaled by a factor 1/2 with respect to the un-projected time scale $\mathcal{T}$, the scale $\mathcal{T}_{\text{dual}}$ feels an inverse logarithmic rescaling, $(1/2)^{-1}$. In total, as compared to the square-root scale, it has a logarithmic rescaling by a factor 4. In order to refer the value of the strong coupling to the square-root scale of the electroweak picture, we must therefore take its fourth root. The present-day "bare" value of the strong coupling is therefore [35]:

$$\alpha_s|_{\text{today}} \sim \left[(\mathcal{T}_0)^{\frac{1}{4}} \right]^{\frac{1}{28}} = \mathcal{T}_0^{-\beta_s} \sim 3,48 . \tag{4.79}$$

As in the case of α_γ and α_w, in order to be compared with the coupling currently inserted in scattering amplitudes also this one has to be "run back" in the way we will discuss in section 4.4.4 .

4.4 The mean mass scale

The mean mass scale corresponds to the inverse of the mean radius of space. In order to have an estimate of this value, it is enough to consider all the coordinates of unit size, apart the three which are of the order of the radius of the classical horizon, R, proportional to the age of the universe $\mathcal{T}$. Indeed, as we will see, the real spectrum of momenta is more

[35] Also in this case we don't need a high precision in the estimate of the age of the universe, whose value appears here rather suppressed.

complicated than just the list of inverse of radii of $3+n$ dimensional subspaces of the string target space, n running from zero to the dimension of the string target space D minus three. Nevertheless, since in the configuration of maximal entropy T-duality is broken we expect the corrections to the geometry of space to be weak, of the order of some (positive) power of the inverse of the age of the universe.

We have seen that three space coordinates are extended up to $\mathcal{T}$, while the other ones are frozen at the Planck scale. Once this is taken into account, expression 4.2 reads:

$$< m > \;\; \sim \frac{1}{\mathcal{T}^{3/D}} \,. \tag{4.80}$$

(here D is the dimension of space, so that the full space-time has dimension $D+1$). In order to obtain the value of the mean mass, it is now a matter of inserting the correct value for D. Until now we have considered, as is usually done, *linearized* realizations of the string space. We know however that from a non-perturbative point of view space-time is not so simply factorized. As we discussed, it seems that we are in the presence of 11 dimensional curved space-time, that, owing to the linearization introduced in the various dual "slices" of the theory, gives the impression to be twelve dimensional. Twelve dimensions are precisely what is required in order to embed in flat space an 11-dimensional space-time, of which 10 are coordinates of the curved space-like part. The "true", intrinsic space-time dimension is therefore 11, not 12, and the mean value of the mass scales as:

$$< m > \;\; \sim \left[\sqrt[10]{\left(\prod_i^{10} R_i = \mathcal{T}^3 \times 1^7 \right)} \right]^{-1} \;\; = \frac{1}{\mathcal{T}^{3/10}} \,. \tag{4.81}$$

If we include also the correct normalization of the mass, which, according to the Heisenberg's Uncertainty relation, $\Delta E \Delta t \geq \frac{1}{2}$, should be proportional to 1/2 the inverse of the space-time length, we conclude that the true value of the mean mass is:

$$< m > \;\; = \frac{1}{2} \, \mathcal{T}^{-\frac{3}{10}} \,. \tag{4.82}$$

In this expression, the time $\mathcal{T}$ is the age of the universe as seen from the string frame. The age of the universe, as derived with interpolations based on the usual Big Bang cosmology, is supposed to range from 11,5 and 14 billions years. Its central value is therefore $\sim 12,75 \times 10^9$ yrs, ($\sim 5 \times 10^{60} \mathrm{M}_{\mathrm{P}}^{-1}$, see Appendix A). Inserting it in 4.82, we obtain:

$$< m > \;\; \sim \; 7,49 \, \mathrm{GeV} \,. \tag{4.83}$$

It may seem surprising that the mean value of the mass, by definition given by:

$$\langle m \rangle \; = \sum_i^n \langle i | m | i \rangle \,, \tag{4.84}$$

is here obtained as a multiplicative average:

$$\langle m \rangle \; = \sqrt[n]{\prod_i^n \frac{1}{R_i}} \,. \tag{4.85}$$

This is due to the multiplicative nature of the phase space, and to the fact that any perturbative construction, in which elementary states can be observed as free states, is a representation "on the tangent space", a logarithmic realization ($g = 1 \to \log g = 0$) in which products are transformed into sums.

Let's see how this works in the case of the mean mass. Mean values of observables can be viewed as obtained by inserting an appropriate operator in 1.2:

$$\langle A \rangle \; = \; \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, A \, \mathrm{e}^{S} \psi \, . \tag{4.86}$$

Similarly to what is usually done in the ordinary path integral, we can imagine to produce the insertion of A in expression 4.86 by switching on, in the exponential of entropy, currents J that couple to the operator. In the case of the mass, these are radii deformations. Since entropy scales as the square of the "radius" (i.e. the age of the universe), eq. 3.2, we have:

$$\exp -S \;\approx\; \exp - \left(\prod^{n} R_i \right)^{2/n} \;\to\; \exp - \left(\prod^{n} (R_i + R_i J) \right)^{2/n} . \tag{4.87}$$

Notice that the integral deformation is $R_i \to R_i(1 + J)$ and not $R_i \to R_i + J$: the latter would be an infinitesimal deformation on the tangent space. The mean mass is therefore given by:

$$< m > \;\approx\; \left[\frac{\delta}{\delta J} \ln \mathcal{Z} \right]_{J=0} \;\approx\; \left(\prod^{n} R_i \right)^{1/n} . \tag{4.88}$$

Had we instead used as deformation the one of the tangent space, $R_i + J$, we would have obtained for the mean mass the additive formula $m \sim \sum 1/R_i$, typical of the traditional perturbative string approach.

4.4.1 The neutron mass

We want now to discuss the physical meaning of the mean mass scale just considered. According to its definition, eq. 4.84, the contribution to the mean value should be provided by the asymptotic stable mass eigenstate(s) of the theory. These are not necessarily elementary mass/interaction eigenstates: in general they will be compounds. Usually, one thinks at the singlets of the strong interactions, because the theory is constructed as a perturbative vacuum around the zero value of the electromagnetic and weak couplings. Here however the situation is different: a finite, non-perturbative functional mass expression, valid at any value of the space-time volume, corresponds to a regime in which not only the strong interactions are non-perturbative, but also the electroweak interactions cannot be considered weak: the perturbative description of electro-weak interactions is an approximation, whose degree of accuracy increases with the age of the universe [36]. The true free mass eigenstates are neutral to both strong and electroweak interactions. The mean value 4.81 corresponds therefore to

[36] The behaviour of these couplings will be discussed in sections 4.3.2 and 4.3.3.

the average value of the mass of stable matter in the universe. Since the time dependence of gauge couplings is much milder than that of masses:

$$\alpha \sim \frac{1}{\mathcal{T}^{1/p}}, \quad m \sim \frac{1}{\mathcal{T}^{1/q}}, \quad p \gg q, \tag{4.89}$$

already outside of a close neighborhood of the Planck scale we rapidly fall into a regime in which the gravitational interaction is weak, while all other interactions are still strong. This is the regime of interest for our problem (at precisely the Planck scale the configuration becomes trivial). In this phase, the only state neutral under strong, electromagnetic and weak interactions is a compound made out of a neutron-antineutron pair at rest, and their decay products, i.e. the proton-electron-neutrino/antiproton-positron-antineutrino system. At the "strong" limit of the weak coupling, family mixings can be neglected because one can assume that all heavier particles have rapidly decayed to the ground family. As it happens for stable matter, the decay probability of the neutron is compensated by an equal probability of the inverse process of neutrino capture, and the system is stable under weak interactions. It is invariant under charge reversal, and stable under electromagnetic interactions as well. This is the *only* singlet under all the above interactions, and therefore the only mass eigenstate at finite volume. At the present age of the universe, the volume of space-time is anyway large enough to assure weakness of the electro-weak interactions. This compound is therefore not necessarily a "bound state", as it has presumably been at earlier times. We expect expression 4.82 to account for the mass of the "composite bound state", i.e. roughly twice as much as the mass of the neutron-antineutron pair. Therefore:

$$m_{\mathrm{n}} = \frac{1}{4} < m > = \frac{1}{8} \mathcal{T}^{-\frac{3}{10}}. \tag{4.90}$$

By inserting in 4.90 the current value for the age of the universe, as obtained by extrapolating data of experimental observations within the theoretical framework of Big Bang cosmology, we obtain a value quite close to the neutron mass. Namely, from 4.83 and 4.90 we obtain:

$$m_{\mathrm{n}} \approx 937\,\mathrm{MeV}, \tag{4.91}$$

quite in good agreement with the value experimentally measured, $939,56563 \pm 0,00028$ MeV [55]. On the other hand, we recall that expression 4.81 is given up to orders $\mathcal{O}(1/\mathcal{T}^p)$, $p > 0$, and the age of the universe itself is not known with great accuracy. Therefore, the only thing we can say here is that our expression is *compatible* with the current experimental extrapolations. A more correct analysis would require a new derivation of the value of the age of the universe completely *within our framework*. Anyway, owing to the degree of approximation applied to the usual computations, we expect the data about the age of the universe, obtained by integrating the equations of motion for the expansion of the universe, to catch at least the correct order of magnitude.

On the other hand, within our theoretical scheme one can reverse the argument, and take the mass of the neutron as the most precise measurement of the age of the universe. In this case, we obtain as its actual value:

$$\mathcal{T}_0 = 12,62028271 \times 10^9\,\mathrm{yr}. \tag{4.92}$$

The fact that our mass formula gives as average mass the mass of the neutron is nicely in agreement with what we would expect from a universe behaving as a black hole. According to the common astrophysical models, a black hole is in fact the step just following the "neutron star" phase of a star at the end of its life. Our considerations of above suggest that the universe, as "seen from outside", can be roughly thought as a kind of neutron star at the point of transition to a black hole.

4.4.2 *The apparent acceleration of the universe*

We are now in the position to come back to the issue of the apparent acceleration of the universe. We have seen that the average mass of the stable matter scales with time as:

$$m \sim \mathcal{T}^{-3/10} \,. \tag{4.93}$$

If we take this mass as the reference for the atomic mass scale, we derive that the above behaviour induces an apparent shift in the frequencies of the light emitted at different distances from the observer, i.e. at different ages of the universe, due to the different scale of the atomic energy levels:

$$\frac{\tilde{\nu}_1}{\tilde{\nu}_2} = \left(\frac{\mathcal{T}_2}{\mathcal{T}_1} \right)^{\frac{3}{10}} \,. \tag{4.94}$$

Once "subtracted" from the bare red-shift 3.22, this gives an apparent, effective red-shift $z_{\text{app.}}$:

$$1 + z_{\text{app.}} = \left(\frac{\nu_1}{\nu_2} \right)_{\text{observed}} = \left(\frac{\mathcal{T}_2}{\mathcal{T}_1} \right)^{\frac{7}{10}} \,, \tag{4.95}$$

as if the universe were expanding with rate $\tilde{R} \sim \mathcal{T}^{7/10}$, normally expected for a matter dominated era.

At the base of what is considered an experimental evidence of the accelerated expansion of the universe is the observed acceleration in the time variation of the red-shift effect. Here, this effect receives a different explanation, in terms of accelerated variation of ratios of mass scales, and therefore of observed emitted frequencies. Indeed, one may question whether a pure expansion of the metric is observable. In the classical approach, the expansion occurs at the level of the overall scale factor of the space part of the Robertson-Walker metric:

$$ds^2 = dt^2 - R^2(t) \left[d\vec{x}^2 \right] \,. \tag{4.96}$$

From a physical point of view, the scale factor $R(t)$ precisely <u>defines</u> the speed of light, obtained from the condition $ds^2 = 0$, therefore $dx/dt = 1/R$. The classical argument is that the Robertson-Walker metric is the metric of the cosmological evolution, not the metric of local physics. This saves things from a formal point of view, but is not satisfactory from a <u>physical</u> point of view: saying that there is an expansion of the overall scale of the metric is equivalent to saying that there is an expansion of the scale in which space lengths are measured in terms of time length. In other words, saying that there is such an expansion means that there is an expansion (more precisely a contraction) of the speed of light. Suppose we want to compare wavelengths between present time and a time at which the scale was 1/2 of the present one. From a physical point of view, what we observe is radiation produced by atomic transitions, and we compare wavelength keeping fixed the period of the light wave. Since in the past time lengths are contracted by 1/2 with respect to today, during each period of the wave light travels twice as much as today. Therefore, the same atomic transition generates a photon with twice the wavelength as today. However, if the space scale is contracted, also energies are different. Energies scale in fact as inverse of lengths (consider for instance the electric potential, $V = e^2/R$). In our specific example, this means that energies were doubled, and, according to $E = h\nu$, also frequencies were doubled, or equivalently periods were halved. The same atomic transition produced therefore photons with twice the frequency, or half the period, as compared to today. This fact, combined with the fact that the speed was doubled, implies that, for the same physica phenomenon, the effective wavelength was the same as today: any such an overall scale of the metric is therefore physically unobservable.

4.4.3 The "unification" of couplings

As one can see, in our framework the fundamental scaling of couplings as a power of the age of the universe does not involve the "field theory gauge strength" g, the strength of the gauge covariant derivative, which couples the gauge field to the matter kinetic term, but the quantity $\alpha = g^2/4\pi$. On the other hand, it is α the physical coupling entering in any expansion, always given as a series of powers in $g^2/4\pi$. As a consequence, for us the quantities which go to 1 and unify, in the specific case at the Planck scale, are not the three couplings g_s, g_1, g_2 introduced through a gauge mechanism, but the effective strengths entering in scattering and decay amplitudes, namely the three couplings α_s, α_w and α_γ:

$$\alpha_i \, = \, \mathcal{T}^{-\beta_i} \quad \Longrightarrow \quad \lim_{\mathcal{T} \to 1} \alpha_i \, = \, 1 \, . \tag{4.97}$$

This in particular means that, at a certain scale, close to but below the Planck scale, the couplings g of the electro-weak interactions will become "strong": $g > 1$. This however doesn't mean that the corresponding interactions are going out of the weak coupling regime. In our framework, strictly speaking there are no gauge interactions, the gauge representation being only a useful approximation, and the gauge connection g doesn't have a particular physical meaning, besides being a useful tool in order to arrive to α. On the other hand, even at the "m_Z" mass scale, α_s is in our case larger than one. What is then the meaning of a $\sim 0,2$ value for this coupling, as predicted by the usual $SU(3)$ colour analysis, and "confirmed" by experiments? In our case, such a value would mean that the strong interactions are not strong at all, but well perturbative, as are the electromagnetic and the weak one. In the next section, we will discuss how the values we have obtained for the three couplings do compare with the parameters of an effective action, and therefore with the data one finds in the literature.

4.4.4 The effective couplings: part 1

The couplings α_γ, $\alpha_{\text{w.i.}}$ and α_s derived in section 4.3.2 and 4.3.3 and 4.3.5 run with time, and therefore with an energy scale, but not in the usual sense of the renormalization group. Namely, they are the *fixed* couplings at a specific age of the universe. The "electron mass scale", or the "Z-boson scale", here would mean a different age, and size, of the universe. The usual running according to the equations of a renormalization group refers instead to the "effective" rescaling in a universe in which the fundamental parameters remain fixed. This is due to the fact that space-time, the space in which the effective action is framed, is normally assumed to be of infinite extension. The infinities which are in this way produced in the effective parameters must be regularized according to certain rules; in practice, by keeping as reference points certain "on shell", "physical" values. If we want to compare our results with the parameters of such an effective action, we must take into account this mismatch in the interpretation of space-time. Namely, we must correct for a finite extension of the universe. This is necessary in order to compare with the experimental data as they are quoted in the literature. Any experimental value is derived in fact through comparison of a certain "scattering amplitude" with an effective formula, expressed in terms of couplings, masses, momenta etc... For instance, the value of the fine structure constant is obtained from a process taking place at the electron's scale. The amplitude is computed by integrating over the momentum/space-time coordinates. In the traditional field theory approach, this is done in an infinitely extended space-time. In our case, instead, the volume of space-time is finite; the fraction of phase space occupied by the process is therefore relatively higher as compared to the case of infinite volume, and the probability of the transition too. We understand therefore how it is possible that, in our framework, the same amplitude is obtained with a smaller electromagnetic coupling as in the usual approach, where the volume of space-time is infinite.

The correction due to the finiteness of space-time involves not only the couplings, such as those of the electro-magnetic and strong interactions, but also, as a consequence, the masses. Indeed, when we say that an elementary particle has a certain mass, it is always intended that this is the "on shell" mass of the particle "at rest", and considered as an asymptotic, free state. For instance, in the case of the electron this means that it is considered "at the electron's scale", and in a phase in which it can be assumed to be decoupled from the other particles, i.e. in a weak coupling regime. From our point of view, this means in a decompactification phase. However, in our scenario the true, physical decompactification occurs only at the infinite future. The decompactification implied in these arguments is therefore an artifact: one takes a limit of weak coupling of some string coordinate, a linearization that corresponds to working on the tangent space, while curing the so produced infinities/zeros and trivializations by imposing a regularization procedure, a renormalization prescription. In our case, we keep on imposing that the neutron's mass is the one given as in 4.90. This means: we treat the neutron's mass as an already *renormalized* value, and consider the relation 4.90 as an "on shell prescription" which we use in order to fix the regularization. The finiteness of the space volume can then be taken into account by considering any mass and coupling in a renormalization group analysis in which we use a finite-volume regularization scheme. The physical masses and couplings become therefore scale dependent: not only they depend on the age of the universe, by prescription fixed by the value of the neutron's mass, but also on the scale at which the process they correspond to takes place. Couplings and masses have therefore a ground time-dependence, as a power of the age of the universe, as a consequence of the time-dependence of the renormalization prescription, and a milder, logarithmic dependence on the cut-off scale of the renormalization.

The electromagnetic and weak couplings

To start with, in this section we consider the correction to the weak "gauge" couplings [37]. The correction to the $SU(2)_{\Delta m}$ couplings, and to the masses, will be considered in a further section 5.3. In the representation at infinite volume, the effective gauge couplings are corrected according to:

$$\alpha_i^{-1} \approx \alpha_i^{-1}|_0 + b_i \ln \mu / \mu_0 \,, \tag{4.98}$$

where b_i are appropriate beta-function coefficients, and μ is the scale of the process of interest (this can be the electron mass in the case of the fine structure constant). In principle, in this vacuum there is no Higgs field, however it is not clear what should be the best linearized representation of the physical configuration: is it non-supersymmetric, as in the "real" picture, or does the process of linearization lift down some supersymmetry? How does one correctly *approximate* the physical situation, which in itself escapes the rules of field theory, with a field theory in which to consistently perform computations? It could be that the introduction of a Higgs field *mimics* with a certain accuracy the effect of masses, in the practical purpose of computing the running of effective parameters in the neighbourhood of a certain scale of the universe. There is therefore an uncertainty in the definition of the running. However, as a consequence of 4.97, in first approximation we can assume that, in the effective representation of the physical configuration, couplings run logarithmically with an effective beta-function such that, starting from their "bare" value at the actual $\mathcal{T}^{-1/2}$ scale, they meet at zero at the Planck scale:

$$\alpha_i^{-1} \approx \alpha_i^{-1}|_0 + b_i^{(\mathrm{eff.})} \ln \mu / \mu_0 \,, \tag{4.99}$$

with $b_i^{(\mathrm{eff.})}$ such that:

$$b_i^{(\mathrm{eff.})} \ln \mu_0 = \alpha_i^{-1}|_0 \,. \tag{4.100}$$

The scale μ_0 is fixed to be the end scale of our symmetry reduction process, the one corresponding to the lowest level attained with the projections, the square root scale:

$$\mu_0 = \frac{1}{2} \mathcal{T}^{-\frac{1}{2}} \,, \tag{4.101}$$

where $\mathcal{T}$ is the age of the universe as fixed by the neutron's formula 4.90. The choice of the square root scale 4.101 as the starting scale is dictated by the fact that this is the fundamental scale of matter states, and their interactions. Matter consists of spinors and their compounds, and a spinor feels a square-root space, in that twice a spinor rotation corresponds to a true vectorial space rotation. From a technical point of view, the square

[37] The strong coupling α_s requires a separate discussion.

root scale is the one produced by the shift giving rise to masses for the matter states in this string scenario. As we will discuss in section 5.3, the exact normalization of the end scale for elementary states is 1/2 of 4.101.

Let's consider the electromagnetic coupling. The value of α_γ given in section 4.3.3 must be considered as a bare value at the scale μ_0. The fine structure constant, which for us is not really a constant, but just the present-day value of this coupling, will correspond to the value of α_γ run from 4.73 at the scale 4.101 to a scale μ_γ, typical of some process related to the electric charge. As it is experimentally given, this quantity refers to the scale of the electron at rest. This is on the other hand the original scale at which historically the electric charge has been referred to. Although in general modern experiments are not performed at the electron's scale, through renormalization techniques their measurements are anyway always reduced to the electron's scale. From the point of view of our theoretical framework, this is the scale at which the "charged world" starts. Below this scale, there are the un-charged particles, and, from a classical point of view, the electric charge effectively ceases to exist. Once recalculated on the electron's mass scale, 4.73 gets corrected to:

$$\alpha_\gamma^{-1} \, : \, \alpha_\gamma^{-1}|_{\mu_0} \; = \; 183,78 \; \to \; \alpha_\gamma^{(0)\,-1}|_{m_e} \; \approx \; 132,85 \, , \tag{4.102}$$

where we used the value 5.36 for the electron's mass. The result 4.102 is definitely closer to the experimental value, nevertheless still quite not right, being out for an amount higher than the error in our approximations. The reason is that the value 4.102 has been calculated by assuming a perfect logarithmic running, without taking into account for an important modification in the volume of the phase space of the charged matter particles around the electron and up quark mass scale, something we will do in section 5.1.2. We postpone therefore a detailed evaluation of the fine structure constant to section 5.4.

For what matters the weak coupling, the contact with experiment is made through the Fermi coupling constant G_F, basically the weak coupling divided by the W-boson mass squared. Any discussion about this must therefore be postponed after we have obtained this mass. However, the W mass too, in order to be calculated, requires a first order estimation of the weak coupling. Indeed, proceeding as in 4.102, we can see that also this coupling undergoes relative corrections of the right magnitude. We will come back to this coupling in section 5.7.

The strong coupling

In the case of the strong coupling, things are, for obvious reasons, more involved, being more model-dependent also the theoretical framework in which its effective experimental value is obtained. A possible "contact with the experiment" is the value α_s at the scale of some typical quark process, for instance the Z-boson mass in a $e^+e^- \to 4\mathrm{J}$ event: $\alpha_s(M_Z) = 0,119$ [55]. As it is usually given, g_s runs logarithmically with the scale. It seems therefore impossible to think that the "on shell" value 4.79 can be effectively corrected to a current value lower than 1 at around 100 GeV. However, not necessarily α_s must admit an effective representation in terms of a logarithmic running *at the same time*, i.e. *in the same picture* as the electromagnetic and weak couplings. Namely, although strongly and weakly coupled sectors are usually described in an effective action that accounts for all of them at the same time, attributing a logarithmic running to all of them in a unified picture, there are good reasons to believe that, especially for low energies, in the case of the effective α_s the logarithmic behaviour is only a first approximation. Indeed, electro-weak and strong coupling are mutually non-perturbative with respect to each other, and, although we don't know what the correct resummed running of α_s should be, and we can only make some speculation, we may expect that its logarithmic behaviour is only the first order approximation of a running that, in the representations in which the electro-weak couplings are linearized, is exponential. If we suppose that the amount of change computed in a certain scale interval should be seen as the first step of an exponential correction, namely, if we suppose that it increases/decreases by a factor ~ 15 for each $\Delta\mu \approx 10^{12\sim 13}\,\mathrm{M_P}$, then it is not impossible that, in passing from the scale $\mu_0 \sim 10^{-30}\,\mathrm{M_P}$ to $\sim 10^{-17}\,\mathrm{M_P}$ (~ 100 GeV) the value of the strong coupling passes from 4.79 to $\sim 0,2$. It appears therefore that an effective value of α_s lower than 1, as it is usually obtained, is not a signal of weakness of the interaction, but the result of working in a "fictitious", infinitely extended space-time.

4.4.5 Running in the "logarithmic picture"

In our analysis, we have used several times the mapping to an artificial logarithmic representation of space-time, in order to investigate properties related to a vacuum that, in its correct, "physical" representation, appears to be strongly coupled. This proves to be particularly useful in the investigation of some properties of leptons and quarks as free states. In the logarithmic picture, a coupling of order one becomes weak, and therefore strongly coupled elementary states become weakly coupled. When considered at early times, namely at a stage in which even the weak interactions are "strong", this representation allows to deal at the same time with leptons and quarks. This is not something so unfamiliar: although not stated in these terms, all the perturbative, either field theoretical or stringy, constructions of a spectrum such as that of the Standard Model of electro-weak interactions are based on the assumption of working in such a kind of representation.

What we did not yet discuss is how does the spectrum of the low-energy theory look in such a representation, as compared to the "exponential" picture, the physical one. In section 2.1.1 we discussed how supersymmetry breaking is related to the appearance of a non-vanishing curvature of space-time. This effect can therefore be viewed as "tuned" by a "coordinate" of the theory, that remains twisted, and therefore frozen, at the Planck scale. Since the strength of the coupling of the theory is related to the volume of the "internal", i.e. non-extended, coordinates, the breaking of supersymmetry is a phenomenon related also to the appearance of strongly coupled sectors, and strongly coupled matter states. Roughly speaking, we could say that strong coupling and supersymmetry breaking go together and are tuned by the same parameter. This is however a "composite" parameter (such as a function of the product of coordinates).

We have seen that logarithmic pictures, and more in general, perturbative constructions, correspond to "decompactifications" of the theory. The decompactification can be either a true flat limit, or be just a singular, non-compact orbifold, which appears flat only locally and perturbatively. In both cases, the theory doesn't appear higher dimensional, because this operation involves an "internal" coordinate, the coupling of the theory. Owing to the flattening, the "trivialization" of this parameter, from a perturbative point of view it appears as a limit of partial restoration of supersymmetry. However, strictly speaking this is not a smooth limit of the string space, which remains basically twisted. That's why we prefer to speak in terms of logarithmic mapping rather than of "limit". The amount of restored supersymmetries depends on the details of the way the "decompactification" is obtained. Indeed, a full bunch of over-Planckian states could in principle come down to a light mass: extended supersymmetries could appear, as well as the extra states lifted by the rank reductions. However, for the investigation of the properties of matter states, the logarithmic mapping of interest for us is the "minimal" one, such that just one parameter, the one responsible for the separation of weakly and strongly coupled sectors, is mapped to zero, while all the other internal coordinates remain "twisted". Depending on the "dual" representation we want to consider, the amount of supersymmetry we are going to recover in the logarithmic picture is therefore either $\mathcal{N}_4 = 1$ or $\mathcal{N}_4 = 2$. We already discussed that $\mathcal{N}_4 = 1$ is a fake, unstable configuration consisting of the projection onto just the perturbative part of the spectrum of a theory which, non-perturbatively, is non-supersymmetric. In the case we are interested in a correct understanding of the beta-functions, mapping to a $\mathcal{N}_4 = 2$ logarithmic representation is more appropriate than to a "fake" $\mathcal{N}_4 = 1$. This is what we have done in section 4.3.3, in order to understand the role played by matter states and gauge bosons in the evaluation of the $U(1)_\gamma$ beta-function as compared to the $SU(2)$ beta-function. In this representation, there is no "parity restoration" in the sense of the $SU(2)_{(\mathrm{R})}$ bosons coming to zero mass. The fact that $\mathcal{N}_4 = 2$ supersymmetry doesn't have a chiral matter spectrum (hypermultiplets include the conjugate states of fermions) simply means that we must expect a doubling of the matter states, due to the fact that both the left and right moving part of a matter state get paired to a conjugate. On the other hand, this is not a problem, because this picture is just a useful representation, in which we can understand certain properties, that must however be appropriately pulled back to the physical picture. In the computation of the beta-function coefficients we don't need to consider this doubling of degrees of freedom, because this is also related to an effective disappearance of one of the projections.

4.4.6 Recovering the "Geometric Probability" tools.

Our method of deriving masses and couplings through an analysis of the associated entropy in the phase space can be viewed as a "lift up" of the methods of deriving these quantities through a computation of the geometric probability of the interaction processes. The idea goes somehow back to the work of Armand Wyler [56], in which the value of the fine structure constant is given as a ratio of volumes, which can be interpreted as phase space volumes. Further developments have shown how, through an appropriate representation of the phase space of particles and interactions, it is possible to obtain couplings and masses which are extremely close to the experimental ones [57, 58, 59, 60]. However, these are given as pure numbers, no dependence on the fundamental scale of the universe seeming to be implied. In order to understand this point, we must consider that, from the point of view of our work, all these computations are performed in *linearized* representations of the physical space. Even in the case space-time, and the associated phase space, is seen as the "tangent space" of a curved embedding space, this higher space is somehow just the "minimal" embedding space, a "first order" departure out of the base, the flat four dimensional space. From our point of view, these analyses are a kind of "logarithmic picture analyses". Let's consider what usually happens to the coupling. In our framework, it scales as a power of the age of the universe; in a logarithmic representation, as a logarithm of the age. Nevertheless, up to a certain extent it is possible, and it does make sense, to reparametrize the scale dependence by approximating a power-law dependence with a logarithmic one, in such a way that:

$$\alpha^{-1} = \left(\frac{1}{\mathcal{T}^{p_\alpha}} \right)^{-1} \quad \leftrightarrow \quad \approx \frac{1}{\alpha_0} + \beta \log \mu \, , \qquad \alpha_0^{-1} \equiv \beta \log \mu_0 \, , \quad \mu \equiv \mathcal{T} \, , \tag{4.103}$$

for some value of α_0 and β. On the left hand side, we have the real running in the physical picture (we can call it the "cosmological picture"), on the right hand side we have the usual running in a perturbative, effective field theory representation. We have discussed in a previous section (4.4.4) how and why this basically works. In sections 4.1–4.3.5 we have also mentioned the problem of computing the exponents to which the age of the universe must be raised, alluding at the possibility of calculating their ratios in a logarithmic representation. Of course, in such an effective representation, the coefficient β is not the same number as the exponent p_α on the l.h.s. of 4.103. For instance, we have seen in 4.68 that the $U(1)$ exponent is $\approx 1/28$, which is quite far from the electro-weak beta-function coefficients of ordinary field theory. This because the r.h.s. of expression 4.103 is an effective reparametrization of the physical problem, not simply the logarithm of the l.h.s. From this point of view, the Wyler's formula for the electromagnetic coupling is the measure, in a linearized, logarithmic representation, of the volume corresponding to this coupling in units of the volume of a five-dimensional space. To be concrete, using the value 4.70 we obtain:

$$\frac{\left[\alpha^{-1} \approx (\mathcal{T}^5)^{\beta} \right]}{V(\mathcal{T}^5)} \quad \overset{\log}{\rightsquigarrow} \quad \simeq \frac{\beta \log(\mu/\mu_0)}{\log(\mu/\mu_0)} \, , \qquad \beta \approx \frac{1}{5} \times \frac{1}{28} = \frac{1}{140} \, , \tag{4.104}$$

where μ/μ_0 corresponds to $\mathcal{T}^5$ and $\beta \approx \frac{1}{140}$ is here our rough approximation of the fine structure constant, what in Wyler's formula indeed comes out closer to the experimental value. This would probably happen also in the present case, after a more accurate inclusion of the corrections discussed in section 4.4.4. The fact that with this procedure one gets a number that corresponds to the electromagnetic coupling, which can be interpreted as a geometric probability in a five-dimensional embedding of the four-dimensional physics [60, 59] is then here no more than a matter of coincidence: the coupling evolves with time, and "measuring" the exponent 4.68 in a five-dimensional space is just a lucky choice, which works because, at present time, $\mathcal{T} = \mathcal{T}_0 \sim 5 \times 10^{60}\, \mathrm{M}_{\mathrm{p}}^{-1}$, indeed $\mathcal{T}_0^{-(1/28)} \sim \frac{1}{5} \times \frac{1}{28}$. However, all this acquires a deep meaning when the so derived couplings and masses are instead seen as ratios of volumes, normalized to a specific scale ([57, 58, 59]). If we consider ratios of couplings in a logarithmic representation, it is easy to see that any scale dependence drops out. At the ground of the disappearance of any scale dependence is the fact that all the quantities we deal with in field-theoretical effective representations are regularized quantities. Namely, one performs the analysis around a starting point, such as α_0 on the r.h.s. of expression 4.103, a value obtained after regularization of the infinities, "endemic" of a representation in an infinitely extended space-time. One deals then with constant, regularized values, and perturbations around the regularization point. The scale dependence appears only as a correction to a scale-independent bare value. Although corresponding to an artificial reparametrization at a certain fixed point of the cosmological evolution, such a representation of the physical space can anyway be very useful. In principle, with our approach one catches the full behaviour of masses and couplings. In particular, we get the running along the history of the universe, something essential in order to understand astronomical experimental observations, or more in general the physics of much earlier times of the history [38]. In practice however, in order to perform fine computations limited to our present time, it may turn out convenient to map to an appropriate "linearized" representation, such as those considered in Refs. [58, 59, 57], in order to carry out a refined calculation of masses and couplings.

[38]See sections 7.2, 8.1, 8.2 and 8.3 for a discussion of these issues.

5 Present-time values of masses and couplings

Now that we have at hand the value of the $SU(2)_{\Delta m}$ coupling, we can proceed to an explicit evaluation of the masses of the elementary particles, listed in table 4. Free elementary particles correspond to our conceptual classification more than to the real world. As we mentioned in section 4.3, asymptotic running mass formulae are naturally given for states which are neutral under the three elementary interactions, all strong at the Planck scale. At present time, leptons are weakly coupled, while quarks feel a coupling which is even stronger than it was at the Planck time. In order to obtain the mass of the elementary particles, intended as free states, we must therefore identify what are the "minimal composite states", i.e. the lightest singlets they can form. We will in the following proceed by discussing the situation case by case.

5.1 "Bare" mass values

We consider here the mass values corresponding to the elementary particles, as they can be computed using the mass formulae given in section 4.3.1. These can be considered the "bare" values, to be corrected in various ways, in order to account for the mismatch between the finite-volume and the usual infinite-volume approach, and for the fact that at any finite time completely free states are an ideal representation, but don't really exist. Mass scales can therefore be perturbed by the "stable" mass scale 4.90. In section 5.2 we will discuss these corrections, and how, in some cases, it is even more appropriate to consider these values themselves as "corrections" of a "bare" mass scale.

5.1.1 Neutrino masses

We start with the less interacting, and therefore lightest, particles. According to the considerations of sections 4.1 and 4.3.1, the lightest mass level must correspond to the lightest electrically neutral particle, the electron's neutrino. Using the value of the present-day age of the universe derived from the neutron's mass, expression 4.92, we obtain the following value for the "square root scale" :

$$\frac{1}{\mathcal{T}^{1/2}} \approx 4,454877246 \times 10^{-31} \mathrm{M_P} \,. \tag{5.1}$$

Following 4.47 and 4.43, the first neutrino mass should be a $\alpha^{-1}_{SU(2)}$ factor above 1/2 this scale. Furthermore, as discussed in section 4.3.1 after the expression 4.64, this procedure, being related by a chain of symmetry reduction factors to the mass of an electrically neutral electron-positron pair, gives twice the mass of the neutrino, or, better, the $\nu\bar{\nu}$ mass. Using the value 4.69, we obtain therefore:

$$2\, m_{\nu_e} \approx 3,279 \times 10^{-29} \mathrm{M_P} \sim 4,0037 \times 10^{-10} \mathrm{GeV} = 0,40 \,\mathrm{eV} \,. \tag{5.2}$$

After multiplication by a further $\alpha^{-1}_{SU(2)}$ factor, we obtain the second neutrino mass:

$$2\, m_{\nu_\mu} \approx 5,89 \times 10^{-8} \mathrm{GeV} = 58,9 \,\mathrm{eV} \,. \tag{5.3}$$

Finally, multiplication by a further $\alpha^{-1}_{SU(2)}$ factor leads us to the tau neutrino:

$$2\, m_{\nu_\tau} \approx 8,677\, \mathrm{KeV}\,. \tag{5.4}$$

These values agree with the experimental indications of possible neutrino oscillation effects at the electronvolt scale.

5.1.2 *The charged particles of the first family*

An $\alpha^{-1}_{SU(2)}$ factor above the mass of the tau-neutrino there is the electron's mass:

$$m_e \sim \alpha^{-1}_{SU(2)} \times m_{\nu_\tau} \sim 0,639\, \mathrm{MeV}\,. \tag{5.5}$$

As discussed in section 4.3.1, this should be the mass of an electron-neutrino compound. However, as we have seen, neutrino masses are negligible in comparison to lepton masses, and with a good approximation the mass of such a compound coincides with the lepton's mass.

Continuing along the lines of section 4.3.1, from 4.55 we should be able to derive then the down and up quark masses, obtaining $m_d \sim 0,48\, \mathrm{MeV}$ and $m_u \sim 0,87\, \mathrm{MeV}$. However, this is not correct, and is contradicted by the experimental observations. The explanation has to do with the way the symmetry breaking is realized in our framework. At low energy, the $SU(2)_{\mathrm{w.i.}}$ symmetry appears as a broken gauge symmetry, with the breaking tuned by a parameter of the order of a negative power of the age of the universe. As we will see in section 5.6, the $SU(2)_{\mathrm{w.i.}}$ gauge boson masses scale in such a way that $\mathcal{T} \to \infty$ is a limit of approximate restoration of the $SU(2)_{\mathrm{w.i.}}$ symmetry. Moreover, remember that the weak force in itself is stronger than the electromagnetic force: $\alpha_w > \alpha_\gamma$ (it is called weak because for low transferred momenta, $p/M_W \ll 1$, effective scattering/decay amplitudes are suppressed by the boson mass: $\alpha^{\mathrm{eff}}_w \approx \alpha_w / M_W$). Therefore the "hierarchy" of matter is prioritarily determined by the $SU(2)_{\mathrm{w.i.}}$ charge, more than by the electric charge. As a consequence, the matter spectrum can be thought as made of two subspaces, the "up" and the "down" subspace, and the trace of the electric charge can be viewed as:

$$< Q_{e.m.} > = \sum_{\ell,\mathrm{q}} < \mathrm{up} | Q_{e.m.} | \mathrm{up} > + \sum_{\ell,\mathrm{q}} < \mathrm{down} | Q_{e.m.} | \mathrm{down} >\,, \tag{5.6}$$

where $\sum_{\ell,\mathrm{q}}$ indicates the sum over leptons and quarks. As we discussed in section 2.2, minimization of entropy requires to choose a particular distribution of the electric charge among the $SU(2)_{\mathrm{w.i.}}$ singlets (up + down), so that we can have an electrically neutral particle. The condition of approximate restoration of the $SU(2)_{\mathrm{w.i.}}$ symmetry, and the dominance of the weak force with respect to the electromagnetic one, require that the two terms of the r.h.s. of 5.6 give an equal contribution to the total mean value of the electric charge. Otherwise, this would explicitly break the $SU(2)_{\mathrm{w.i.}}$ invariance. This imposes that the trace of the electric charge has to vanish separately on the "up" and "down" multiplets. In practice, both of them must vanish.

For the validity of this argument it is essential that the weak force ends up by dominating the more and more over the electric one, and that the symmetry is restored at infinitely extended space-time; therefore, the full space must be essentially thought as separated in two $SU(2)_{\rm w.i.}$ eigenspaces. Compatibility of the theory at any finite time with the situation at the limit tells us that:

$$\mathrm{tr}\,(\nu, d) \;=\; 0\,. \tag{5.7}$$

Since the ν charge vanishes, we have that:

$$\mathrm{tr}\,(d) \;=\; 0\,. \tag{5.8}$$

This is only possible if, for one family, the roles of the up and down quarks, for what matters the electric charge, are exchanged, so that we have $\mathrm{tr}\,(d) \;=\; 3 \,\times\, \left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) \;=\; 0$. Correspondingly, the trace of the "ups" is also vanishing:

$$\mathrm{tr}\,(\mathrm{e}, \mu, \tau, \mathrm{u}) \;=\; -1 \,-\, 1 \,-\, 1 \,+\, 3 \times \left(-\frac{1}{3} + \frac{2}{3} + \frac{2}{3}\right) \;=\; 0\,. \tag{5.9}$$

Therefore, in one of the three quark families the role of up and down is interchanged: the quark with electric charge +2/3 is indeed the "down", while the one with charge -1/3 is the "up". In the ordinary field theory approach, this argument does not apply because the symmetry remains broken also at infinitely extended space-time [39].

Simple entropy considerations allow us to identify in which family the flip occurs. Let's consider the $SU(3)_c$-singlet made out of the three quarks, one per each family, with higher electric charge, and the one made in a similar way out of the three quarks with the lower electric charge. Clearly, the first one is the most interacting singlet we can form by picking one quark from each family, and conversely the other one is the less interacting one we can form. The first must therefore also be the most massive out of all the possible $SU(3)$-singlets formed by one quark per each family, while the second one must be the lightest. The only possibility we have to achieve this condition is when the flip between charge +2/3 and -1/3 quarks occurs in the lightest family, i.e., for the quarks we usually call the up quark and the down quark. Therefore, approximately the value of the mass of the up quark is the one we computed for the lightest "down" quark states, and conversely the mass of the down quark is the one we assigned to the lightest "up". However, now the lightest quark has a higher electric charge. Namely, from charge $|Q| = \frac{1}{3}$ we pass to $|Q| = \frac{2}{3}$. This transformation is *not* a rotation of the group $SU(2)_{\rm w.i.}$, but a pure electromagnetic charge shift. Therefore, here it does not matter that the former down had negative charge, so that the charge *difference* is $\Delta Q = \frac{2}{3} - \left(-\frac{1}{3}\right) = 1$: a charge conjugation is a symmetry for what matters the occupation in the phase space, or equivalently the mass. What counts is the pure increase in the absolute value of the charge, which implies an increasing of the strength of the interaction of a particle, therefore the probability of interaction, and as a consequence also its volume of occupation in the phase space, that is, the mass. Indeed, doubling the charge means logarithmically doubling, i.e. squaring, the interaction probability, $P \propto \alpha \propto g^2$. Since in the present case we increase $|Q|$ by $\frac{1}{3}$ of the unit electric charge, we expect that, in passing from the electron to the lightest quark, besides the factor 4.54, we approximately gain an extra $(\alpha_\gamma)^{-1/3}$ factor [40].

[39] Notice that the usual charge assignment breaks the $SU(2)$ symmetry explicitly.

[40] No further 1/3 normalization factors are needed, because in this operation we are leaving unchanged the $SU(3)$ indices.

The upper quark of the $SU(2)$ pair passes on the other hand from $|Q| = \frac{2}{3}$ to $|Q| = \frac{1}{3}$, but it does not acquire mass shifts (in the sense either of expansion or of contraction of its volume in the phase space) other than what already inherited by the expansion in the phase space of the lower partner quark. The two are in fact separated by an $SU(2)$ rotation, and the absolute value of their mass difference remains the same: the electric charge modification $|Q| : \frac{2}{3} \to \frac{1}{3}$ has to be seen as the result of an $SU(2)_{\mathrm{w.i.}}$ rotation from the lower member of the pair, therefore a $|\Delta Q| = 1$ rotation, not as a charge shift by $|Q| = \frac{1}{3}$. If, in order to better compare with experimental data, instead of using the inverse of 4.73, we consider the current value of the fine structure constant at the MeV scale we will obtain in section 5.4, putting everything together we get:

$$m_d \approx 4,39\,\mathrm{MeV}\,, \tag{5.10}$$
$$m_u \approx 2,50\,\mathrm{MeV}\,, \tag{5.11}$$

so that:

$$\delta m_{u/d} = m_u - m_d \approx 1,89\,\mathrm{MeV}\,. \tag{5.12}$$

5.1.3 The charged particles of the second family

The masses of the charged particles of the second family are obtained from 4.56, 4.57, 4.58. At present time, they are:

$$m_\mu \approx 94\,\mathrm{MeV}\,; \tag{5.13}$$
$$m_s \approx 167\,\mathrm{MeV}\,; \tag{5.14}$$
$$m_c \approx 1,539\,\mathrm{GeV}\,. \tag{5.15}$$

5.1.4 The charged particles of the third family

The masses of the charged particles of the third family are obtained from 4.59, 4.60, 4.61:

$$m_\tau \approx 13,85\,\mathrm{GeV}\,; \tag{5.16}$$
$$m_b \approx 56\,\mathrm{GeV}\,; \tag{5.17}$$
$$m_t \approx 2749\,\mathrm{GeV}\,. \tag{5.18}$$

One can see that, up to the second family, the mass values, although all more or less slightly differing from those experimentally measured, are anyway of the correct order of magnitude. The values obtained for the third family, instead, seem to be hopelessly wrong. In the next sections we will discuss how these “bare” values get corrected by a refinement in our approximation.

5.2 Corrections to masses

Some of the mass values we have obtained are close to the experimental ones. Other masses, in particular those of the charged particles of the third family, are decidedly out of their experimental value by almost one order of magnitude. In any case, no one really coincides with its known experimental value. Indeed, as we already pointed out, free elementary particles correspond to a conceptual classification of the real world, that makes sense only in the case of weakly coupled states. In our scenario, for the leptons this condition is better and better satisfied as the universe expands. Quarks are instead strongly coupled, and for us their coupling will become stronger and stronger as time goes by. In order to disentangle the properties of the elementary states as "free states", we have mapped to a logarithmic representation of the string vacuum. In this picture, owing to the linearization of the string space, it was easier to consider the ratios of the volumes occupied in the phase space by the various particles, and their interactions. However, as we pointed out, this mapping works only at times close to the Planck scale, where the "logarithmic world" becomes weakly coupled. At a generic finite time, the entire spectrum of particles is "strongly coupled": not only the "colour" interactions are strong, but even the electro-weak symmetry can hardly be expanded around a vanishing value of the coupling. As we discussed in section 4.4.1, in such a world, the only true "asymptotic" state is neutral to all the interactions. We have identified this as a bound state made of neutron, proton, electron and its neutrino and their antiparticles. The corresponding mass scales as $\mathcal{T}^{-3/10}/2$. Strictly speaking, at finite time this is the only true "bare" state of our theory, and its mass scale can be used in order to set the scale of the universe. The problem of correctly computing masses is then twofold:

1) first of all there is the fact that masses, as they are experimentally derived, correspond to a theoretical framework of scale-running, finite volume regularization of a theory basically defined in an infinitely extended space-time. The values we gave in the previous sections must be corrected in such a scheme, taking as starting point the "regularized" value of the fundamental scale, related to the neutron's mass;

2) besides this correction, we must also consider that the masses below the $\mathcal{T}^{-3/10}$ scale should be treated as perturbations of this scale. This is the case of the proton and the neutron, which are made of up and down quarks of the first family, but have a mass much closer to the GeV scale than to the one of the quarks they are made of. By consistency, we should apply the same argument also to the electron. The electron's mass too should be considered as a perturbation of the mean scale. And indeed, strictly speaking it is: free electrons exist for a short time, until they "recombine" into atoms or anyway they bound into some materials. However, since the strength of their interaction (as well as that of neutrinos) decreases with time and at present is sufficiently small, we can safely speak of electrons as free states;

3) for masses above the $\mathcal{T}^{-1/3}$ scale, things are reversed: it is $\mathcal{T}^{-1/3}$ which is rather a perturbation of the bare mass scale.

When we consider the corrections to the masses we want to compute, it is therefore of primary importance to distinguish, at least from an ideal point of view, what are the corrections with respect to what: with respect to the mass of a stable state, or to that of an unstable phase, whose existence can anyway be indirectly detected? This problem is of particular relevance when we talk about quarks. For instance, the "bottom" or "top" mass. In this case, what are indeed measured are the masses of the quark compounds, mesons that exist for a short time: transitory states, that we mostly know through their decay products.

It is currently assumed that the mass of the compound reflects with a good approximation the mass of the heavy quark. However, for us the question is: how is this mass related to the "bare" mass quoted in 5.17, 5.18?

5.3 Converting to an infinite-volume framework

As discussed in section 4.4.4 for the couplings, also for elementary masses, in order to convert their values to on-shell values at the appropriate scale, in a framework of infinitely extended space-time, we must treat them in a renormalization scheme based on a finite-volume regularization. In this picture they are converted to running values, fixed to reduce to the "bare" values at a certain age $\mathcal{T}$ at the fundamental mass scale for spinors, the $\mathcal{T}^{-1/2}$ scale. The "regularization prescription" is that the effective value of the age of the universe $\mathcal{T}$ is adjusted on the neutron mass through its relation to the $\mathcal{T}^{-3/10}$ scale 4.82.

The evaluation of masses proceeds therefore along a sequence of perturbative steps: at first we roughly determine, as in sections 5.1.1–5.1.4, the energy scale "at rest" of the a certain particle. Then we improve the computation by letting the mass to run from the fundamental $\mathcal{T}^{-1/2}$ scale to the specific scale, obtaining thereby an improvement in the perturbation process. Exactly knowing the running of masses entails a detailed knowledge of the interaction and decay processes the particle is involved in: these in fact decide what is the weight of a particle in the phase space. This investigation too can be viewed as part of a sequence of perturbative steps. At the first step, the logarithmic running of masses can be inferred from 4.22: by differentiation of this equation one obtains a renormalization group equation in which the running of mass ratios results to be the opposite of the running of couplings. In this case, the coupling concerned is $\alpha_{SU(2)_{\Delta m}}$, that, according to 4.72, runs more or less like the electromagnetic coupling α_γ, just a bit slower. It is a kind of "non-chiral weak coupling", and the correct evaluation of its beta function suffers of the same theoretical problems of the other gauge couplings, namely of the lack of exact knowledge of the most appropriate effective theory (non-supersymmetric? minimally supersymmetric? with an effective Higgs field?...). In section 4.4.4 we assumed that, in first approximation, in the effective representation of the physical configuration, couplings run logarithmically with an effective beta-function such that, starting from their "bare" value at the actual $\mathcal{T}^{-1/2}$ scale, they meet at zero at the Planck scale. We can here assume that this holds for the $\alpha_{SU(2)_{\Delta m}}$ coupling too. From 4.22 we derive then that the relative variation of a mass along a certain scale variation is opposite to the one of the $SU(2)_{\Delta m}$ coupling:

$$\frac{\Delta m}{m} = -\frac{\Delta \alpha_{SU(2)_{\Delta m}}}{\alpha_{SU(2)_{\Delta m}}}. \tag{5.19}$$

Notice that, while the inverse couplings decrease to zero, and therefore couplings increase when going toward the Planck scale, masses instead decrease. This is correct, because what we are giving here are relative corrections to mass ratios, not masses in themselves. It must be kept in mind that this linearized representation makes only sense reasonably away from the Planck scale. In first approximation the mass corrections are of order:

$$\frac{\Delta m_i}{m_{\mathrm{i}}} \approx \frac{\ln \mu_0 - \ln \mu_i}{\ln \mu_0}, \tag{5.20}$$

where μ_i are the mass scales given in sections 5.1.1–5.1.4, $\mu_0 = \left(\frac{1}{2}\right)^2 \mathcal{T}^{-1/2}$, μ and μ_i are expressed in reduced Planck units, an appropriate Planck mass rescaling in the argument of each logarithm being implicitly understood. Indeed, since masses are obtained from the expressions of mass ratios, the higher mass of a pair is obtained as a function of an inverse coupling times the lower mass, which sets the scale of the process. Effectively, expression 5.20 is therefore shifted to:

$$\frac{\Delta m_{\rm i}}{m_{\rm i}} \approx \frac{\ln \mu_0 - \ln \mu_{\rm i} + \ln \mu_{\nu_e}}{\ln \mu_0} \,. \tag{5.21}$$

The first neutrino mass remains unvaried. For the other masses, we obtain:

$$m_{\nu_\mu} : \quad 2,945\,\mathrm{eV} \to 2,739\,\mathrm{eV}\,; \tag{5.22}$$
$$m_{\nu_\tau} : \quad 4,3385\,\mathrm{KeV} \to 3,731\,\mathrm{KeV}\,; \tag{5.23}$$
$$m_e : \quad 0,639\,\mathrm{MeV} \to 0,505\,\mathrm{MeV}\,; \tag{5.24}$$
$$m_\mu : \quad 94\,\mathrm{MeV} \to 67,7\,\mathrm{MeV}\,; \tag{5.25}$$
$$m_\tau : \quad 13,85\,\mathrm{GeV} \to 8,99\,\mathrm{GeV}\,; \tag{5.26}$$
$$m_u : \quad 2,50\,\mathrm{MeV} \to 1,93\,\mathrm{MeV}\,; \tag{5.27}$$
$$m_d : \quad 4,39\,\mathrm{MeV} \to 3,35\,\mathrm{MeV}\,; \tag{5.28}$$
$$\delta m_{u/d} : \quad 1,89\,\mathrm{MeV} \to 1,42\,\mathrm{MeV}\,; \tag{5.29}$$
$$m_c : \quad 1,539\,\mathrm{GeV} \to 1,048\,\mathrm{GeV}\,; \tag{5.30}$$
$$m_s : \quad 167\,\mathrm{MeV} \to 118,9\,\mathrm{MeV}\,; \tag{5.31}$$
$$m_t : \quad 2749\,\mathrm{GeV} \to 1582\,\mathrm{GeV}\,; \tag{5.32}$$
$$m_b : \quad 56\,\mathrm{GeV} \to 35,3\,\mathrm{GeV}\,. \tag{5.33}$$

The only elementary particle mass we can here use for a precise comparison with experimental data is the one of the electron: neutrino masses are not yet known, and the other masses will undergo further corrections (see next sections).

The correction 5.24 must be considered as a first order correction: once determined at "order zero" the bare mass, $0,639\,\mathrm{MeV}$, we have rescaled it according to 5.21, by recalculating the effective coupling on the zero order electron's scale. Now that we have the first order electron's mass scale, $\sim 0,505\,\mathrm{MeV}$, we can improve our approximation by recalculating the effective coupling on this new scale, and using this newly obtained relative mass correction in order to correct the scale of the $0,639\,\mathrm{MeV}$. We obtain in this case:

$$m_e|_{2^{nd}}\,;\ 0,505 \ \to\ 0,5069397\ldots\,\mathrm{MeV}\,. \tag{5.34}$$

This is still about 1% lower than the experimental value. Indeed, in order to get the *physical* mass of the electron, to the "bare" mass 5.24 we must add also the masses of the lighter states. The reason is the following. In the derivation of the mass ratios of section 4.3.1, namely proceeding from 4.22, there is the implicit assumption that all particles lighter than a certain one belong to a subspace of its phase space. Suppose we have just two particles, particle A with mass m_A, and particle B, with mass $m_B = \alpha\, m_A$, $\alpha < 1$. When we say that α is the ratio of the two volumes in the phase space, we also imply that particle A is heavier than particle B in that the space of B has been obtained by a process of symmetry reduction, by truncating the space of A. Particle A has more interaction/decay channels than B, because the space of A contains the space of B. Let's now consider the full phase space of a sub-universe consisting of A and B. The full volume is:

$$V(A) + V(B) = V(A) + \alpha V(A) . \tag{5.35}$$

Now, in our specific case A is the electron, and B is basically the τ-neutrino (we neglect here the other neutrinos, that give corrections of order $\mathcal{O}(\alpha^2)$). When we measure the mass of the physical electron, what we look at is the modification to the geometry of the space-time produced by the existence of the electron. For what we just said, deriving the electron's mass from 4.22 implies considering that, when generating the electron, we generate also the τ-neutrino and the lighter particles. They also interact, and the modification to the whole phase space produced by the existence of the electron is indeed the full $V(A) + V(B) = V(A) + \alpha V(A)$. This implies that what we call the physical electron mass is the sum of the bare electron mass 5.24 *plus*, in first approximation, the mass of the τ-neutrino. Summing to 5.34 the ν_τ mass 5.23, we obtain then:

$$m'_e|_{2^{nd}} ; \; 0,50694 \; \to \; 0,51057 \ldots \text{MeV} . \tag{5.36}$$

Of course, we can correct to the second order also the ν_τ mass, and further refine our evaluation. At this order the ν_τ mass gets increased, thereby increasing also the estimate of the electron's mass. A further recalculation of the coupling at the new scales leads on the other hand to a subsequent lowering of all masses. The approximation of the electron's mass proceeds through a converging series of "zigzag" steps of decreasing size, below and above the final value. One can easily see that in this way we better and better approximate the experimental value of the electron's mass (see [55]). However, we don't want here to go into a detailed fine evaluation of mass values, because $\sim 1\% \div 0,1\%$ is our best precision in many steps of our analysis of masses.

In general, accounting for the shifting of phase space 5.35 amounts in a small ($\mathcal{O}(\alpha^{-1})$) correction to mass values, but for the quarks of the first and second family the relative change is much higher ($\mathcal{O}(\sqrt[3]{\alpha^{-1}})$ and $\mathcal{O}(\sqrt{\alpha^{-1}})$ respectively). Once this is taken into account, the masses of the up and down quarks get further corrected to:

$$m_u : \quad 1,93\,\text{MeV} \to 2,435\,\text{MeV} ; \tag{5.37}$$

$$m_d : \quad 3,35\,\text{MeV} \to 5,785\,\text{MeV} ; \tag{5.38}$$

$$\delta m_{u/d} : \quad 1,42\,\text{MeV} \to 3,35\,\text{MeV} . \tag{5.39}$$

5.4 The fine structure constant: part 2

Let's now come back to a more precise determination of the fine structure constant. As discussed in section 4.4.4, the fine structure constant is the value of α_γ^{-1} at the electron's scale, the scale that can be considered as the reference for the operational definition of the electric charge. According to our analysis of section 4.3.1, summarized in the diagram 4, the phase space of the elementary particles divides into two subspaces, the electrically uncharged and the charged space, the latter being the upper one in the sense that all charged particles are heavier than the uncharged ones. This second subspace starts at the electron's scale. As we discussed at page 80, section 5.1.2, after the up-down flip in the quarks of the first family, the phase space gets further expanded by a $\sqrt[3]{\alpha_{\gamma}^{-1}}$ factor. This shift modifies the effective strength of the projections applied in order to get the mass hierarchy of section 4.3.1 in the sub-volume of the phase space corresponding to the first charged family. As a consequence, it modifies also the effective weight of the corresponding states, and the ratio of the effective $U(1)_{\gamma}$ and the $SU(2)_{\Delta m}$ beta-functions *around this scale*. The effect is that, as the states weight more, the effective running of the coupling is faster, or, equivalently, the one of its inverse slower. Namely, as the volumes of the matter phase space are expanded (or, logarithmically, shifted), the value of the electromagnetic coupling at the scale m_e effectively corresponds to the value of the coupling *without correction* at a run-back scale, $m_e^{\text{eff.}}$. The amount of running-back in the scale of the logarithmic effective coupling is equivalent to the amount of the forward shift in the logarithmic representation of the volumes of particles in the phase space. If volumes get multiplied by a factor, their logarithm gets shifted, and so gets shifted back the scale at which the coupling in its logarithmic representation is effectively evaluated. From an effective point of view, we can therefore derive the value of the fine structure constant by evaluating the electromagnetic coupling proceeding as in 4.99, but at a scale a factor $\sqrt[3]{\alpha_{\gamma}^{-1}}$ below the electron's scale, rather than precisely at the electron's scale as we did in 4.102 (see also Appendix D). To have a first rough estimate, we can use 4.102) to calculate that the effective scale μ_{γ} is lower than $0,511\,\text{MeV}$ by a factor $\sim 5,102549027\ldots$. In this case we obtain:

$$\alpha_{\gamma}^{(1)\,-1}|_{m_e} \;=\; 137,0700548\,. \tag{5.40}$$

In order to improve our evaluation, we need a better approximation of the shape and size of the effective shift of the phase space of the first family. If we consider that the $\sqrt[3]{\alpha_{\gamma}^{-1}}$ shift on the up quark translates also to the down quark, the heavier in this case, we should conclude that the scale at which to evaluate $\sqrt[3]{\alpha_{\gamma}^{-1}}$ is around the down quark mass scale. Using the value 5.28 for the point of evaluation, we obtain:

$$\alpha_{\gamma}^{(2)\,-1}|_{m_e} \;=\; 137,0366167\,. \tag{5.41}$$

In order to further improve the estimation, one should then proceed as we did for the electron, by iterated steps of corrections of the down and electron scale, recalculating the $\alpha_{SU(2)}$ factors at the new scales to obtain improved estimations of m_d and of $\alpha_{\gamma}^{(0)}$ at the down mass scale, and so on, obtaining a series of converging "zigzag" steps. The first step corresponds to a slight increasing of the effective down mass, thereby lowering the factor $\sqrt[3]{\alpha_{\gamma}^{(0)\,-1}}$, eventually resulting in a slight, higher order decrease of the value of the inverse of the fine structure constant. The value 5.41 is around $0,0005\,\%$ above the current experimental one, $\alpha^{-1} \sim 137,035999\ldots$ [61], and these considerations induce to expect that the further steps of the approximation do improve the convergence toward the experimental value. However, one should not forget the major point of uncertainty, namely that we are here attempting to parametrize the effective modification of the size of the projections applied to the phase space, and therefore the value of the fine structure constant, due to a local dilatation of the phase space. A true fine evaluation of α_{γ}^{-1} requires first of all a better approximation of this effect. Last but not least, there is the question whether, and at which extent, a convergence toward the official "experimental value" of this parameter should be expected and desired. Beyond a certain order of approximation, current evaluations heavily rely on QED techniques, and are extrapolated within a theoretical scheme that only at the first orders corresponds to the one discussed in this paper . A mismatch beyond this regime of approximate correspondence does not necessarily implies and indicates that the values here obtained are wrong, provided the effective computation of physical amplitudes nevertheless produces correct results.

Finally, we repeat and stress that, in our framework, the electric charge is time-dependent, and 5.41, possibly corrected at any desired order, only represents the present-day value of this parameter. The rate of the time variation at present time can be easily derived from the very definition. From 4.68 and 4.72 we obtain:

$$\frac{1}{\alpha}\frac{d\alpha}{dt} \;=\; \frac{1}{28}\times\frac{47}{45}\times\frac{1}{\mathcal{T}}\,. \tag{5.42}$$

In one year, the expected relative variation is therefore of order $\approx 3 \times 10^{-12}$. This is a rather small variation, however not so small when compared with the supposed precision with which α is obtained. Indeed, the most recent measurements give for its inverse a number with precisely 12 digits, a number whose variation could be observed by repeating the measurement at a distance of some years. Since however a fine experimental determination of α depends, through the theoretical framework within which it is derived, on time-varying parameters such as lepton masses etc..., it would not be an easy task to disentangle all these effects to get the "pure α time-variation". This kind of effects can be better detected when expanded on a cosmological scale, as we will discuss in section 8.1.

5.5 The Heavy Mass Corrections

Any perturbative approach is based on the identification of a "bare" configuration, a set of states, which serve as starting point for a series of corrections. For the sake of consistency, these must be "small" as compared to the main contribution, the "bare" quantity. The selection of a "bare" set is in general not uniquely determined: precisely in string theory, a complete investigation requires a full bunch of "dual constructions", built as perturbations around different sets of bare states. In our case specific case, several physical quantities can be viewed both from the the physical ("exponential") picture, and from the point of view of a logarithmic representation of the vacuum they are mapped to. As we have discussed, a comparison with experimental data presented within a framework of infinite space volume and free particles may require to map quantities to this picture. The advantage of a logarithmic representation resides in that, in the cases of interest, the logarithm maps large quantities into small ones, and vice-versa. In particular, in the case of masses which are below the Planck scale, since everything is measured in units of the Planck scale, higher masses are mapped into smaller ones:

$$m_1 > m_2 \qquad \Rightarrow \quad |\ln m_1| < |\ln m_2|\,, \qquad (|m_1|\,,\,|m_2| < 1)\,. \tag{5.43}$$

When we evaluate the corrections to the masses, we must therefore consider the perturbations to the bare values as seen from both the point of view of the "exponential picture", where for instance the mass of the up quark is smaller than the mean mass scale $m_{3/10}$, which has then to be considered as the "bare" scale around which to perturb, and the point of view of the "logarithmic picture", in which, according to 5.43, this hierarchy is reversed, and it is the quark mass which is going to be perturbed. Once resummed, depending on the picture one considers, the mass corrections may therefore be quite large, larger than the scale they are going to correct.

We will in the following consider two types of corrections to the bare quark masses, depending on whether they refer to a perturbation of the stable non-perturbative mass scale, the $m_{3/10}$ (the case of the proton mass), or to unstable particles, whose existence in itself is a perturbation of the mean configuration of space-time.

5.5.1 *Stable particles*

At large volume-age of the universe, $1 \ll \mathcal{T} \to \infty$, electro-weak interactions are very weak, while strong interactions become stronger and stronger. Asymptotically, the universe settles to a configuration in which only the lightest particles of the decay chain are normally present, the probability of producing higher mass, unstable ones becoming lower and lower. The particles charged under the strong interaction tend to form bound states, "attracted" by the mean mass scale "$m_{3/10}$", defined in eq. 4.82. In practice, this means that the universe tends toward a world with matter made out of up and down quarks, electrons and neutrinos [41]. The masses of these objects as *free* particles are those computed in sections 5.1.1, 5.1.2, and corrected in 5.22–5.33. Quarks however are not present as free particles, but form the bounds we call proton and neutron, which are stable in the sense that the neutron's decay into proton+electron+neutrino is balanced by the inverse process of electron and neutrino capture by the proton. As discussed, the stable mass scale $m_{3/10}$ corresponds to the rest energy of this system, namely the neutron + proton-electron-neutrino plus their antiparticles. If we look at the masses of the quarks and leptons constituting this system, we see that the neutron is made of an heavier quark set than the proton, and that the quark mass difference between neutron and proton is higher than the sum of the electron and neutrino masses. This means that the neutron decay into proton+electron+neutrino leaves some amount of energy. As far as the $m_{3/10}$ compound at equilibrium is concerned, this energy must be included in the account. This allows us to deduce that "at equilibrium" $(1/2)\, m_{3/10}$ corresponds to four times the mass of the neutron. Furthermore, neutron and proton differ for their electromagnetic charge, i.e. for "weak" interaction properties as compared to the strong coupling; it is therefore reasonable to expect that their mass difference is basically due to the mass difference between the up and down quark. On the other hand, the $m_{3/10}$ scale is much higher than the down-up mass difference, and, as it corresponds to a stable scale, it can be considered weakly coupled. This implies that the quark mass difference can be treated as a small perturbation of the $m_{3/10}$ scale. We can therefore write:

$$m_{\mathrm{p}} \approx \frac{1}{4}\, m_{3/10} + \mathcal{O}(\delta m_{u,d}) \,. \tag{5.44}$$

[41] We will comment later about muon- and tau- neutrinos.

Indeed, since the proton is a stable particle, the difference in the volume occupied in the moduli space by neutron and proton is entirely due to the difference of the volumes occupied by the quarks they are formed of. Differently from what discussed at pages 5.3 and 5.3, a correction to the down quark mass does not require summing the mass of the lighter states, as it was the case for instance of the electron, whose bare mass had to be corrected by adding the ν_τ mass. The reason is that, in this case, once bound to form the heavy, strong-coupling-singlet compound, the lighter particle, the quark up, does not interact anymore: it does not have an independent phase space, as it was the case of the τ-neutrino. For what concerns the physical mass of the proton and the neutron, namely, as long as, like for the case of the electron, we look at the modification caused to the geometry of space-time by the existence of the proton and the neutron, the phase space of the up quark does not add to the phase space of the "bare" down quark. The quark mass difference entering in this game is therefore the (corrected) bare quark mass difference 5.29. *At the quark scale*, this is $\delta m_{u/d} = 1,42\,\text{MeV}$. However, for what matters the mass difference between neutron and proton, the scale at which the quark masses have to be run is the proton/neutron scale. At this scale, once recalculated according to 5.21, the quark masses are:

$$m_u|_{E=m_n} = 1,7189\,\text{MeV} \tag{5.45}$$

$$m_d|_{E=m_n} = 3,0183\,\text{MeV} \tag{5.46}$$

that imply:

$$m_d - m_u|_{E=m_n} = 1,299\,\text{MeV} \approx m_{\text{n}} - m_{\text{p}}\,, \tag{5.47}$$

quite in good agreement, apart the usual $\mathcal{O}(1\,\%)$ mismatch, with the experimental value of the neutron-proton mass difference [55]. Notice that, in their logarithmic running, masses, and mass differences, decrease when increasing the scale. This is the opposite of what happens in the real, cosmological scaling. The point is that in the real scaling they all tend to 1 in Planck units. As it happens for the couplings, also masses tend to the logarithm of 1, namely, to zero, at the Planck scale.

Expression 5.44 accounts with good approximation for the behaviour of the proton-to-neutron mass relation far away (i.e. well below) the Planck scale. As we get close to this scale, this approximation looses its validity.

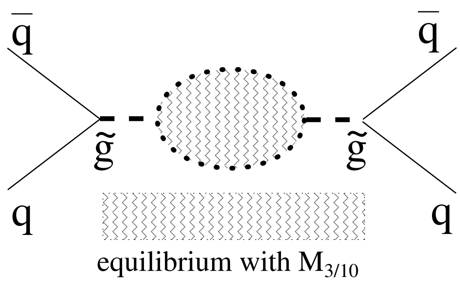
5.5.2 Unstable particles

Let's now consider the particles that exist only for a short time: the leptons μ, τ and the mesons. As seen from the point of view of the universe along the running of its history, at a late stage, as it is today, the existence, i.e. the production and decay, of these particles can be seen as a fluctuation out of a "vacuum" characterized by the mass scale $m_{3/10}$, of which they are a perturbation. Indeed, since we are going to compare masses with values given in an infinite-volume theoretical frame corresponding to a logarithmic picture, in practice it is the lower mass what is going to be seen as the "bare" value to be corrected. This may be the $m_{3/10}$ mass itself in the case of particles with a bare mass higher than the $m_{3/10}$ scale, as is the case of the particles of the third family (the quarks top, bottom and the τ). Or it can be the mass of the particle, as is the case of the second family (charm, strange and muon). In any case, the correction is of the form:

$$M_0^2 \ \rightsquigarrow \ M^2 \ \approx \ M_0^2 \left(1 + \alpha \, \frac{m^2}{M_0^2} \right) , \tag{5.48}$$

where M_0 and M are the bare and the corrected mass, and m is the perturbing mass, the mass scale with which the state of mass M is in contact through an interaction with strength $\alpha = g^2/4\pi$.

In the case of particles of the third family, M is the $m_{3/10}$ scale, that from the point of view of a logarithmic picture is the higher scale, and m the bare quark or lepton mass. For the second family, things work the other way around: M is the bare mass of the particle, and m corresponds to the $m_{3/10}$. When we say "the bare quark mass" here we intend something different from the usual concept of bare quark mass. As they are usually given, quark masses are in general directly derived from the mass of mesons they form, possibly subtracted of the mass of the partner quark they are bound with, and corrected within the framework of an $SU(3)$-colour symmetry based model of hadrons. Apart from the case of the up and down quarks, the quark mass turns out to be, although not really coinciding and sometimes considerably different [42], anyway of the same order of magnitude of the meson mass. In our case, bare mass means instead the value given in 5.27–5.33. The coupling α is in general the electromagnetic coupling, which provides the strongest interaction between the two mass scales. Masses enter in expression 5.48 to the second power, because this mass correction can be viewed as a propagator correction of an effective boson, as here illustrated



where q and $\bar{q}$ stand for a quark-antiquark pair, in the simplest case for instance in a π-meson. Indeed, an expression similar to 5.48 could be considered also for the stable barions considered in section 5.5.1. In that case, the strongest contact between the two scales, the up and down quark scale and the $m_{3/10}$ scale, is given by the strong coupling itself, of order one. The correction ends up therefore to the $m_{3/10}$ scale itself. For the π-mesons, or the other mesons, K, C, B etc... (these last ones more or less "by definition" in direct relation to the mass of their heaviest quark), although their constituents interact strongly, this interaction involves the quarks within each meson. The strongest contact between the two scales is

[42] See for instance the case of the strange quark and the K mesons.

however given by the electromagnetic interaction, and α is basically the electromagnetic coupling.

In the case of neutrinos, their only contact with the $m_{3/10}$ scale occurs through the weak coupling. In itself, $\alpha_{\text{w.i.}}$ is even a bit stronger than the electromagnetic coupling. However, the effective strength of the interaction is of order:

$$\alpha^{\text{eff.}} \approx \alpha_{\text{w.i.}} \times \frac{m_\nu^2}{M_W^2}, \tag{5.49}$$

where $\alpha^{\text{eff.}}$ already takes into account typical energies of neutrino processes, and should not be confused with G_F, the Fermi coupling constant. The neutrino mass corrections are therefore extremely suppressed.

The correction 5.48 reduces the effective mass of the t or b quarks and the τ lepton by around one order of magnitude, producing values close to those experimentally measured. More precisely, the top mass gets corrected to:

$$m_t \to \sim 164\,\text{GeV}, \tag{5.50}$$

where, besides the value 5.32, we have also used the value of the electromagnetic coupling logarithmically corrected to the bare top scale (α_γ^{-1} : $183,78 \to 92,91$) [43]. As in the case of the electron, this value too should be corrected at higher orders, by recalculating the "bare" top mass, from the 1582 GeV of the first order, to a second order value, to be used as starting point for the correction, to be plugged in 5.48. Then, as we did for the electron, in order to catch the full phase space of the physical top particle, we must add the lighter masses, the heaviest of which are the bottom, tau, and charm masses. Then, here too one can easily see that these higher order corrections better and better approximate the experimental value of the top mass. Let's see the first steps of this correction. First of all, we recalculate the relative mass correction, or equivalently the relative coupling correction, run at the new corrected top mass, 1582 GeV. We obtain:

$$m_t^{(0)} = 2749\,\text{GeV} \to 2749 - (2749 \times 0,4167) = 1603,5298\,\text{GeV}. \tag{5.51}$$

To this, we must sum the non-negligible contributions of the bottom, τ and charm bare masses, obtaining:

$$m_t' \approx 1603,53 + 35,3 + 8,99 + 1,048 = 1648,87\,\text{GeV}. \tag{5.52}$$

Of course, to be more precise we should re-correct at the second order also the bottom, τ and charm masses, something we are not doing here. Re-plugging 5.52 in 5.48, we obtain:

$$m_t'' \approx 171,07\,\text{GeV}, \tag{5.53}$$

[43] In principle, this value could be affected by the shift in the effective beta function, centered to the electron's scale, we discussed in section 5.4. However, we don't have a recipe in order to derive the full non-linear effective running of the electromagnetic coupling. We suppose that the local modification has its peak at around the electron/up/down scale, and tends to vanish both toward the $\mathcal{T}^{-1/2}$ and the m_t scale. Therefore, we neglect it in this and the following computations, already affected in themselves by possibly larger uncertainties.

quite more in agreement with the experimental value, which is around $\sim 171,4 \pm 1,7\,\mathrm{GeV}$ [62]. We don't go further in the refinement of 5.53, because, to start with, we should recalculate also the bottom, τ and charm bare masses. Then, to be more precise, we should also take into account the modifications to the effective coupling and bare mass logarithmic scales, as due to the $SU(3)$ normalization factors of the quark mass ratios, 1/3 and 1/9 for the bottom and the top of each $SU(2)$ doublet. All these corrections contribute for at most an order $\sim 1\,\%$, therefore an uncertainty lower than the error in the experimental value of the top mass. More importantly, we must warn here that the agreement we obtain between our estimate and the experimental value has to be taken more as the indication of the plausibility of our analysis, rather than a real fine test. We are trying to evaluate the ratios of the volumes in the phase space of the particles in a rather complicated part of the spectrum, where the regions of validity of our simple perturbative dual approaches meet. For instance, it is not completely clear whether the best approximation is obtained by summing to the top phase space the lower masses *before* the correction through the $m_{3/10}$ scale, or *after* it. Here and in the following we choose the first option. In the case of the top quark, since the top scale is well above all these scales, this does not make such a big difference. Things become however more critical when looking at the corrections to the lower masses, such as the one of the bottom quark, the τ or the charm quark.

For the bottom, the effective coupling we use is the inverse electromagnetic at the bottom scale, $\alpha_{\gamma}^{-1}|_b \sim 102,95$. We obtain:

$$m_b \to \sim 3,61\,\mathrm{GeV}\,. \tag{5.54}$$

This scale too should then be corrected in a way similar to the top mass. Adding the tau and charm masses, we obtain:

$$m_b \to \sim 4,57\,\mathrm{GeV}\,. \tag{5.55}$$

This value is slightly above the average experimental estimate. However, the latter is basically *extrapolated* from the B-meson width, and 5.55, although above the extrapolated value, is actually still compatible with the mass of the B-meson. A serious comparison would require a better understanding of the theoretical uncertainties underlying the entire derivation, both on the side of our evaluation of volumes in the phase space, and on the side of the experimental derivation: for consistency, the extrapolation from experimental data should be done entirely within the light of our theoretical scheme.

For the τ lepton we use a value of the electromagnetic coupling run to the lepton's scale, $\alpha_{\gamma}^{-1}|_{\tau} \sim 106,55$, and obtain:

$$m_{\tau} \to \sim 1,28\,\mathrm{GeV}\,. \tag{5.56}$$

For the further corrections to this value, analogous arguments apply also here, with the difference that, being the τ mass so close to the $m_{3/10}$ scale, the final result is more sensitive to these corrections than in the top and bottom case. For instance, at the second order the corrected bare τ mass, instead of 5.26, is $m_{\tau}|_{2^{nd}} \sim 9,52\,\mathrm{GeV}$, that gives $1,32\,\mathrm{GeV}$. Adding the charm mass, we get a further correction by some 5%, leading to:

$$m'_{\tau} \sim 1,39\,\mathrm{GeV}\,. \tag{5.57}$$

As it is also the case of the quarks of this family, in particular the bottom quark, it doesn't make however sense to go on with refinements of scale evaluations, as it is already clear that something more fundamental is here missing, in order to explain the gap between the values we obtain and the so-called experimental one ($\sim 1,78\,\mathrm{GeV}$ [55]). As we said, a better understanding of the corrections to the volumes of phase spaces around the $m_{3/10}$ scale for unstable particles is in order. In the case of the bottom quark, the experimental value too is strongly affected by model-dependent considerations, and things are even more complicated.

When we pass to the second family, analogous considerations hold for the charm quark, whose mass is extremely close to $m_{3/10}$. In first approximation, by inserting the renormalized value of the electromagnetic coupling at the bare charm mass scale, $\alpha_\gamma^{-1}|_c \sim 113,5$, we obtain a slight decrease of the quark mass:

$$m_c \,:\, 1,048 \to 0,946\,\mathrm{GeV}\,. \tag{5.58}$$

However, as it is already evident from the τ mass evaluation of above, as the bare scale approaches the $m_{3/10}$ scale, our perturbation method starts showing its limitations. Indeed, in the case of the charm quark, it would be also possible to invert the role of bare mass and perturbing mass, using the charm bare mass 5.30 as the mass M in the expression 5.48, and, for m, the neutron mass, obtaining:

$$m_c' \,:\, 1,048 \to 1,051\,\mathrm{GeV}\,. \tag{5.59}$$

Including the strange-quark mass shift, we would obtain a light increase to:

$$m_c' \sim 1,170\,\mathrm{GeV}\,. \tag{5.60}$$

Similar considerations as for the bottom and τ masses are in order here too, and we leave any further analysis for the future.

For the strange quark and the μ-lepton, they are below the $m_{3/10}$ scale, and, as we start to get far away from it, the reliability of our estimate starts to improve again. For the strange quark, we use $\alpha_\gamma^{-1}|_s \sim 117,94$, to obtain, if we don't consider the μ-mass shift:

$$m_s \to \sim 147\,\mathrm{MeV}\,, \tag{5.61}$$

and, when including the muon mass shift:

$$m_s \to \sim 205,7\,\mathrm{MeV}\,. \tag{5.62}$$

A comparison with what is known as the experimental value of the strange quark mass is affected by theoretical considerations. In itself, the strange quark mass is extrapolated via $SU(3)_c$-related techniques from the width of the K-mesons. Surely, in the space of the K-mesons there is also the μ- channel. However, when the "bare" s-quark mass is disentangled from the total width, does this mean that also the μ- shift gets decoupled? In this case, the value to be considered for a comparison should not be the second one, 5.62, but the μ-unshifted one, 5.61. The difficulties rely here also on the fact that we are comparing *extrapolated* values, not true "experimental" ones.

Finally, for the muon we use $\alpha_{\gamma}^{-1}|_{\mu} \sim 119,42$, that leads to:

$$m_{\mu} \to \sim 109,4\,\mathrm{MeV}\,, \tag{5.63}$$

and, when including the electron mass shift:

$$m_{\mu} \to \sim 109,8\,\mathrm{MeV}\,. \tag{5.64}$$

One may notice that our mass corrections become the less and less precise as we get closer to the $m_{3/10}$ mass scale. Indeed, our approximation of the correction works better when the bare scale of the particle is far away from $m_{3/10}$, so that we can either treat the particle's scale, or the $m_{3/10}$ scale, as the perturbing or the perturbed scale. When they are close, other "non-linear" effects become important, and with our approximation we systematically obtain an overestimate for the particles with a mass below $m_{3/10}$ (muon and s-quark), and an underestimate for the particles that are above (charm, tau, (bottom ?)).

5.5.3 The π and K mesons

The π^0 mesons are bound states of the up and down quarks, that, differently from the proton and the neutron, "interact" with the $m_{3/10}$ scale through the electroweak coupling felt by their quarks, instead than directly through the strong force. As a consequence, the relation of the meson to the quark mass is given as according to 5.48, with α the electromagnetic coupling. We expect therefore:

$$\begin{aligned} m_{\pi}^2 &\sim \mathcal{O}(m_{\mathrm{q}}^2) \times \left\{ \alpha_{e.m.}\, \mathcal{O}(m_{3/10}^2) + \mathcal{O}(1) \right\} \\ &\approx \mathcal{O}(m_{\mathrm{q}}^2) \times \left\{ \alpha_{e.m.}\, (2m_{\mathrm{n}})^2 + \mathcal{O}(1) \right\} . \end{aligned} \tag{5.65}$$

This leads to a $\sim$ 100 MeV scale.

As we already observed, in principle the s-quark mass corrected by the $m_{3/10}$ scale as given in 5.62 is somehow already the effective mass "corresponding" to the K meson. It is not our scope here to enter into the details of the relation between the effective quark and meson mass, that, according to the common framework in which experimental data are interpreted, and therefore masses are derived, are supposed to be linked through $SU(3)$-colour-splitting relations. We want here only point out that, for what matters the charged mesons $\pi^{\pm}$ and $K^{\pm}$, they occupy a different phase space volume than the corresponding neutral ones; since the difference is due to the $U(1)_{\gamma}$ transformation properties, i.e. to the quark content, we expect the mass difference between charged and neutral mesons to be of the order of the mass difference of the component quarks. However, differently from the case of the neutron–proton mass difference, here we don't have stable particles. While for proton and neutron the phase space is basically the same (in they sense that they stably transform the one into the other), so that their differences simply reflect the differences in the properties of the bare particles they are formed of, for the π and K mesons charged and neutral ones have access to completely different decay and interaction chains. Their phase spaces are therefore really different. As a consequence, although of the order of the mass difference of their quarks, the mass difference of the mesons are further modified by the modifications of the volumes of their effective phase spaces, and should be investigated as higher order corrections, after a recalculation of the phase spaces obtained by correcting the bare ones according to the meson interactions.

5.6 Gauge boson masses

We already discussed how, in any perturbative realization of a string vacuum with gauge bosons and matter states charged under a symmetry group, gauge and matter originate from T-dual sectors. For instance, in heterotic realizations the gauge bosons transforming in the adjoint of the group originate from the currents, while the fermions transforming in the fundamental representation originate from a twisted sector. As a consequence, any projection producing a non-vanishing mass for the matter states as the result of a shift on the windings, produces also a mass for the projected gauge bosons as a consequence of a shift on the momenta. Therefore, the mass of the projected matter states scales in a T-dual way to that of the gauge bosons. After the projections involved in the breaking of the internal symmetry into a set of separate families of particles, matter acquires a light mass, below the Planck scale, while the gauge bosons of the broken symmetry acquire a mass above the Planck scale.

As anticipated in section 2.2, the gauge bosons of the $SU(2)_{\rm w.i.}$ interaction don't follow this rule: they are lifted by a shift "on the momenta", not on the windings, and acquire an under-Planckian mass, of the same order as the matter states. As discussed in section 2.2, on the space-time coordinates act two shifts. One of them produces the breaking of parity, and gives a light mass to the matter states ($m \sim \mathcal{T}^{-1/2}$) while lifting in a T-dual way the mass of the gauge bosons coupled to the right-moving degrees of freedom. The other shift produces instead the lifting of the left-moving gauge group, the group of weak interactions. As discussed in section 2.2, this operation does not act, like the previous one, through "rank-reducing level-doubling", and its effect is not simply a further, equivalent shift. However, from the analysis of section 4.3.1, we see that more or less the effect of this ' 'Wilson-line like" operation is the one of producing a mass lift whose typical length is approximately a fourth root power, $\sim \mathcal{T}^{-1/4}$. The space-time gets in fact doubly contracted, the root being the lift up of what in a perturbative, logarithmic picture appear as projection coefficients, $\frac{1}{2}$ for the first shift and $\frac{1}{2} \times \frac{1}{2}$ from the second operation. Indeed, what we didn't say in section 2.2, is why parity should be broken by the first shift, and be related to the "square root" scale, and the weak group by the second shift, resulting approximately in a $\sim \mathcal{T}^{-1/4}$ scale [44]. Why not the opposite? In principle, there seems to be no ground for this choice of ordering. Once again, what explains this choice is the minimization of the volume of the symmetry group, resulting in the maximization of the entropy of the configuration [45]. In the case of the first shift the volume of the symmetry is reduced by the strength of the symmetry breaking driven by the mass of the right moving bosons, scaling as a positive power of the

[44] Indeed, as it can be seen from the mass expressions given in section 4.3.1, the highest mass scale is a $\sqrt{\alpha_{SU(2)}}^{-1} = \mathcal{T}^{1/56}$ factor above the the $\mathcal{T}^{-1/4}$ scale.

[45] We recall that the volume of occupation of the whole configuration, in the phase space of configurations, is dual to the volume of the symmetry group: the smaller the second one, the larger the first one.

age of the universe. The higher this power, the higher the reduction of the volume of the symmetry group ($1/2 > 1/4$). In the case of the second shift, the boson masses scale as negative powers of the age of the universe, and therefore the higher reduction of the volume of the symmetry, obtained with a higher boson mass, is achieved with a lower exponent ($1/4 < 1/2$).

The scale at which first the breaking of the $SU(2)_{\text{w.i.}}$ symmetry takes place approximately corresponds the scale of the top-bottom mass difference, and, like the latter is higher than the experimental hadron mass scale, this one is around one-two orders of magnitude above the experimental mass scale of the $SU(2)_{\text{w.i.}}$ bosons. It is reasonable to think that this one too is subjected to the same kind of renormalization as the other scales which are above the $m_{3/10}$ scale. However, as it is the case of the other masses, here too thinking in terms of shifts and elementary orbifold projections is a too simplified picture to get the fine details of mass differencies; in order to understand the mass of these bosons it is convenient to follow an approach similar to the one we have used for the masses of elementary particles, and use in our formulae the mass values already corrected according to section 5.5.2.

Let's first concentrate on what happens to the W bosons. The computation of the mass of these bosons proceeds similarly to that of the elementary particles. The basic diagram for a broken symmetry is again 4.18 of section 4.3, where, in this case, m_1 and m_2 refer to the masses of the heaviest $SU(2)_{\text{w.i.}}$ doublet, the top-bottom quark pair. However, this time we are not interested in evaluating the ratio of the volumes occupied in the phase space by the up and down particle of the doublet. The relation between the two particles we want to consider is therefore not a $SU(2) = SU(2)_{\Delta m}$ symmetry, but the one established through the $SU(2)_{\text{w.i.}}$ symmetry. Being produced by a shift "on the momenta", differently from the case of the broken symmetries discussed in section 4.3, here the W mass is T-dual to the mass of the over-Planckian bosons . Therefore, instead of 4.20, here the cut-off energy of the space of the process is:

$$< p >_{\text{w.i.}} \; \sim \; \left(\frac{m_t m_b}{M_W^2} \right)^{1/4} , \tag{5.66}$$

and the relation 4.22 is replaced by:

$$\alpha \, \frac{3 m_t m_b}{M_W^2} \; \approx \; 1 \, , \tag{5.67}$$

where $\alpha \equiv \alpha_{SU(2)_{\text{w.i.}}}$ The factor 3 can be understood in this way: each $SU(2)_{\text{w.i.}}$ transformation rotates one quark colour; we need therefore three such rotations in order to pass from the bottom to the top quark. Notice that the relation 5.67 can be viewed as the integral form of a renormalization group equation. Differentiated and mapped to a logarithmic (and therefore also supersymmetric) representation, it roughly corresponds to the usual expressions of the beta-function:

$$\alpha \, \frac{m_t \, m_b}{M_W^2} \; \approx \; 1 \quad \overset{\partial, \log}{\rightsquigarrow} \quad b \approx T(R) - C(G) \, , \tag{5.68}$$

where b is the gauge beta-function coefficient and $T(R)$, $C(G)$ are the contributions of matter and gauge, entering with opposite sign. Inserting the mass values obtained in section 5.1.4, as corrected in section 5.3, namely 5.32 and 5.33, and the value of the weak coupling 4.75, run at the bottom scale, $\alpha^{-1} \sim 24,1$ [46], we get:

$$M_{W^{\pm}} \sim 83,4\,\mathrm{GeV}\,. \tag{5.69}$$

In order to obtain this mass, we used for the top and bottom mass the “bare” values of page 84, not the values after the correction that brings them to their actual experimental value. Indeed, the relation 5.67 involves in its “bare” formulation bare particles. As it was for the quarks, also W are unstable and their mass is corrected by their interaction with the $m_{3/10}$ scale. However, for gauge bosons things go differently than for matter states, and their corrected mass cannot simply be obtained by plugging in 5.67 the corrected values of m_t and m_b. Gauge bosons behave T-dually with respect to particles; therefore, in their case, we must use an expression like 5.48 in its T-dual form:

$$\frac{1}{M_W^2} \to \frac{1}{M_W^2}\left(1 + \alpha \times \frac{1}{M_W^2}\int \frac{d^4 p}{(p + m_{3/10})^2}\right), \tag{5.70}$$

where $m_{3/10}$ here basically stays for the neutron’s mass, and the integral is intended up to the W-boson energy. Since $M_W > m_{3/10}$, $1/M_W < 1/m_{3/10}$, and, as in section 5.5.2, we correct the lower (inverse) scale $1/M_W$ with the higher (inverse) scale $1/m_{3/10}$. Moreover, the effective W-boson contact interaction is not suppressed by W-boson transfer propagators, and the strongest interaction they have with the $m_{3/10}$ scale occurs through the weak coupling. Therefore, here $\alpha = \alpha_{\mathrm{w.i.}}$. Owing to the different type of effective loop correction to the boson interaction with matter, as compared to the one of matter with matter, the term that multiplies the coupling is of order 1. We have therefore:

$$\frac{1}{M_W^2} \to \frac{1}{M_W^2}\,(1 + \alpha_{\mathrm{w.i.}})\,, \tag{5.71}$$

or, T-dualized back:

$$M_W^2 \to\approx M_W^2\,(1 - \alpha_{\mathrm{w.i.}})\,. \tag{5.72}$$

Inserting the value of $\alpha_{\mathrm{w.i.}}$ at the W mass scale, $\alpha^{-1}_{\mathrm{w.i.}}|_{M_W} \sim 23,46$, we obtain:

$$M_{W^{\pm}} \to \approx 81,6\,\mathrm{GeV}\,. \tag{5.73}$$

All the above expressions, 5.70, 5.71 and 5.72, neglect terms of order $\mathcal{O}(\alpha^2)$ (according to [61], the fit of the current experimental values of the $W^{\pm}$ mass is around $80,399 \pm 0,023$

[46] In principle, also the weak coupling should undergo an effective beta-function modification similar to the one of the electromagnetic coupling discussed in section 5.4. However, as discussed in section 5.4 and in the footnote at page 91, this is expected to be a local modification, that tends to vanish toward the upper end scale of the matter sector, the scale that at present time is around the TeV scale. At the W-boson scale, $\alpha_{\mathrm{w.i.}}$ should have almost regained its “regular” value. However, we cannot exclude a slight modification toward a lower effective value, which could explain why we get a boson mass slightly higher than the experimental one. If we assume a “linear” decrease of the effect, from the MeV to the TeV scale, we should find that, if at the MeV scale the weak coupling undergoes a shift proportional to the one of the effective electromagnetic coupling: $\alpha_{\mathrm{w.i.}}|_{1\,\mathrm{MeV}} \to \alpha_{\mathrm{w.i.}}|_{1\,\mathrm{MeV}} \times (132,8/137)$, at the 80 GeV scale it should have lost $\sim 2/3$ of its effect, leading to a $\sim 83,0\,\mathrm{GeV}$ W-boson mass (see Appendix D).

GeV; its difference with respect to our estimate is therefore of the order of the corrections we are neglecting).

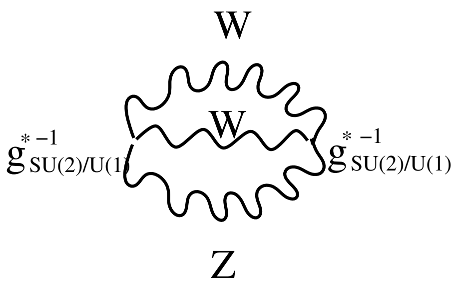
The mass of the Z boson cannot be directly derived in a similar way, by simply substituting m_t to m_b in 5.67: when $m_1 = m_2$ the symmetry is not broken, and the boson is massless! In first approximation, we expect the Z mass to be of the order of the mass of the W bosons: at the $SU(2)_{\rm w.i.}$ extended symmetry point the shift lifts all the three bosons by the same amount. However, as discussed in section 2.2, at the point of minimal symmetry, a bit away from the orbifold point, the group is broken to $U(1) = U(1)_Z$ through a parity-preserving, left-right symmetric operation. This means that what distinguishes the mass of the Z boson from the one of the chiral $W^{\pm}$ bosons is the fact that it acquires a "right moving" component: while the charged bosons interact only with a left-handed chiral current, the neutral boson has now a certain amount of coupling with a right-moving current [47]. This "misbalance" should be related to the mass difference between the Z and the $W^{\pm}$ bosons. In turn, as it is for all the other massive excitations, also the Z mass should be related to the volume it occupies in the phase space. The disagreement between the W and the Z mass should then be tuned by the strength of $SU(2)_{\rm w.i.}$ as compared to $U(1)_Z$. In order to derive the mass of the Z boson, consider therefore once again the diagram 4.18, this time with Z, W^- and W^+ replacing respectively the top, bottom quarks and the W boson: in this case we view the process as a transition between W^- and Z, produced by an element of the "group" $SU(2)_{\rm w.i.}/U(1)_Z$ (more precisely not a group but a coset) [48]. At the vertices, g is now the "coupling" of $SU(2)_{\rm w.i.}/U(1)_Z$. More precisely, since, as we discussed in section 4.3.3, the relation between "width" in the phase space and mass, in the case of gauge bosons, is the inverse with respect to the case of matter states (higher probability = lower boson mass), the relation of diagram 4.18 has to be "T-dualized" in the space of couplings; namely, "S-dualized". The coupling appearing at the vertices is therefore the inverse of the "coupling" g^* of $SU(2)_{\rm w.i.}/U(1)_Z$. This on the other hand is precisely what we should expect. If we set:

$$\alpha_{SU(2)_{\rm w.i.}} = \alpha^*_{SU(2)_{\rm w.i.}/U(1)_Z} \times \alpha_{U(1)_Z} \, , \tag{5.74}$$

being the $U(1)_Z$ coupling smaller than the one of the unbroken group, we obtain that $\alpha^* > 1$, and the relation 4.18 must be dualized in order to reduce to the ordinary weak coupling diagram:

[47] This is not a general property of string vacua: when going to the $U(1)$ point, it is in general not true that vector fields get mixed up. In the present case, these fields are massive, already lifted by a shift, after which the left and right chiral components of fermions combine to give rise to massive matter. From a "pure" string point of view the $SU(2)$ bosons don't exist as massless fields, and therefore not as gauge bosons. Here we are discussing about the properties of massive string excitations, that we account among the field degrees of freedom only because their mass is small, lower than the Planck scale. Since the parity-breaking shift reflects in a "level-2" realization of the weak group, there is no surprise that a displacement toward the $U(1)$ point indeed involves the breaking of two $U(1)$'s, the left and right, as a matter of fact "patched together".

[48] Notice that we are not defining here Z as a linear combination of W^0 and the field B_μ, associated to the hypercharge.



(5.75)

Instead of a momentum to the fourth power, now the loop integral pops out a momentum squared, and at the place of 4.20, the typical momentum of the space of the process is here:

$$< p > \quad \sim \quad \frac{M_Z}{M_W} \, . \tag{5.76}$$

The W mass, the mass of the boson mediating the process, appears in the denominator, as in 5.66, T-dually to the case of 4.20. From the diagram 5.75 we obtain therefore:

$$\left(\frac{M_Z}{M_W} \right)^2 \quad \approx \quad \alpha^*_{SU(2)_{\text{w.i.}}/U(1)_Z} \ , \tag{5.77}$$

and, using the relation 5.74,

$$M_Z \ \sim \ \sqrt{\frac{\alpha_{SU(2)_{\text{w.i.}}}}{\alpha_{U(1)_Z}}} \, M_W \, . \tag{5.78}$$

In order to obtain $\alpha_{U(1)_Z}$ we can proceed as in section 4.3.3, this time by determining the fraction with respect to the volume occupied by $SU(2)_{\text{w.i.}}$ at the place of $SU(2)_{\Delta m}$. This means that the coupling of $U(1)_Z$ should stay to the coupling of $U(1)_\gamma$ in the same ratio as the coupling of $SU(2)_{\text{w.i.}}$ stays to the one of $SU(2)_{\Delta m}$. Therefore, we expect:

$$\frac{\alpha_{U(1)_Z}}{\alpha_{SU(2)_{\text{w.i.}}}} \approx \frac{\alpha_{U(1)_\gamma}}{\alpha_{SU(2)_{\Delta m}}} \, . \tag{5.79}$$

At present time, 4.69 and 4.73 and the W-boson mass 5.73 tell us that the Z boson mass should be approximately:

$$M_Z \sim 1,127 \, M_W \quad \approx 91,96 \, \text{GeV} \, . \tag{5.80}$$

If we proceed as in the footnote at page 97, by assuming a linear decrease of the local correction to the effective beta-function, this time of the electromagnetic coupling discussed in section 5.4, till its vanishing at the top scale of the charged matter phase space, 5.18, we get that at the 80 GeV scale the shift should have been reduced to around 1/4 of its size, producing a relative modification of the electromagnetic coupling at this scale of a factor $\sim 1,00791$, leading to a modification of the ratio 5.78 by a factor 1,00394553, i.e. a Z to $W^\pm$ mass ratio:

$$\frac{M_Z}{M_W} \ : \ 1,127 \ \to \ 1,132 \ , \tag{5.81}$$

a number that should be compared with the experimental ratio of these masses, $\sim 1,134$ [61]. Owing to the theoretical uncertainties implicit in our derivation, it does not make sense to refine the calculation, although it seems that the linear approximation of the effective beta-function is not quite far from the real behaviour.

Let's now come back to see how, for what concerns the neutral and charged currents, the low-energy action looks like. According to the results of section 2.2, the matter states feel:

i) a long range force, mediated by a massless field, with the strength of the non-anomalous traceless $U(1)$ group discussed in section 4.3.3,

ii) a short range force, mediated by the bosons obtained from the breaking of $SU(2)_{(\mathrm{L})}$ ($= SU(2)_{\mathrm{w.i.}}$), and

iii) a would-be "very short" range force, "mediated" by the boson corresponding to T_3^{R}. As we said, this third is not an interaction in the sense of field theory, because the mass of this boson is higher than the Planck mass. On the other hand, its contribution is highly suppressed.

Our analysis tells us that $SU(2)$ *singlets, both left and right moving*, couple to a massless boson in a diagonal way with the strength of the group $U(1)_{\tilde{Y}}$, whose present time value is given by 4.73. The electric charge corresponds to a certain choice of charge distribution among the degrees of freedom constituting these singlets, as dictated by minimization of symmetry. It can be viewed as a linear combination of the three $U(1)$ charges T_3^{L}, T_3^{R} and $\tilde{Y}$, resulting in a traceless "shift" in the definition of the states. The total charge remains of course the same as the "hypercharge" $\tilde{Y}$, and therefore also the strength of the coupling, which is related to the "width" of the interaction [49]. We conclude therefore that this is the strength of the electromagnetic interaction, mediated by a massless boson that we identify with the photon.

The left-handed current couples in non-diagonal way to the W^+ and W^- bosons associated to the "raising" and "lowering" generators of the $SU(2)_{\mathrm{w.i.}}$ group, and diagonally with the $U(1)_Z$ resulting after the breaking of this group. Owing to the fact that this symmetry is broken, the coupling and the mass of this boson is not the same as the one of the $W^\pm$ bosons: they are related as given in 5.78. If we now consider the values of the couplings of these terms, "run back" to an infinite-volume effective action in order to make possible a comparison with the standard description of low-energy physics, we can see that approximately α_γ, $\alpha_{\mathrm{w.i.}}$ and "α_Z", the total coupling of the Z boson, are related by ratios that can be written as:

$$\sqrt{\alpha_\gamma} \approx \sqrt{\alpha_{\mathrm{w.i.}}}\,\sin\theta\,; \tag{5.82}$$

$$\sqrt{\alpha_Z} \approx \sqrt{\alpha_{\mathrm{w.i.}}}\,\cos\theta\,, \tag{5.83}$$

[49] This in first approximation: mass differences lead in fact to a differentiation of the decay widths, depending on the mass/charge assignments and distribution.

for a certain angle θ. The value of α_γ, the coupling of the electromagnetic current $J^{e.m.}$ to the photon, numerically approximately coincides with the one of the fine structure constant. $\alpha_{\text{w.i.}}$ is the coupling of the $W^\pm$ and Z bosons to the axial current $J^{\pm,0}$, and numerically approximately coincides with the standard weak coupling α_w. Notice that in our framework also the Z boson couples to this current by the same amount as the charged bosons. In the Standard Model effective action, at the tree level the couplings of the $J^{\pm,0}$ are instead:

$$W^\pm \;\to\; g\, J^\pm_\mu W^{\mp\,\mu}\,, \tag{5.84}$$

$$Z \;\to\; \frac{g}{\cos\vartheta_w} J^0_\mu Z^\mu\,. \tag{5.85}$$

However, the effective amplitude is weighted also by the boson mass, in such a way that:

$$\alpha_{\text{eff}}(W) \;\sim\; \frac{\alpha_w}{M_W^2}\,, \tag{5.86}$$

$$\alpha_{\text{eff}}(Z) \;\sim\; \frac{\alpha_w}{\cos^2\vartheta_w} \times \frac{1}{M_Z^2} = \frac{\alpha_w}{\cos^2\vartheta_w} \times \frac{\cos^2\vartheta_w}{M_W^2} = \frac{\alpha_w}{M_W^2}\,. \tag{5.87}$$

Effectively, the two couplings are therefore the same. This agrees with the fact that, in our framework, couplings are related to the effective interaction/decay width, not to the microscopical description in terms of gauge strengths and connections. In the Standard Model, the Z boson couples also to the electromagnetic current, in such a way that its total interaction with matter is:

$$\frac{g}{\cos\vartheta_w}\left(J^0_\mu - \sin^2\vartheta_w J^{e.m.}_\mu\right) Z^\mu\,. \tag{5.88}$$

This is a consequence of the fact that the boson mass eigenstates are obtained from the hypercharge and the Cartan of the $SU(2)_{(\mathrm{L})}$ group through an orthogonal transformation. The "total" width of the Z boson is therefore corrected by the fact that it couples also to a left-right symmetric current, whose trace is the same as the one of our "hypercharge". Effectively, the size of the coupling is:

$$\alpha_Z^{\text{eff}} \sim \frac{1}{4\pi}\frac{g^2}{\cos^2\vartheta_w}(1-\sin^2\vartheta_w)^2 \;=\; \alpha_w\cos^2\vartheta_w\,. \tag{5.89}$$

In practice, this means that, for what concerns the axial coupling, the Z boson behaves analogously to the charged bosons, namely, it couples with the same strength and it "propagates" with the same mass. However, it is also "displaced" by a left-right symmetrically interacting term, in such a way that its total width corresponds to a higher mass and lower coupling, as given in 5.89. These relations should be compared with 5.82 and 5.83. The empiric parametrization we gave in terms of the angle θ corresponds then to the expressions in terms of the Weinberg angle ϑ_w:

$$\theta \cong \vartheta_w\,. \tag{5.90}$$

Therefore, for what concerns the electromagnetic and the charged and neutral axial currents, we have effectively reproduced, although in a completely different way, the coupling terms of the effective action of the Standard Model. Things turn out to correspond, although the underlying basic description doesn't at all. On the other hand, perhaps it is not so dramatic that things don't exactly match with all the details of the Standard Model description. It should be clear that our way of approaching all the issues related to the low energy parameters is completely different from the approach of the Standard Model, or its field theoretical extensions, even "string inspired" ones. In our case, there is no field theoretical Higgs mechanism at work; masses are generated, and symmetries are broken, in a different way, and consequently different is also the parametrization of the elementary interactions. Our aim is not to show how in detail a Standard-Model-like description of the "microscopical physics" is recovered. The usual effective theory is anyway an approximation, a "choice of linearization", which, although if appropriately restricted to a certain region of the space of parameters it can nicely fit the experimental data, on a larger scale it seems to not work. Certainly, fitting of data shows a rather good agreement of many Standard Model predictions with the experiments. However, this agreement is achieved by advocating, "predicting", the existence of degrees of freedom that continue to escape any attempt of detection. We can almost perfectly fit the Weinberg angle and boson masses, but at the expense of introducing a Higgs field with a mass such that it should have been already detected somewhere, and still some things don't match. We can further adjust the misprediction by enlarging the number of degrees of freedom, which allow to "improve the direction", pushing forward the problem. For instance, detecting supersymmetric partners at low energy etc. In this situation, what is the meaning of statements like "the Standard Model correctly predicts the one or the other quantity", if at the end this is the result of a fit which assumes the existence of non-detected degrees of freedom? Of course, these questions are well known since a long time, and the answer still open. We recall the problem here just to remind the reader that there is nothing illegal in the fact that we are not reproducing all the terms of the low-energy Standard Model action, apart from those aspects that one can safely consider as "experimentally detected", and not just "fitted".

A remark on the meaning of "massless" in this framework

In the usual field theoretical representation of particles and interactions in infinitely extended space-time, a free photon can not be "localized" as can a particle, and its energy can be as small as we want. In our framework, this corresponds only to an asymptotic situation, closer and closer approached but concretely never realized. At present time, a photon can not be more extended than the distance from us to the horizon of the universe. Masslessness does not translate therefore in the energy beeing arbitrarily small, but just in the property that "the minimal energy corresponds to the inverse of the radius of the universe":

$$E_{\gamma} \sim \frac{1}{2}\frac{1}{\mathcal{T}} \,. \tag{5.91}$$

Massive particles and fields are characterized by a minimal energy that departs from 5.91 in that it scales as a lower power of the inverse of the universe, and therefore they have an extension smaller than its size:

$$E_{\text{massive}} \sim \frac{1}{2}\frac{1}{\mathcal{T}^{p}} \,, \qquad 0 < \, p \, < 1 \,, \tag{5.92}$$

so that the spread in space is:

$$< \Delta X >_{\text{massive}} \;\sim\; \mathcal{T}^{p} \;<\; \mathcal{T}\,. \tag{5.93}$$

For a massive object, the exponent p is necessarily always smaller than 1.

5.7 The Fermi coupling constant

We are now in a position to make contact with the experimental value of the weak coupling. This is measured through the so-called Fermi coupling constant G_F, a dimensional ($[m^{-2}]$) parameter defined as the effective coupling of the weak interaction at low transferred momentum [50]:

$$\frac{G_F}{\sqrt{2}} \;=\; \frac{g^2}{8M_W^2} \;=\; \frac{\pi\alpha_w}{2M_W^2}\,. \tag{5.94}$$

From section 5.6 we know that we can identify $\alpha_{\text{w.i.}}$ with the usual weak coupling α_w of the literature Inserting our results for the W-boson mass, 5.73, and the value of the weak coupling at the W-boson scale, given at page 97, we obtain:

$$G_F|_{M_W} \;=\; 1,4221 \times 10^{-5}\,\text{GeV}^{-2}\,. \tag{5.95}$$

As it was for the case of the fine structure constant, once again we are faced with the problem of understanding what is the meaning of a physical quantity, whose value is always related to a certain experimental process at a certain scale. Here, from an experimental point of view the Fermi coupling is obtained by inspecting the pion into muon decay. The effective, infinite-volume renormalization of G_F to the pion–muon scale is obtained in our framework in the same way as for the other couplings, namely treating G_F as a generic coupling, whose behaviour is represented through an effective linearization as in 4.99. The relative variation from the W to the $\mu \div \pi$ scale [51] is of order:

$$\frac{\Delta G_F}{G_F}|_{M_W \to m_\pi} \;\approx\; 0,81\,, \tag{5.96}$$

and we get:

$$G_F|_{\pi/\mu} \;\approx\; 1,1519 \times 10^{-5}\,\text{GeV}^{-2}\,, \tag{5.97}$$

a value about 1 % away from the effective experimental value [55]. The percent is on the other hand the order of the precision we have in our estimate of the W-boson mass, and as a consequence we cannot hope to get something better for the Fermi coupling.

⋆⋆⋆

This brief survey does not pretend to exhaust the argument of masses. In particular, certainly not the topic of meson and baryon masses. However, we hope to have at least shed a bit of light onto the subject. As we mentioned in section 4.4.6, for the practical purpose of computing the fine corrections to masses and couplings it may turn out convenient to

[50] Low means here negligible when compared to the W-boson mass.

[51] Within our degree of approximation, it does not make such a difference the choice of one scale or the other, between muon and pion.

map to an appropriate “linearized representation” of the string space, in which the issue of the computation of these quantities with the methods of geometric probability can be approached with a more evolved technology. What however is missing in these approaches is a “cosmological perspective”, which would give the possibility of “fixing the scale”. For instance, the agreement of (almost) all the quantities computed in Ref. [57] with experiments is impressive, and, according to what we discussed in section 4 (in particular, 4.4.6), in many cases it is not just mere coincidence or numerology; however, at least one input has to be supplied from outside; e.g. the electron’s mass, or any other scale, to serve as the “measure” to which to compare all the ratios of volumes. Moreover, we expect that in order to refine the results such as those of [57], [59] or [60], a better understanding of how the non-perturbative vacuum is mapped, and its degrees of freedom are represented in such a linearized approximation, is in order.

6 Mixing flavours

As is well known, mass eigenstates are not weak-interaction eigenstates: the weak currents cross off-diagonally the elementary particles, and, besides diagonal up/down decays, there is a smaller fraction of non-diagonal, flavour changing decays that mix up the three families. Experimentally, this phenomenon is well known for what matter quarks. Still hypothetical is however whether it occurs also for leptons: its detection would go in pair with a clear indication of non-vanishing neutrino masses. Indeed, in this case one tries to go the other way around, looking for neutrino oscillations as a signal of non-vanishing neutrino masses. In the case of quarks, the standard parametrization of these phenomena is made through the introduction of a matrix V_{CKM}, the Cabibbo-Kobayashi-Maskawa matrix, which encodes all the information about the "non-diagonal" propagation of elementary particles. It is defined as the matrix which rotates the base of "down" quarks of the $SU(2)$ doublets, allowing to express the currents eigenstates in terms of mass eigenstates:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{6.1}$$

V_{CKM} accounts for the mixing among different generations, as well as the CP violations.

Despite the elegance of the formal treatment, and the intriguing relation between number of quarks and the existence of a phase, from the point of view of the Standard Model the entries of the CKM matrix remain external inputs, chosen to fit experimental data: there seems to be no deep reason why a mixing of quark generations should exist at all, nor why there should be a phase responsible for CP violations. The ordinary theoretical treatment simply provides a parametrization of the quark mixing, for which the number of quark families results to be precisely the minimal one allowing the existence of a a phase giving rise to CP violations. In our approach, non-vanishing off-diagonal CKM entries, and CP violation, are not simply allowed, but necessarily implied. As such, they come out as a prediction of the theoretical framework. The reason is that any non-trivial mass ratio corresponds to the coupling of a broken symmetry; in particular, the CKM matrix expresses the mixing due to what remains of the broken symmetry between quark families (the symmetry among the three orbifold planes of the internal string space). CP violations are related to the breaking of parity induced by the shift along the extended space that gives origin to all sub-Planckian masses. Therefore, both the family mixing and the parity violation have their origin in the mechanism that produces masses, and is necessarily implied by symmetry minimization/maximization of entropy. Nevertheless, from this point of view it could seem that there is no relation between the existence of a CP violating phase, and the number of quarks, as is instead the case of the standard CKM approach. However, even though apparently less tightly bound, in our framework these aspects are related in an even stronger way, because all this proceeds from a unique condition, the minimization of symmetry, necessary to produce the configuration of highest entropy, i.e. the configuration that dominates in the physical world. In our case, also the number of quarks is a consequence of symmetry minimization and the uniqueness of the representation of the combinatorial scenario, encoded in 1.1, in terms of string theory.

6.1 The effective CKM matrix

Although quark mixing and CP violations in our case are in principle not related to a description in terms of a CKM matrix, in order to make contact with the usual theoretical organization of experimental results, we will derive the entries of an effective CKM within our theoretical framework. In our framework, masses have a different explanation than in ordinary field theory. Nevertheless, it is still possible to refer to an effective field theory description, once accepted that this has to be considered only as a tool useful for practical purpose, without any pretence of being a (self-)consistent theory. Although in our approach we directly consider decay amplitudes rather than effective terms of a Lagrangian, in order to make possible an easy comparison between our predictions and the usual literature it is therefore convenient building an effective Lagrangian, that, within the rules of field theory, will reproduce, or "mimic", our amplitudes.

As is well known, the CKM matrix is a unitary matrix, in which all phases except one are reabsorbed into a redefinition of the quarks wave functions. Therefore, its nine entries are parametrized by nine real coefficients and one phase, responsible for parity violation. In our framework, the analysis of the spectrum has been carried out by classifying the degrees of freedom according to their charge. This means that what we got are the "current eigenstates" (section 2). Subsequently, we have considered a "perturbation" of this configuration, obtained by switching-on so-far neglected parameters, in order to investigate their masses (section 4). In section 4.3 we have put mass ratios in relation with ratios of sub-volumes of the phase space, which is divided into several sectors by the breaking of the initial symmetry. Mass ratios are then related to the couplings of the broken symmetries. As we anticipated, there are two kinds of breaking: a "strong breaking", in which the would-be gauge bosons acquire a mass above the Planck scale, and a "soft breaking", in which the gauge bosons acquire a mass below the Planck scale. Only in this second case the transition appears as an ordinary decay, mediated by a propagating massive boson. Otherwise, the boson of the broken symmetry works somehow like an external field: we don't see any boson propagating, and we interpret the phenomenon as a "family mixing". The off-diagonal entries of the CKM matrix precisely collect the effect of this type of "non-field-theory decay": off-diagonal entries account for transitions from one generation to another, non mediated through gauge bosons as in the case of ordinary decays.

According to our previous discussion, the ratios between entries of the CKM matrix should be of the same order of the mass ratios, normalized to the full decay amplitude: mass ratios correspond in fact to squares of "elementary" couplings: $m_f/m_i \sim \alpha_{i\to f}$ $(\sim g^2_{i\to f})$. If α_{ab} is the coupling for the flip from family a to family b, the decay amplitude of a $a \to b$ flavour changing decay is expected to be proportional to α^2_{ab}.

Any decay amplitude depends on masses, both of initial and final states of the process. With our non-perturbative methods we have direct access to the full decay amplitudes. In order to make contact with the ordinary description of the mixing mechanism, we must consider that, as it is defined, the CKM matrix is unitary, and collects the information about flavour changing, subtracted from any dependence on masses: in expressions of amplitudes, this dependence is carried by other terms. This allows to normalize the matrix in such a way that, owing to the fact that off-diagonal elements are much smaller than diagonal ones, the diagonal elements are close to 1:

$$|V_{ud}|, \ |V_{cs}|, \ |V_{tb}| \ \approx \ 1 \, , \tag{6.2}$$

and, with a good approximation,

$$|V_{us}| \ \approx \ |V_{cd}| \tag{6.3}$$
$$|V_{ub}| \ \approx \ |V_{td}| \tag{6.4}$$
$$|V_{cb}| \ \approx \ |V_{ts}| \tag{6.5}$$

As for the computation of masses, a detailed evaluation of the CKM matrix entries would require taking into account all processes contributing to the determination of the phase space. Here we want just to make a test of reliability of our scheme; we are therefore only interested in a first approximation. To this purpose, it is reasonable to work within the framework of the simplifications 6.2–6.5. Owing to these simplifications, we can restrict our discussion to the off-diagonal elements $|V_{ts}|$, $|V_{td}|$ and $|V_{cd}|$. A direct, non-diagonal $t \to s$ decay should have an amplitude of order m_s/m_t, normalized then through m_b/m_t in order to reduce to the scheme 6.2. A rough prediction for V_{ts} is therefore:

$$V_{ts} \ \approx \ \frac{m_s}{m_b} \ \sim \ \frac{0,147 \, \mathrm{GeV}}{3,6 \, \mathrm{GeV}} \ \sim \ 0,04 \, , \tag{6.6}$$

where we have used the values 5.61 and 5.54. Similarly, we obtain:

$$V_{td} \ \approx \ \frac{m_d}{m_b} \ \sim \ 0,001 \, , \tag{6.7}$$

and

$$V_{cd} \approx \frac{m_d}{m_s} \sim 0,027 \, . \tag{6.8}$$

While 6.6 basically agrees with the commonly expected value of this entry (see Ref. [55]), 6.7 and 6.8 are away by a factor ~ 4 in the first case, and ~ 8 in the second. An adjustment of the value is not a matter of "second order" corrections. Here the problem is that for these mixings, experimental results are mostly obtained through branching ratios of meson (π, K) decays. In these quark compounds, the strong non-perturbative resummation is highly sensitive to the GeV scale. Indeed, an experimental value $|V_{cd}| \sim 0,22$ seems to be much influenced not by the mass ratio of the bare quarks, but of the K and π mesons:

$$V_{cd} \approx \frac{m_\pi}{m_K} \ \sim \ \mathcal{O}(0,22) \, . \tag{6.9}$$

Although in a lighter way, the meson scale seems to modify also the ratio of the bottom to down quark transition. As we already said, here it is not a matter of determining a physical quantity: only decay amplitudes are physical, the CKM matrix doesn't have a physical meaning in itself. It is therefore crucial to see how do we refer to this effective tool: how much "resummation" we want to attribute to a correction to be applied to "bare" decay amplitudes computed from a "bare" CKM matrix, and how much of it we prefer to already include in the CKM matrix. As long as the final products we consider are just meson amplitudes, the two approaches are equivalent.

By comparing eqs. 6.8 and 6.9, we are faced with something at the same time reasonable and which nevertheless sounds somehow odd. On one hand, the fact that 6.9 gives a higher ratio is not surprising: it is in fact quite natural to think that a heavier particle has a larger decay probability than a lighter one. On the other hand, when applied to the $|V_{cd}|$ transition, this argument seems to lead to a contradiction: the basic degrees of freedom of a K- and π-mesons are the quarks; nevertheless, the Kaon has a larger decay probability than the quarks it is made of. Indeed, it is not in this way that the enhancement of the V_{cd} entry due to the passage from quarks to mesons has to be interpreted. The free quark "does not exist", Pions and Kaons are the lightest strong-interaction singlets containing the d and s quark. Once inserted in the computation of a decay amplitude, the values we are proposing for the entries of the CKM matrix must be corrected by some overall "form factor", of the order of m_K/m_s for the initial state, and of m_π/m_d for the final state. In practice, this is equivalent to the introduction of an "effective" CKM matrix entry, $V_{cd}^{\rm eff} \sim (m_K/m_s)/(m_\pi/m_d) \times V_{cd}$. This rescaling eats the factor ~ 8 of disagreement between our prediction and the usual value of this entry, as reported in the literature.

Differently from the case of $|V_{cd}|$, $|V_{ts}|$ turns out to be in agreement with what reported in the literature, because the latter is derived by unitarity from $|V_{cb}|$, measured through $B \to D$ decays. Both these mesons have a mass of the same order as the b and c quark respectively. To be more precise, in these cases the quark mass itself, as is given in the literature, corresponds to the "corrected mass", basically coinciding with the mass of the meson of which it constitutes the heaviest component. The matrix entry is therefore "by definition" almost the same as the "bare" one.

The case of the $|V_{td}|$ (and V_{ub}) entries is even more involved, being much higher the uncertainties in the experimental derivation of the transition elements.

A more detailed derivation of the CKM entries as a function of masses can be found in Ref. [57]. As we said, this is perfectly fine in a neighbourhood of our present time. In our case, the entries of 6.1 are however time-dependent, and the branching ratios of various decays vary along the evolution of the universe. The various particles tend toward a higher relative separation: although the curvature of space-time tends to a "flat" limit, and the absolute value of masses decrease with time, the ratios of the various masses increase, thereby lowering the probability of mixing among families. This goes together with the T-dual increase with time of the mass of the "would be gauge bosons" of the broken symmetry among generations, the non-field-theoretical decay we discussed in section 4.3.

In our framework, neutrinos are massive, and we expect that the CKM matrix has a leptonic counterpart. The "leptonic CKM" entries should however be more suppressed, as a consequence of the rearrangement of the phase space, so that all the three neutrinos are lighter than the lightest charged lepton, and their spaces have a higher separation. According to the leptonic mass values derived in section 5.1.1, we expect at present time approximately:

$$\begin{aligned} V_{\rm CKM}^{\rm leptons} & \equiv \left(\begin{array}{ccc} V_{e\nu_e} & V_{e\nu_\mu} & V_{e\nu_\tau} \\ V_{\mu\nu_e} & V_{\mu\nu_\mu} & V_{\mu\nu_\tau} \\ V_{\tau\nu_e} & V_{\tau\nu_\mu} & V_{\tau\nu_\tau} \end{array} \right) \\ & \approx \left(\begin{array}{ccc} \sim 1 & \sim 0,007 & \sim 0,00005 \\ \sim 0,007 & \sim 1 & \sim 0,007 \\ \sim 0,00005 & \sim 0,007 & \sim 1 \end{array} \right) . \end{aligned} \tag{6.10}$$

Non-diagonal lepton decays are therefore more difficult to observe than those of quarks, perhaps more difficult to detect than neutrino masses themselves.

6.2 CP violations

In our framework, CP (and T) violation is already implemented in the construction: it is a consequence of the shifts in the space and time coordinates, that break parity (in the case of space), and time reversal symmetries. As we have seen, this is related to entropy, and to the fact that also the second principle of thermodynamics is automatically implemented in this scenario.

Let's consider the decay of a particle, e.g. $K \to \pi \, (+\gamma)$, or $B \to K \, (+\gamma)$. The decay probability is proportional to the square "coupling": $\alpha^2_{K \to \pi\gamma}$, $\alpha^2_{B \to K\gamma}$. The square effective couplings represent volumes in the phase space: they are in fact proportional to mass ratios, and can be viewed as the inverse of a proper time, the proper time of the decaying particle measured in units of the decay product, raised to the fourth power. The larger is this volume, the higher is the decay probability.

However, if we consider the problem more in detail, we see that not the entire length, given as the inverse of the mass, is at disposal for increasing the entropy through the decay process: part of this phase space is occupied by the rest energy of the product particle(s), that we collectively indicate with m_f. As for all masses, the origin of m_f is a shift along space-time. We want now to see what happens if we invert the shifted coordinate. Let's consider the simplified case of just one space-time coordinate, t. In this case, this inversion is a time reversion. Under $t \to -t$, the shift operation turns out to act in the opposite direction, and results in an expansion of the volume. In our framework, time is compact: $t \in [0, \mathcal{T}]$. A mass corresponds to a shift $t \to t + a$, $a \approx \frac{1}{m_f}$, such that now the zero point has been displaced to a: $0 \to a$. If we perform a time reversal operation, $t \to -t$, we obtain that the shift acts now as: $(-t) \to (-t) + a$. By overall sign reversal, this is equivalent to a mirror situation, $t \to t - a$, in which we have a "particle" with mass $-m_f$. Roughly speaking, we can think that the volume in the phase space occupied by this particle is "stolen" from the correct time evolution, and goes "in the opposite direction". The asymmetry between a "straight" decay and its "time-reversed" one is given by the fact that in the first case the phase space volume is proportional to $[(m_i + m_f)/m_f]^4$, after the inversion to $[(m_i - m_f)/m_f]^4$. The general result with four space-time coordinates is then

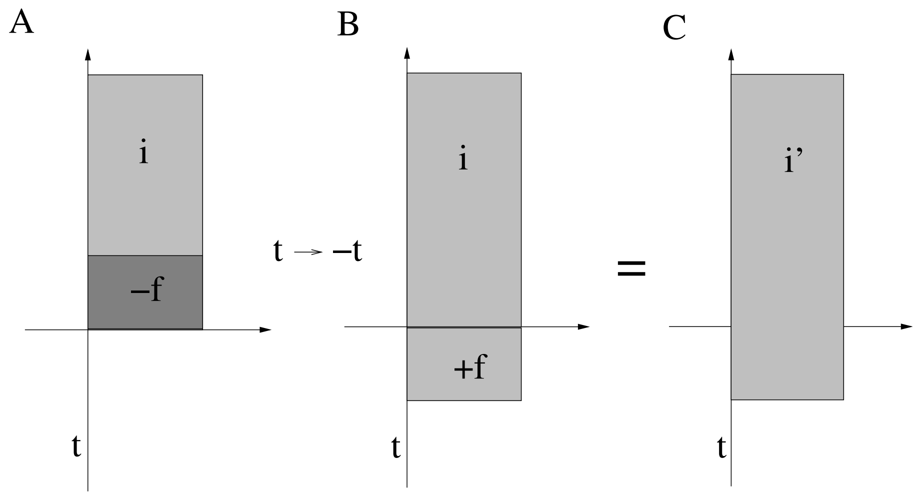


Figure 5: The increase in the decay amplitude as produced by a time reversal. While in picture A the phase space volume of the final state f has to be subtracted from the volume at disposal of the initial state, after a time flip it adds up (picture B), ending in an increased decay probability for the initial state (picture C).

a simple consequence of the fact that a global inversion of all the space-time coordinates: $t \to -t, \vec{x} \to -\vec{x}$ is a symmetry of the system.

In decays involving transitions from neighbouring generations. e.g. $b \to s$, $s \to d$, the mass ratio is at present time $m_i/m_f \sim 10$ and $\sim 3,8$ respectively (as in the case of the CKM angles, the mass ratios we have to consider are not those of free quarks, but of the particles effectively involved in the decays. In this specific case, the B, K and π mesons). Therefore, at present we get an asymmetry of order $\sim (1/3,8)^4 \approx 4,8 \times 10^{-3}$ for the K decays, and $\sim 10^{-4}$ for B decays.

In the case of D mesons ($(c\bar{d})$, $(c\bar{u})$ and conjugates), we get a $\mathcal{O}(10^{-2})$ decay asymmetry. This D result is of particular relevance, because in this case there is no reliable prediction based on the Standard Model effective parametrization. Indeed, while in our framework we obtain an asymmetry in the range of the experimental observations, the Standard Model prediction fails to account for the magnitude of the observed effect (for a review, see for instance [55]). Because of this, CP violations in the D mesons decays are often considered a test for "new physics".

7 Astrophysical implications

Expression 1.1 contains in principle all the information about the universe, at any time of its evolution. In section 3 we have already seen how it is possible to derive information also about "astronomical" data, such as the cosmological constant and the energy density of the universe. We have then investigated the masses and interactions of the elementary particles. Masses and couplings are of interest for the physics of accelerators. However, a perhaps larger potential source of stringent tests seems to come from astrophysics, a field which appears the more and more as one of the most exciting domains of present and future investigation. This is particularly true for our framework, in which the present day physics is tightly related to the history and the evolution of the universe. Therefore, after having gained a better insight into the details of the elementary particles, here and in section 8 we come back to consider some important issues addressed by astrophysical observations.

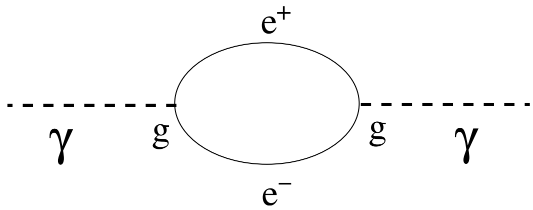
7.1 The CMB radiation

An important experimental cosmological observation is the detection of a "ground" cosmic electromagnetic radiation with the typical spectrum of a black body radiation, with a temperature of about 2,8 ^0K [63, 64]. This phenomenon is often claimed to constitute a proof of the theory of the Big Bang: this radiation would consist of photons cooled down during the expansion of the universe, and at the origin they should have possessed an energy corresponding to a microwave length, as expected from energy exchange due to Compton scattering through the plasma at the origin of the universe. Here we want to discuss how these issues are addressed in our scenario. In the framework we are proposing, the history of the universe is already implied in the solution of 1.1; its expansion and cooling down are already "embedded" in the framework, which is in itself a cosmological scenario, and in principle don't need to be added as separate inputs. As a consequence, the 3 Kelvin temperature of this radiation can be justified by directly using the present-day data of the universe. This does not mean that the CMB radiation does not have any relation with the universe at early times of its expansion: simply, the present configuration of the universe is in our framework a particular case of a cosmological solution accounting for any era of its evolution, and therefore already "contains the information" about the earlier times. In particular, all energies and mass scales of our present time are the primordial ones, cooled down.

According to the discussion of section 2.2, the spectrum of the universe "stabilizes" into neutrons/protons+electrons (the visible matter) plus free neutrinos ν_e, photons and, of course, gravitons. The CMB radiation consists of photons of "ground energy", i.e. they are not produced by interactions and decays of visible matter. This background radiation can be viewed as a bath of photons in contact with a "thermal reservoir", a neutral vacuum, whose energy corresponds to the inverse square root scale of the age of the universe, the ground energy scale of matter [52]. Particle's pair production out of this vacuum leads eventually to

[52] Notice that we must use the ground energy scale, not the "mean" mass scale, roughly corresponding to the neutron mass scale. Here we are not interested in the non-perturbative mass eigenstate at any finite time, but in the ground energy step of a universe evolving toward a "flat" limit at infinity (flat in the sense

photon production. In order to understand what the average energy of this "photon sea", the "temperature", is, let's look at a typical photon/vacuum transition process, taking into account that at equilibrium the photon propagator, intended as "the mean propagator", is not renormalized. As we already discussed several times, in the physical picture Feynman-like diagram corrections act multiplicatively, a passage from logarithms of quantities such as couplings and coordinates to their exponential being subtended. The traditional series expansion corresponds to a linearization, to working with algebras, on the "tangent space", of a phenomenon that, once resummed, has to be viewed as working at the level of the group. The diagram representing the class of processes of interest for us is sketched in figure 7.1.



(7.1)

As seen from the "exponential picture", diagrams like this one contribute to a multiplicative correction to the photon propagator, such that:

$$\frac{1}{p_\gamma^2} \to \frac{1}{p_\gamma^2} \times \left[\frac{\alpha \int (\text{loop})}{p_\gamma^2} \right] , \tag{7.2}$$

where $\alpha = \alpha_{\text{e.m.}} \equiv \alpha_\gamma$ and the loop integral pops out a $[m^2]$, corresponding to the energy of our black body reservoir E_0:

$$\int (\text{loop}) \approx E_0^2 \approx \left(\frac{1}{2\sqrt{\mathcal{T}}} \right)^2 . \tag{7.3}$$

In other words, we can say that the ground electromagnetic radiation consist of a gas of photons at equilibrium, interacting with a matter vacuum, whose ground energy fluctuations are, according to the Uncertainty Principle, of the order of 1/2 the inverse of the typical length of the matter sector, the square root scale of the age of the universe. Since at equilibrium, as we have said, the average photon propagator is not renormalized, we conclude that the term under square brackets in 7.2 must contribute by a factor 1, and therefore:

$$< p_\gamma > \sim \sqrt{\alpha_\gamma} \, \frac{1}{2} \frac{1}{\sqrt{\mathcal{T}}} . \tag{7.4}$$

Using the value of α_γ at the electron's scale, derived through an effective running of the type 4.99 from the initial value 4.73 at the $\mathcal{T}^{-1/2}$ scale to the 0, 5 MeV scale, $\alpha_\gamma^{-1}|_{m_e} \sim 132, 3$,

of the non-compact orbifolds discussed in the previous sections).

4.92 for the present age of the universe, and converting energy into temperature through the Boltzmann constant, we obtain:

$$T_\gamma \equiv k^{-1} < p_\gamma > \; = k^{-1} E^0_\gamma \sim 2,72\,^0 K \,. \tag{7.5}$$

A signal that the present day phenomenon results from the cooling down of a primordial situation comes from the spatial inhomogeneity of this radiation over a solid angle of observation [65]. We have seen that in our framework, that describes the physics "on shell", the invariance under space rotations is broken by the same mechanism that gives rise to non vanishing masses for the matter states. The scaling of masses, that, going backwards in time, tend to the Planck scale, tells us that in the primordial universe also the space inhomogeneity must have been higher than what it is today. Toward the Planck scale, the spatial inhomogeneities are expected to become of a size comparable with the size of the universe itself.

7.2 The fate of dark matter and the Chandra observations

A discrepancy between our framework and the common expectations is the absence in our scenario of dark matter. According to our analysis, the universe consists only of the already known and detected particles. Of course, there can be regions of the space in which a high concentration of neutrinos, which for us are massive, increases the curvature without being electromagnetically detected. But this is not going to change dramatically the scenario: there is no hidden matter acting as an extra source able to increase the gravitational force by around a factor ten over what is produced by visible matter, as it seems to be required in order to explain a gravitational attraction among galaxies much higher than expected on the base of the estimated mass of the visible stars. The problem arises in several contexts: Big-Bang nucleosynthesis, rotational speed of galaxies, gravitational lensing. All these points would require a detailed investigation, beyond the scope of this work. We will also not attempt to rediscuss a huge literature, and limit ourselves here to mention some hypotheses. The first remark is that the discrepancies between theoretical expectations and the observed effects, which are found in so different issues as primordial universe, nucleosynthesis and galaxy phenomenology, don't need necessarily to be explained all in the same way.

About nucleosynthesis we will spend some words in section 8.3. Let's consider here the problems related to the motion of external stars in spiral galaxies, where for the first time the issue of dark matter has been addressed, and the "anomalous" gravitational lensing, with reference to the recently observed effect in the 1E0657-558 cluster [66].

It is since 1933 (Fritz Zwicky) that, by looking at the amount of red-shift in the light emitted by the stars in the wings of a spiral galaxy, it has been noticed how, differently from what expected, the rotation speed does not decrease with the inverse of the square root of the radius: it is a constant [67, 68]. Presence of invisible matter has been advocated, in order to fill the gap between the mass of the observed matter and the amount necessary to increase the gravitational force. Indeed, the expectation that the rotation speed of stars in the external legs should decrease is based on the assumption that almost the entire mass of the galaxy is concentrated in the bulge at the center of the spiral. Any star on the wings would therefore feel the typical gravitational field due to a fixed, central mass.

In the framework of our scenario, masses have been in the past higher than what they are now. Moreover, owing to the fact that, as we discuss in [1], the universe "closes up", in such a way that the horizon we observe corresponds to a "point", the space separation between objects located at a certain cosmic distance from us appears to be larger than what actually is. All this could mean that the mass of the center of a galaxy, as compared to the wings, has been systematically overestimated. It would be interesting to see, by carrying out a detailed re-examination of the astronomical observations, whether the behaviour of the center of a galaxy still requires to advocate the presence of a heavy black hole, in order to explain a gravitational force higher than what expected on the base of the estimated mass of the visible stars. In any case, it is possible that, once the downscaling of length and upscaling of masses has been appropriately taken into account, a better approximation of a spiral galaxy is the one sketched in figure 6. In part A of the picture the galaxy is (very roughly) represented with wide wings, with stars relatively "broadened" on the plane of the galaxy. Part B shows the same figure, simply with much narrower arms. In picture A the broad lines have been shadowed in a way to make evident that the higher star density of the bulge is largely due to the "superposition" of the various arms. Nevertheless, as it is clear from picture B, the problem remains basically "one-dimensional": the wings are one-dimensional lines coming out of the center of the galaxy. Under the hypothesis that all the stars have the same mass, the linear density of a wing is constant, and, once integrated from the center up to a certain radius R, the total mass M_R of the portion of galaxy enclosed within a distance R from the center is roughly proportional to R:

$$\rho = \frac{dM}{dr} \sim \text{const.} \quad \Rightarrow \; M_R \sim \text{const} \times R \, . \tag{7.6}$$

In the expression of the gravitational potential, the linear R dependence of the mass cancels against the R appearing in the denominator (the potential remains the one of a Coulomb force). The gravitational potential energy is therefore a constant times the mass of the star in the wing. Conservation of energy implies therefore that also the velocity of the star does not depend on the radius R. We stress that this is only an approximation: it would be exact if the arms were not those of a spiral but straight legs coming out radially from the center, and under the assumption that all the stars of the bulge correspond to the superposition of the arms.

In the case of the 1E0657-558 cluster, the Chandra observatory has detected a gravitational lensing higher than what expected on the base of the amount of luminous matter. Moreover, the highest effect corresponds to two dark regions close to the cluster, rather than to places where the visible matter is more dense. In the framework of our scenario, a possible explanation could be that what is observed is the effect of a "solitonic" gravitational wave, produced as a consequence of the separation of one sub-cluster from the other one. This could increase the gravitational force by an amount equivalent to the displaced cluster mass, for a length/time comparable to the cluster size, therefore a time much higher than the few hours during which the effect has been measured ($\sim$ 140 hours). It remains that the lensing is around 8-9 times higher than what expected on the base of the amount of visible mass. However, the cluster under consideration is at about 4 billion light years away from us. This is around 1/3 of the age of the universe. This time distance is large

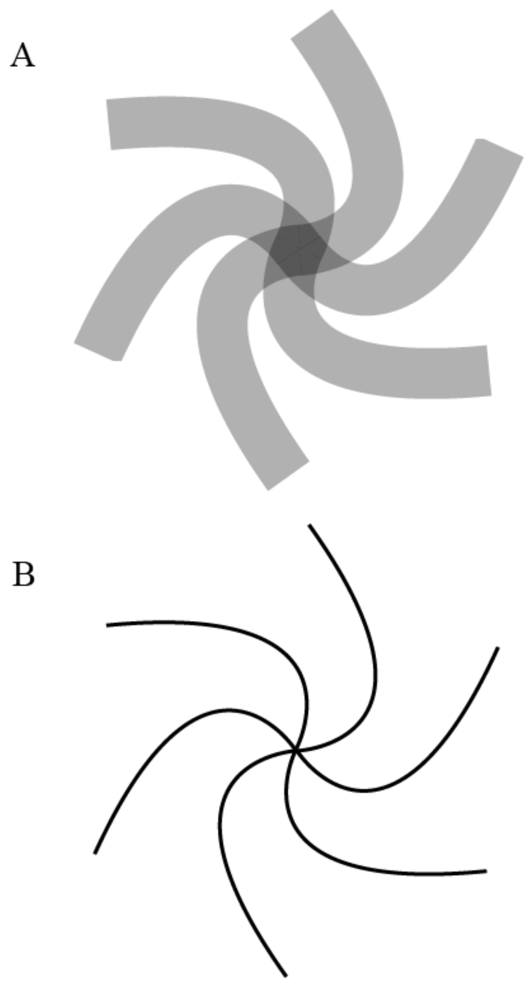


Figure 6: Picture A is the rough sketch of a spiral galaxy, in which the arms are broad and shadowed in a way to highlight the increasing mass density due to their superposition at the center. Figure B represents the same object, with the arms narrowed down, in order to highlight the one-dimensional nature of the physical problem, for what concerns the mass density.

enough to make relevant the effects due to a change of the curvature of space-time along the evolution of the universe, as well as a change of masses, according to 3.10–3.12 and 4.82. Furthermore, as we discussed above, the apparent space separation between objects located at a certain cosmic distance from us must be appropriately downscaled, in order to account for the curving up of space-time into a sphere, with the horizon "identified" with the origin. Putting all this together, we obtain that the effective gravitational force experienced on the 1E0657-558 cluster is (or, better, it was) indeed 8-9 times higher than what it appears to us on the base of the expected mass of the objects in the cluster, i.e. precisely the amount otherwise referred to dark matter.

8 Cosmological constraints

Cosmology addresses two kinds of problems for what concerns the "running back" of a theory, or an "early time" model. Namely, 1) the possible non-constancy of what are commonly called "constants", and 2) the agreement with the expected origin/evolution of the early universe (baryogenesis, nucleosynthesis etc...). In our framework, these issues are put in a light quite different from the usual perspective: there are in fact indeed no constants; therefore, a variation of couplings, masses, cosmological parameters, and, as a consequence, energy spectra, is naturally implemented. However, there is a peculiarity: all these parameters scale as appropriate powers of the age of the universe. As a consequence, a "number" close to one at present day has a very mild time dependence:

$$\mathcal{O}(1) \approx \mathcal{T}^{\epsilon} \Rightarrow |\epsilon| \ll 1 \,, \tag{8.1}$$

and therefore varies quite a little with time. Oklo and nucleosynthesis bounds, being given as ratios of masses and couplings that cancel each other to an almost "adimensional" quantity, are precisely of this kind. In our case they don't provide therefore any dangerous constraint.

For what concerns the non-constancy of "constants", there are not enough data enabling to test our prediction about a time variation of the cosmological constant, whose measurement is still too imprecise. A more stringent test of the variation of parameters comes from the observations on the light emitted by ancient Quasars. In this case, the spectrum shows an "anomalous" red-shifted spectrum. This shift should not be confused with the usual red-shift, of which we have discussed in section 3.2. The effect we consider here persists once the "universal" red-shift effect has been subtracted. As an explanation, it is often advocated a possible time variation of the fine structure constant α. We already devoted a paper to this subject [69], at a time in which many issues concerning our framework were not enough clear. We therefore rediscuss here the argument, in the light of a better understanding of the theoretical framework we are proposing.

8.1 *The "time dependence of α"*

The question of the possible time variation of the fine structure "constant" arises in the framework of string theory derived effective models for cosmology and elementary particles. Various investigations have considered the possibility of producing some evidence of this variation, or at least a bound on its size. To this regard, astrophysics is certainly a favoured field of research, in that it naturally provides us with data about earlier ages of the universe. A possible signal for such a time variation could be an observed deviation in the absorption spectra of ancient Quasars [70, 71, 72, 73]. This effect consists is a deviation in the energies corresponding to some electron transitions, which remains after subtraction of the background effect of the red-shift, and is obtained with interpolations and fitting of data.

What is observed is a decrease of the relativistic effects in the energies of the electrons cloud, with respect to what expected on the base of present-day parameters (in particular, the fine structure constant). Indeed, while the atomic spectra are universally proportional to the atomic unit $me^2 \propto m\alpha^2$, the relativistic corrections depend on the coupling α. After subtraction of the "universal" red-shift effects, their variation should then be directly related to a variation of α. In our framework, the explanation comes from considering both the scaling of α and the one of masses at the same time: going backwards in time α increases, as also the proton and the electron mass do, but *the ratio of α to the mass scales decreases*. This is the "variation of α after subtracting the universal red-shift" which is usually considered in the discussions of the literature. Namely, if we measure the variation of α with respect to the electron's mass scale (whether the true electron mass or the "reduced" mass doesn't make a relevant difference [53]), i.e. if we rescale quantities in the frame in which masses are considered fixed, we indeed observe a decrease of the coupling α. Indeed, what is done in the literature (see Refs. [71]) is not only to consider masses fixed, but to exclude from the evaluation also the effect of the red-shift. With current experimental methods, based on the interpolation of spectral data in order to find out the "background" and the variations out of it, this subtraction is somewhat unavoidable.

In order to obtain what the prediction in our scenario is, and how it compares with the literature, let's first see how the decrease of the relativistic effects, when going backwards in time, turns out to be a prediction of our framework. Consider the effective scaling of α in terms of $m\alpha^2$ units, the "universal" scaling of emission/absorption atomic energies. We have that:

$$\bar{\alpha} \stackrel{\text{def}}{\equiv} \frac{\alpha}{m\alpha^2} \approx \mathcal{T}^{\frac{1}{3}+\frac{1}{28}} . \tag{8.2}$$

The "effective" coupling $\bar{\alpha}$ scales as a positive power of the age of the universe: going backwards in time, it decreases. According to the literature, atomic energies have an approximate scaling of the type [54]:

$$E_n \approx K_n (m\,\alpha^2) + \Gamma_n \, \alpha^2 (m\,\alpha^2) , \tag{8.3}$$

where K_n and Γ_n are constants and the second term, of order α^2 with respect to the first one, accounts for the relativistic corrections. Investigations on the possible variation of α use interpolation methods to disentangle the second term from the first one. Since the universal part is reabsorbed into the red-shift, the relative variation should give information on just the variation of α. Expression 8.3 is of the form:

$$E_n \approx E_n^0 (1 + a_1 \, \alpha^2) . \tag{8.4}$$

It is derived by considering the first terms of a field theory expansion around the fine structure constant (the electric coupling). Indeed, since we are interested in the correction subtracted of the universal part reabsorbable in the red shift, we can separate the $\mathcal{O}(\alpha^2)$ term in 8.3 as:

$$K_n = (K_n - \Gamma_n) + \Gamma_n . \tag{8.5}$$

This allows to reduce the part of interest for us to:

$$E_n^{\text{eff}} \approx E_n^0 (1 + \alpha^2) . \tag{8.6}$$

[53]In the hydrogen atom this is given by $m_e = \frac{m_e m_p}{m_e + m_p}$. The possibility of referring to a change of this quantity the effect measured in Ref. [71] can be found in Ref. [74, 75, 76].

[54]See for instance Ref. [71]

As we already observed several times along this work, perturbative expressions involving elementary particles are naturally defined and carried out in a logarithmic representation of the physical vacuum. In particular, when writing expressions like 8.3 it is intended that the coupling α scales logarithmically. An expression like 8.6 should be better viewed as accounting for the first terms of a series that sums up to an expression scaling as a certain power of the age of the universe:

$$\alpha_{\text{eff}} \equiv 1 + \alpha^2 \approx 1 + \alpha^2 + \mathcal{O}(\alpha^4) \rightsquigarrow \exp\alpha \sim \mathcal{T}^{\beta}\,, \tag{8.7}$$

where α is then not the full coupling, intended in the non-perturbative sense of 4.39, but its logarithm. According to 8.2, in the hypothesis of keeping masses fixed, this term should then effectively scale as a positive power of the age of the universe: $\beta > 0$. The exponent β can be fixed by comparing values at present time:

$$\alpha|_{\text{today}} \approx \sqrt{5 \times 10^{-5}}\,. \tag{8.8}$$

We obtain therefore:

$$1 + \alpha^2 \approx \mathcal{O}(1 + 5 \times 10^{-5}) \approx \mathcal{T}^{\beta} \;\Rightarrow\; \beta \sim \mathcal{O}(10^{-6})\,, \tag{8.9}$$

and a relative time variation:

$$\frac{\dot{\alpha}_{\text{eff}}}{\alpha_{\text{eff}}} \approx \beta\,\mathcal{T}^{-1} \approx \mathcal{O}\left(10^{-16}\,\text{yr}^{-1}\right)\,. \tag{8.10}$$

This is the relative variation of the relativistic correction subtracted of the universal part (reabsorbed in the red-shift), to be compared with the results of [70], as reported also in [71]:

$$\frac{\langle\dot{\alpha}\rangle}{\alpha} = -2.2 \pm 5.1 \times 10^{-16}\;\text{yr}^{-1}\,. \tag{8.11}$$

Since the deviation of the resummed function 8.6 from a pure exponential is of order $\alpha^4 \sim 2 \times 10^{-9}$, four orders of magnitude smaller than the dominant term, the inaccuracy in our computation is much lower than the order of magnitude of the result.

8.2 *The Oklo bound*

Data from the natural fission reactor, active in Oklo around two billions years ago, are today considered one of the most important sources of constraints on the time variation of the fundamental constants. By comparing the cross section for the neutron capture by Samarium at present time with the one estimated at the time of the reactor's activity, one derives a bound on the possible variation of the fine structure constant, and on the ratio $G_F m_{\text{p}}^2$, in the corresponding time interval. The interpretation of the experimental measurements and their translation into a bound on the variation of the capture energy resonance is not so straightforward, and depends on several hypotheses. In any case, all these steps are sufficiently under control. More uncertain is the translation of this bound on the energy variation into a bound on the variation of the fine structure constant and other parameters: this passage requires strong assumptions about what is going to contribute to the atomic energies. This analysis was carried out in Ref. [77], basically on the hypothesis that the main contribution to the resonance energy comes from the Coulomb potential of the electric interaction among the various protons of which the nucleus of Samarium consists. According to [77], after a certain amount of reasonable approximations, the energy bound translates into a bound on the variation of the electromagnetic coupling. A simple look at expression 4.68 shows that, in our scenario, the variation of this coupling over the time interval under consideration violates the Oklo bound. This bound seems therefore to rule out our theoretical framework. However, things are not so simple: the derivation of a bound on a coupling out of a bound on energies works much differently in our framework, and we cannot simply use for our purpose the results of [77]. Indeed, in our framework what varies with time is not only the fine structure constant, but also the nuclear force, and the proton and neutron mass as well. Of relevance for us is therefore not a bound on a coupling, derived under the hypothesis of keeping everything else fixed, but the bound on the energy itself [77]:

$$-0,12 \text{ eV} \; < \; \Delta E \; < \; 0,09 \text{ eV} \,. \tag{8.12}$$

In order to give an estimate of the amount of the energy variation over time, as expected in our framework, we don't need to know the details of the evaluation of the resonance energy starting from the fundamental parameters of the theory. To this purpose, it is enough to consider that, whatever the expression of this energy is, it must be built out of i) masses, ii) couplings (electro-weak and strong) and iii) the true fundamental constants (the speed of light c, the Planck constant $\hbar$, and the Planck mass $\mathrm{M_p}$). Working in units in which the latter are set to 1 (reduced Planck units), all parameters of points i) and ii) scale as a certain power of the age of the universe. As a consequence, the resonance energy itself mainly scales as a power of the age of the universe:

$$E \; \sim \; a \mathcal{T}^{-b} \,. \tag{8.13}$$

(More generically, it could be a polynomial: $E \sim a_1 \mathcal{T}^{-b_1} + a_2 \mathcal{T}^{-b_2} + \ldots + a_n \mathcal{T}^{-b_n}$. In this case, to the purpose of checking the agreement with a bound, it is enough to look at the dominant term). We can fix the exponent b by comparing the expression, evaluated using the present-day age of the universe, with the value of the resonance, that we take from [77]:

$$E \; \sim \; a \mathcal{T}^{-b} \; = \; 0,0973 \, \text{eV} \; \times \; 1,2 \, \times \, 10^{-28} \; = \; 1,2 \, \times \, 10^{-29} \, \mathrm{M_P} \,. \tag{8.14}$$

In order to solve the equation, we would need to know the coefficient a, something we don't. However, as long as we are just interested in a rough estimate, it is reasonable to assume that, since this coefficient mostly accounts for possible symmetry factors, it may affect the value of the result for about no more than one order of magnitude. Inserting the value $\mathcal{T} \sim 5 \times 10^{60} \mathrm{M_P^{-1}}$ for the age of the universe, we obtain:

$$b \; \sim \; \frac{1}{2} \,, \tag{8.15}$$

and finally:

$$|\Delta E| \; \sim \; \frac{1}{10} E \; \sim \; 0,01 \text{ eV} \,. \tag{8.16}$$

over a time of two billion years. This is compatible with the Oklo bound, eq. 8.12.

From the Oklo data one tries also to derive a bound on the adimensional quantity

$$\beta \equiv G_F m_{\mathrm{p}}^2 (c/\bar{h}^3) \,. \tag{8.17}$$

In this case, our discussion is easier, because we know the scaling of all the quantities involved [55]. Once again, we have to deal with a quantity that scales as a power of the age of the universe. At present time, we have:

$$\beta \sim \mathcal{T}^{-b_\beta} = 1,03 \times 10^{-5} \,. \tag{8.18}$$

Inserting the actual value of the age of the universe, we obtain $b_\beta \sim \frac{1}{12}$. Over a time interval of around 1/5 of the age of the universe, this gives a relative variation:

$$\frac{\Delta\beta}{\beta} \sim 0,017 \,, \tag{8.19}$$

to be compared with the one quoted in Ref. [77]:

$$\frac{|\beta^{\mathrm{Oklo}} - \beta^{\mathrm{now}}|}{\beta} < 0,02 \,. \tag{8.20}$$

Both results 8.16 and 8.19, although still within the allowed range of values, seem to be quite close to the threshold, beyond which the model is ruled out. One would therefore think that a slight refinement on the measurement and derivation of these bounds could in a near future decide whether it is still acceptable or definitely ruled out. Things are not like that. Indeed, as we already stressed in several similar cases, the *entire* derivation of bounds and constraints, involving at any level various assumptions about the history of the universe and therefore of its fundamental parameters, should be rediscussed within the new theoretical framework: it doesn't make much sense to compare pieces of an argument, extracted from an analysis carried out in a different theoretical framework, with different phenomenological implications. To be explicit, in the case of the derivation of the Oklo bounds, one should reconsider all the derivation of absorption thresholds and resonances. We should therefore better take into account from the beginning the time variation of all masses, and in particular the neutron and proton masses, as well as couplings. Perhaps a more meaningful quantity is then not anymore the pure resonance shift, but this quantity rescaled by the neutron mass. In this case, the effective variation of interest for our test is not 8.16, but:

$$\frac{\Delta(E/m_n)}{(E/m_n)} \approx \frac{\Delta\mathcal{T}^{-\frac{1}{9}}}{\mathcal{T}^{-\frac{1}{9}}} \sim 0,02 \,, \tag{8.21}$$

a variation one order of magnitude smaller than 8.16 ($\Delta E/E \sim 0,1$). Analogous considerations apply also to the case of the second bound 8.19, basically equivalent to the nucleosynthesis bound.

[55] We recall that $G_F/\sqrt{2} = g^2/8M_W^2$. Therefore, $\beta = \pi\alpha m_{\mathrm{p}}^2/\sqrt{2}M_W^2$. For times much higher than 1 in reduced Planck units, the proton mass can be assumed to scale approximately like the mean mass scale 4.82.

Bounds derived from nucleosynthesis models are even more questionable: they certainly make sense within a certain cosmological model, but, precisely because of that, they cannot be simply translated into a framework implying a rather different cosmological scenario. Once again, the only anchor points on which we can rely are the few "pure" experimental observations, to be interpreted in a consistent way in the light of a different theory. The point of nucleosynthesis is that there is a very narrow "window" of favourable conditions under which, out of the initial hot plasma, our universe, with the known matter content, has been formed. Of interest for us is the very stringent condition about the temperature (and age of the universe) at which the amount of neutrons in baryonic matter have been fixed. As soon as, owing to a cooling down of the temperature, the weak interactions are no more at equilibrium, the probability for a proton to transform into a neutron is suppressed with respect to the probability of a neutron to decay into a proton. Owing to their short life time, comparable with the age of the universe at which the equilibrium is broken, basically almost all neutrons rapidly decay into protons, except for those that bound into ^{4}He. Nucleosynthesis predicts a fraction of ^{4}Helium and Hydrogen baryon numbers ($\sim 1/4$) in the primordial universe, which is in good agreement with experimental observations. The formula for the equilibrium of neutron/proton transitions is given by:

$$\frac{\mathrm{n}}{\mathrm{p}} = \mathrm{e}^{-\frac{\Delta m}{kT}} \sim 1 \,, \tag{8.22}$$

where $\Delta m = m_{\mathrm{n}} - m_{\mathrm{p}}$. In the standard scenario, this mass difference is a constant, and the temperature runs as the inverse of the age of the universe. The equilibrium is broken at a temperature of around $0,8$ MeV, when $(\mathrm{n}/\mathrm{p}) \simeq 1/7$. In our framework too the temperature runs as the inverse of the age of the universe, but the mass difference Δm is not a constant: all masses run with time. At large times ($\mathcal{T} \gg 1$ in Planck units), we are in a regime in which we can use the arguments of section 5.5, in order to conclude that, being the u and d quark masses much lighter than the neutron mass scale (which is related to the "$m_{11/36}$" mass scale), we can consider Δm as a perturbation of $m \simeq m_{\mathrm{n}}$. In this regime, the neutron-proton mass difference is basically of the order of the constituent quark mass difference, and we have reasons to expect that it also runs accordingly. It would therefore seem that, in our case, going backwards in time, the ratio (n/p) remains lower than in the standard case, and the equilibrium 8.22 is attained at a temperature much higher. However, to the purpose of determining the processes of the nucleosynthesis, essential is not just the scaling of the equilibrium law of the neutron-to-proton ratio, but also that of the mean life of the neutron. It is the combined effect of these two quantities what determines the primordial baryon composition. In the standard case, the neutron mean life is assumed to be constant. Being related to the neutron decay amplitude, i.e. to the volume occupied by the neutron in the phase space, in our framework this quantity too is not constant. In order to see what in practice changes in our scenario with respect to the standard one, instead of attempting to guess what the scaling behaviour of the neutron mean life could be, we can proceed by considering, instead of the pure running of the equilibrium equation, the *reduced running at fixed neutron mean life*. Certainly the mean life is constant if the neutron mass is constant.

The quantity of interest for us is therefore the scaling of the mass difference, as measured in units of the neutron mass itself. According to our considerations of above, we have:

$$\Delta m_{\rm red}(\mathcal{T}) \equiv \frac{\Delta m}{m_{\rm n}} \sim \frac{\mathcal{T}^{p_{(u-d)}}}{\mathcal{T}^{p_{\rm n}}}, \tag{8.23}$$

where $p_{(u-d)}$ and $p_{\rm n}$ are exponents corresponding to the up-down quark mass difference and to the neutron mass respectively. This running is expected to hold not only at present time but also at a temperature of ~ 1 MeV, which is anyway much lower than the Planck scale. We can therefore compare our prediction with the standard one by simply considering the relative deviation of equation 8.22 from its standard value, as obtained by replacing the constant mass difference Δm with $\Delta m_{\rm red}(\mathcal{T})$:

$$\frac{\rm n}{\rm p} = {\rm e}^{-\frac{\Delta m}{kT}} \;\to\; \left(\frac{\rm n}{\rm p}\right)_{\rm red} \equiv {\rm e}^{-\frac{\bar{m}_{\rm n}\Delta m_{\rm red}(\mathcal{T})}{kT}}, \tag{8.24}$$

where $\bar{m}_{\rm n}$ is the *fixed*, time-independent present-day value of the neutron mass. Therefore, in the standard case $({\rm n/p})_{\rm red}$ coincides with (n/p). According to the mass values given in section 4, we have:

$$\Delta m_{\rm red}(\mathcal{T}) \approx \mathcal{T}^{-\frac{1}{24}}. \tag{8.25}$$

Considering that the time variation between the point $\mathcal{T}_f$ of the breaking of equilibrium and the present day is of the order of the age of the universe itself, $\Delta T \equiv \mathcal{T} - \mathcal{T}_f \sim \mathcal{T}$, we obtain approximately that the integral variation of $x \equiv \Delta m_{\rm red}(\mathcal{T})$ over this time interval is:

$$\Delta x \sim \frac{1}{24} x. \tag{8.26}$$

The "reduced value" of (n/p), $({\rm n/p})_{\rm red}$, is now modified to:

$$\left(\frac{\rm n}{\rm p}\right)_{\rm red.} : \frac{1}{7} \;\to\; \sim \frac{1}{7}\left(1 - \frac{\ln 7}{24}\right) \approx 0,131. \tag{8.27}$$

This value leads to a ratio X_4 of helium to Hydrogen of around:

$$X_4 \sim 0,232, \tag{8.28}$$

still in excellent agreement with what expect from today's most precise determinations (for a list of results and references, see Ref. [55]).

As mentioned above, there is here no reason to push the discussion into further detail, because the entire issue, as well as all the extrapolations from experimental observations, should be rediscussed within the framework of this scenario, something well beyond the scope of this work. We want however to point out another aspect of the problem, which arises in our theoretical framework. A peculiarity of our scenario is that, at any time, there is a bound on the number of particles that can exist in the universe. At any time the volume of space-time is finite; the maximal amount of energy is fixed by the Schwarzschild black hole relation, $2M = R$, where R is here identified with $\mathcal{T}$, and the lowest particle's mass is the one of the electron's neutrino, given in 5.2. The number of matter states is therefore finite, and cannot exceed the Schwarzschild mass of the universe divided by the lowest neutrino mass. On the other hand, this statement has the "mean value" validity of any other statement related to 1.1. As we already discussed, only in an "average" sense we can in fact talk of geometry of space-time, and make contact with "classical" objects such as black holes, and in general with the Einstein's equations and their Scharzschild's solution. One may then ask if the "black hole bound" on the total mass of the universe can be "violated" in a quantum way, for a time $\Delta t \sim 1/\Delta m$, also along the minimal entropy solutions: although they are the configurations with a space-time more close to a classical geometry description, they are nevertheless string vacua and, as such, quantum vacua. The point is however how do we observe such a violation: in order to measure a mass fluctuation $\Delta m \sim 1/\Delta t$, we need precisely a time $\Delta t > 1$ in Planck units. According to the black hole bound, in a time Δt the universe increases its energy from $E \sim \mathcal{T}$ to $E' \sim \mathcal{T} + \Delta t$, i.e. $\Delta E \sim \Delta t$. Since $\Delta t > 1$, we have $\Delta E > \Delta m$. In other words, the mass/energy fluctuation implied by the Uncertainty Principle is always lower than the total energy increase due to the time evolution of the universe: in the time we need in order to measure a possible quantum fluctuation out of the ground value of the energy, this value changes by a much larger amount. Quantum fluctuations are smaller than the "classical" increase due to the expansion.

Appendix

A Conversion units for the age of the universe

We give here some conversion factors from time units to Planck mass units.

$$1 \text{ year (yr)} \;=\; 3,1536 \;\times\; 10^7 \text{ s}$$

In order to convert this value to eV units we divide by $\hbar = 6,582122 \times 10^{-22}$ MeV s. We obtain:

$$1 \text{ yr} \;=\; 4,791160054 \;\times\; 10^{28} \text{ MeV}^{-1}$$

Considering that the Planck mass $\mathrm{M_P} \;=\; 1,2 \;\times\; 10^{19}$ GeV, we have also the relation:

$$1 \text{ yr} \;=\; 3,992633379 \;\times\; 10^{50}\, \mathrm{M_P^{-1}}\,.$$

The age of the universe $\mathcal{T}$, estimated to be around 11,5 to 14 billion years, reads therefore:

$$\mathcal{T} \;\approx\; \left\{ \begin{array}{l} 4,59152839 \\ 5,58968673 \end{array} \right. \;\times\; 10^{60} \;\mathrm{M_P^{-1}}$$

If instead we take the neutron mass as the most precise way of deriving the age of the universe, from expression 4.90 and the present-day measured neutron mass, we obtain:

$$\mathcal{T} \;\approx\; 5.038816199 \;\times\; 10^{60} \;\mathrm{M_P^{-1}} \quad (=\; 12,6202827 \times 10^9 \text{ yr})$$

B The type II dual of the $\mathcal{N}_4 = 1$ orbifold

We discuss here the type II dual construction of the $\mathcal{N}_4 = 1$ orbifold vacuum of section 2.1.1. On the heterotic side, this appears as a supersymmetric construction. We claimed that $\mathcal{N}_4 = 1$ supersymmetry exists only perturbatively, but when the full, non-perturbative construction is considered, one sees that this symmetry is broken. From the heterotic point of view, the breaking is non-perturbative, being produced by a "twist" along the coupling-coordinate around which the perturbative expansion is built. The only signal of the supersymmetry breaking is then indirectly provided by the way couplings of non-perturbative matter and gauge sectors (parametrized by perturbative fields of the heterotic string) enter in the expressions of threshold corrections of effective couplings. Namely, with the "wrong" power, as if these couplings were "inverted", from a < 1 to a > 1 value. Indeed, these couplings are parametrized by moduli only at the $\mathcal{N}_4 = 2$ level (see Ref. [15]). When the perturbative supersymmetry is reduced to $\mathcal{N}_4 = 1$, these fields are twisted. This however only means that the expectation value is not anymore a running parameter, but is fixed. We can nevertheless trace the fate of the couplings by investigating the so-called "$\mathcal{N} = 2$" sectors.

In order to follow the operation of supersymmetry breaking from the the type II side, let's first consider the starting point, the $\mathcal{N}_4 = 2$ construction. In order to make easier the investigation of the projections, it is convenient to express the degrees of freedom in terms of free fermions (Ref. [42]). In the case of type II strings, these constructions have been extensively analysed in Ref. [12]. Indeed, the cases we are referring to are embedded in a infinitely extended space-time, a situation deeply different from the one considered in this paper, where space-time is compact. However, as we have seen, in practice this reflects in a different interpretation of the results (e.g. the fact that densities become global quantities), whereas from a technical point of view the usual analysis carries over from a scenario to the other one with minor, obvious changes (the substitution of a continuum of modes along the space-time coordinates with a discrete lattice of momenta/energies). For simplicity, we use therefore here the same notation for the string modes as in the cited works. The set of all fermions is therefore:

$$F = \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_I, y^L_I, \omega^L_I \\ \psi^R_\mu, \chi^R_I, y^R_I, \omega^R_I \end{array} \right\}, \quad (\mu = 1, 2;\ I = 1, ..., 6), \tag{2.1}$$

where $\psi^{L,R}_\mu$ indicate the left and right moving fermion degrees of freedom along the transverse space-time coordinates, while $\chi^{L,R}_I$ those along the internal coordinates. $y^{L,R}_I \omega^{L,R}_I$ correspond instead to the internal fermionized bosons. The basic sets of boundary conditions are S and $\bar{S}$, which contain only eight left- or right-moving fermions, and distinguish the boundary conditions of the left- and right- moving world-sheet superpartners:

$$S = \left\{ \psi^L_\mu, \chi^L_1, \ldots, \chi^L_6 \right\}, \qquad \bar{S} = \left\{ \psi^R_\mu, \chi^R_1, \ldots, \chi^R_6 \right\}. \tag{2.2}$$

In order to obtain a $Z_2 \times Z_2$ symmetric orbifold, we need then the two sets b_1 and b_2:

$$b_1 \;=\; \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_{1,2}, y^L_{3,...,6} \\ \psi^R_\mu, \chi^R_{1,2}, y^R_{3,...,6} \end{array} \right\}, \tag{2.3}$$

$$b_2 \;=\; \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_{3,4}, y^L_{1,2}, y^L_{5,6} \\ \psi^R_\mu, \chi^R_{3,4}, y^R_{1,2}, y^R_{5,6} \end{array} \right\}. \tag{2.4}$$

These sets assign Z_2 boundary conditions and break the $\mathcal{N}_4 = 8$ supersymmetry to $\mathcal{N}_4 = 2$. The lowest entropy configuration is then obtained by further partial shifting of some states of the twisted sectors. We will not consider these further operations: they commute with the projection we want to consider in the following, namely the one that leads to the breaking of supersymmetry; considering them complicates the construction without altering the conclusions. As discussed in Ref. [12] and [15], depending on the relative phase of the projections introduced by b_1 and b_2, we obtain two mirror configurations which, according to [15], are two slices of the same model: in one we see only the vector multiplets, in the other only the hypermultiplets, of the same $U(16)$ model.

We want now to introduce another projection, dual to the $Z_2^{(2)}$. In order to understand what we have to expect from this further operation, we must take into account that 1) in order to preserve the pattern of duality with the heterotic and type I string established at the $\mathcal{N}_4 = 2$ level, i.e. the identification of the geometric moduli of the type II space with those of the heterotic space and the type I coupling moduli, also this third projection must act symmetrically on left and right movers; 2) it must twist all these moduli. On the other hand, we cannot pretend to see the extended space-time represented in a similar way in both the heterotic/type I and the type II dual: a further symmetric, independent twist on the type II space must necessarily act also on the coordinates with index "μ". This means that, in order to see the action of the heterotic $Z_2^{N=2\to 1}$ projection, on the type II side we must trade the space-time coordinates for internal ones. From the type II point of view the heterotic space-time will therefore be entirely non-perturbative, and the type II construction will look perturbatively compactified to two dimensions. Being two coordinates hidden in the light-cone gauge, we see therefore no transverse non-compact coordinates. As we discussed in section 2.1.3, representing the "11-th coordinate" of string theory in orbifolds entails its linear realization through an embedding in a two-dimensional toroidal space. The space gives therefore the fake impression of being "12-dimensional". This is however an artifact of the perturbative representation.

Compactifying the "μ" indices implies that we can now fermionize the bosons also along these coordinates. The boson degrees of freedom ∂X_μ and $\bar{\partial} X_\mu$ will be now represented as $y^L_\mu \omega^L_\mu$ and $y^R_\mu \omega^R_\mu$. (By the way, we remark that in the scenario discussed in this work, all string coordinates are always compactified. Therefore, in principle fermionization of the space-time degrees of freedom is always possible. On the other hand, when considering explicit string constructions, perturbation is always possible only around a decompactified coordinate, that works as the vanishing coupling around which to perturb. The very fact of writing a perturbative representation of a string vacuum implies the assumption that a certain limiting procedure toward a non-fermionizable point of some coordinates has been taken.)

From this two-dimensional point of view, the $\mathcal{N}_4 = 2$ type II construction contains only scalar fields: the space-time is non-perturbative, and therefore so are all indices (vector, spinor and tensor) running along space-time coordinates. The type II construction is therefore blind to the distinction between gauge and matter, whose degrees of freedom have a space-time vector or spinor index, and an internal, scalar index: only this last index is visible on the type II, and these states appear all as scalars. There is no trace of the graviton, because it bears only space-time indices. Moreover, the fields T^i and U^i, $i = 1, 2, 3$, usually appearing in one-loop expressions of threshold corrections, don't correspond now to geometric moduli of two-tori. Indeed, for any twist what remains untwisted is a four-torus. In practice, we have added a two-torus. However, as we discussed, this is an artifact of the linearization of the space; there is indeed no twelve-dimensional theory, and the appearance of the two-torus is due to an "over-dimensional" representation of a curved space with just one more coordinate, the one that served as the coupling on which to expand in the four-dimensional vacuum. The 12-th coordinate is instead a curvature. There is no surprise that, in this representation, the former moduli T^i and U^i are now multiplied by what was the coupling of the theory: its dependence was simply "frozen" by construction. For what matters duality with the heterotic construction, nothing changes, because the value of these fields was not fixed. We can recover a description in terms of moduli of two-tori by introducing independent boundary conditions for the "complex planes" (1,2), (3,4), (5,6), (7,8) (see Ref. [12] for a detailed discussion of these sets). This allows to disentangle the two-torus moduli, by factorizing the space in four two-tori. On the type II side we see then that, besides the T^i and U^i, $i = 1, 2, 3$, we have now one more field, corresponding to what was the (hidden) coupling of the four-dimensional construction. It misleadingly appears as a pair of torus moduli, T^4, U^4, respectively corresponding to the volume form and the complex structure. Owing to the symmetry of the construction under exchange of the three tori with the fourth one, a $T^4 \leftrightarrow U^4$ reflection exchanges the two $\mathcal{N} = 4$ mirror constructions (the one with only vectors with the one with only hyper multiplets). It is worth to consider more in detail this property. The "fourth torus" volume form is the product of two radii, that we call R_{11} and R_{12} for obvious reasons. The moduli T^4 and U^4 are related to these radii by: $\mathrm{Im}\, T^4 = R_{11}R_{12}$, $\mathrm{Im}\, U^4 = R_{11}/R_{12}$. As we said, one of the two radii is indeed not a real further coordinate, but a curvature. When seen from the "four dimensional point of view", an inversion of this radius corresponds to an inversion of the full string coupling. Therefore, the $T^4 \leftrightarrow U^4$ mirror exchange that relates the two constructions is an "S-duality" of the "normal representation" of the type II vacuum.

We already discussed in Ref. [15] how the heterotic construction, containing both vector and hyper multiplets, corresponds to a slice, built around a corner of the moduli space, of the "union" of both the type II mirror models. From this point of view it is therefore "self-mirror". Here we understand that this mirror symmetry is indeed a strong-weak coupling duality of the type II string, an operation which is perturbative on the heterotic dual [56]. For the rest, it is important to observe that, although we cannot explicitly verify it on the base of the carried space-time indices, all hidden, the identification of the degrees of freedom allows anyway to see the S and $\bar{S}$ as the generators of space-time supersymmetry. This time they are to be intended as a representation of the "internal part" of the supersymmetry sets.

From the above considerations, we conclude that, on the type II side, the new projection, corresponding to the step $\mathcal{N}_4 = 2 \to \mathcal{N}_4 = 1$, must be represented by a set b_3 given, up to a permutation of the three complex planes corresponding to the indices $I = 1, \ldots, 6$, by:

$$b_3 = \left\{ \begin{array}{l} \chi_{3,\ldots,6}, y^L_\mu, y^L_{1,2} \\ \chi^R_{3,\ldots,6}, y^R_\mu, y^R_{1,2} \end{array} \right\} . \tag{2.5}$$

The condition 2) of above tells us however that, differently from the case of b_1 and b_2, the "GSO phase" of this set must be [57]:

$$\delta_{b_3} = -1 \,, \tag{2.6}$$

(we recall that $\delta_{b_1} = \delta_{b_2} = 1$ and $\delta_S = \delta_{\bar{S}} = -1$). This condition projects out all the states of the type $\phi^L \otimes \phi^R$, for whatever indices and $\phi \in \{\psi, \chi, y, \omega\}$, i.e. all the states of the untwisted sector. The moduli "T" and "U" are now "twisted", and the only massless states come from the twisted sectors. The projection coefficients of the fermionic construction are

[56] On the heterotic side, matter and gauge sectors are exchanged by an exchange of the twisted and the untwisted sectors. This corresponds to an inversion of the world-sheet parameter τ: $\tau \to -1/\tau$. This parameter is integrated out, and it never appears explicitly in the effective theory. On the other hand, we have seen that the world-sheet coordinates are roughly "identified" with the two longitudinal coordinates of the light-cone gauge. Any trace of the moduli of this symmetry is therefore hidden by the gauge fixing.

[57] We refer the reader to [42] for an explanation of this coefficient and its role.

given in the following table:

	F	S	$\bar{S}$	b_1	b_2	b_3
F	1	-1	-1	1	1	1
S	-1	1	1	-1	-1	-1
$\bar{S}$	-1	1	1	-1	-1	-1
b_1	1	1	1	1	1	1
b_2	1	1	1	1	1	1
b_3	1	1	1	1	1	1

(2.7)

together with the conditions: $\delta_S = \delta_{\bar{S}} = \delta_{b_3} = -1$, $\delta_\phi = \delta_{b_1} = \delta_{b_2} = 1$. Observe that, with this choice, b_3, although a type II symmetric twist as b_1 and b_2, projects the states with the same phase as a heterotic Z_2 orbifold projection, as we precisely wanted. Notice also that, differently from how it appears on the heterotic side, the projection introduced by b_3 is not exactly symmetrical to the one introduced by b_2. For instance, it seems that it would project out all the T and U fields even when acting alone, i.e. before the introduction of b_2. This impression is however misleading, in that it neglects that, as we have seen, from the point of view of this two-dimensional compactification, these fields are no more moduli of a torus, but have a more complicate expression as functions also of the former coupling coordinate, here "embedded" in the further, fourth torus. And indeed, if we want to introduce the "planes" as in Ref. [12, 15] in order to lower the rank of the twisted sectors, the sets which introduce separate boundary conditions for the coordinates must be defined in order to include more than one bosonic coordinate. Namely, they must contain also the "coupling plane". In the $\mathcal{N}_4 = 2$ model constructed with just $\{b_3, b_1\}$ (or $\{b_3, b_2\}$) the moduli T^i, U^i are no more built from the states:

$$\delta_{ij}\, x_i \bar{x}_j |0> \tag{2.8}$$

but as combinations of states of the type:

$$x_i \bar{x}_j |0> \qquad i \neq j\,,\ \{i,j\} \in (\{3,4\},\,\{5,6\},\{7,8\}) \ \cup \{11,12\}\,. \tag{2.9}$$

The partition function of this orbifold is given by the integral over the modular parameter τ, with modular-invariant measure $(\,\mathrm{Im}\ \tau)^{-2} d\tau d\bar{\tau}$, of:

$$Z^{\rm string} = \left(\frac{1}{2}\right)^3 \sum_{(H_1,G_1,H_2,G_2,H_3,G_3)} Z^F_{\rm L}\ Z^F_{\rm R} \sum_{(\gamma,\delta)} Z_{8,8}\begin{bmatrix}\gamma\\ \delta\end{bmatrix}\,, \tag{2.10}$$

where $Z^F_{L,R}$ contain the contribution of the world-sheet fields $\psi^{L,R}_\mu$, $\chi^{L,R}_a$ (the sets S and $\bar{S}$); $Z_{8,8}$ substitutes what in four dimensional constructions is $Z_{6,6}$, the $c = (6,6)$ internal space. Now this space spans all bosonic degrees of freedom and has $c = (8,8)$, corresponding to the fields $\omega^{L,R}_I$, $y^{L,R}_I$, $I = 1, \ldots, 8$. Notice that we don't have now the factor $1/(\,\mathrm{Im}\,\tau|\eta(\tau)|^4)$, the contribution of the space-time transverse bosonic degrees of freedom, now accounted in

$Z_{8,8}$. We have:

$$Z_{\rm L}^F = \frac{1}{2}\sum_{(a,b)} \frac{{\rm e}^{i\pi\varphi_{\rm L}\left(a,b,\vec{H},\vec{G}\right)}}{\eta^4} \vartheta\begin{bmatrix} a+H_3 \\ b+G_3 \end{bmatrix} \vartheta\begin{bmatrix} a+H_2-H_3 \\ b+G_2-G_3 \end{bmatrix} \vartheta\begin{bmatrix} a+H_1 \\ b+G_1 \end{bmatrix} \vartheta\begin{bmatrix} a-H_1-H_2 \\ b-G_1-G_2 \end{bmatrix} , \quad (2.11)$$

$$Z_{\rm R}^F = \frac{1}{2}\sum_{(\bar{a},\bar{b})} \frac{{\rm e}^{i\pi\varphi_{\rm R}\left(\bar{a},\bar{b},\vec{H},\vec{G}\right)}}{\bar{\eta}^4} \vartheta\begin{bmatrix} \bar{a}+H_3 \\ \bar{b}+G_3 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}+H_1-H_3 \\ \bar{b}+G_1-G_3 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}+H_2 \\ \bar{b}+G_2 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}-H_1-H_2 \\ \bar{b}-G_1-G_2 \end{bmatrix} , \quad (2.12)$$

with:

$$\begin{aligned} \varphi_{\rm L} &= a+b+ab \, , && (2.13) \\ \varphi_{\rm R} &= \bar{a}+\bar{b}+\bar{a}\bar{b} \, . && (2.14) \end{aligned}$$

The contribution of the compact bosons is:

$$\begin{aligned} Z_{8,8}\begin{bmatrix} \gamma \\ \delta \end{bmatrix} &= {\rm e}^{i\pi(H_3+G_3+H_3G_3)} \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_3 \\ \delta+G_3 \end{bmatrix}\right|^2 \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_2+H_3 \\ \delta+G_2+G_3 \end{bmatrix}\right|^2 \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_1 \\ \delta+G_1 \end{bmatrix}\right|^2 \qquad (2.15) \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_1+H_2 \\ \delta+G_1+G_2 \end{bmatrix}\right|^2 . \end{aligned}$$

The pairs (a,b) and $(\bar{a},\bar{b})$ specify the boundary conditions, in the directions $\mathbf{1}$ and τ of the world-sheet torus, of the sets S and $\bar{S}$, while (γ,δ) refer to the set of all fermionized bosons; (H_1,G_1), (H_2,G_2) and (H_3,G_3) refer to the sets b_1, b_2 and b_3. Notice the presence of the phase ${\rm e}^{i\pi(H_3+G_3+H_3G_3)}$, corresponding to the choice $\delta_{b_3}=-1$.

In this model there are nine massless sectors, corresponding to the previous b_1, b_2, b_1b_2, the new ones, b_3, b_3b_1, Fb_3b_2, $b_3b_1b_2$, $S\bar{S}b_3b_2$, and the $S\bar{S}$ sector. Only three sectors have a perturbative dual on the heterotic side, and correspond to a tern generated by a pair of intersecting projections. Here $b_3\cap b_1 \neq \emptyset$ and $b_2\cap b_1 \neq \emptyset$, while $b_3\cap b_2 = \emptyset$, therefore the pair is either $\{b_3,b_1\}$ or $\{b_2,b_1\}$. On the sets generated by one of these pairs, the third independent projection doesn't impose any further constraint. The third projection is already "built-in" by construction in the heterotic string, which starts with half the maximal supersymmetry of the type II string. Therefore, apart from the supersymmetry reduction, from the heterotic point of view the further projection triplicates the structure of the $\mathcal{N}_4=2$ model. However, on the type II side, where we have access to all the sectors, we can see that some of the sectors hidden for the heterotic string are not supersymmetric: owing to the δ_{b_3} GSO torsion, the $S\bar{S}$ states are here supersymmetric to nothing, and the same is true for the states of the Fb_3b_2 and $S\bar{S}b_3b_2$ sectors: their superpartners are massive. This is a representation in terms of free fermions of what more generally is a mass shift (see Ref. [12] for a discussion of the translation of the fermionic language in terms of orbifold operations).

With different choices of the relative GSO projections of one sector to the other one, the coefficients $(b_3|b_j)$ in table 2.7, we obtain mirror configurations in which supersymmetry is broken in a different way: a negative projection of b_3 to b_1 and b_2 implies that all the twisted sectors are projected out. Some of them, not as a consequence of a shift, but due to incompatibility of the selected chiralities of the spinors of the twisted sectors. It seems therefore that the model is empty unless the S and $\bar{S}$ projections are removed from the definition of the basis: only the pure Ramond-Ramond sector survives (the projections $(b_3|S)$ and $(b_3|\bar{S})$ remain unchanged). These mirror models seem to exist only at a "delta-function" point in the string moduli space.

C The supersymmetry-breaking scale

The string vacuum whose type II dual has been discussed in appendix B shows that, once the string space attains the maximal amount of twisted coordinates, supersymmetry is broken and the space is necessarily curved. In section 2.1.1 we pointed out that, in a compact space, supersymmetry is always broken, as a consequence of the missing invariance under space-time translations, which are part of the super-Poincaré group ($\{Q,\bar{Q}\} \sim P$). Indeed, minimization of entropy requires that all coupling moduli, and in particular the heterotic dilaton field, are twisted, and frozen at the Planck scale. All this means that the heterotic construction discussed in sections 2.1, 2.1.1 doesn't correspond to a true perturbation around a decompactified coordinate, but is a kind of non-compact orbifold, in which, in order to be able to build the states around a small/vanishing value of a coordinate, we artificially neglect its being twisted, and proceed as if along this direction we would not have fixed points. In some sense, this is also what is done in Ref. [34, 35]. However, in that case, as long as one is not interested in further curling of the string space, and remains at the level of maximal supersymmetry, the game may appear basically innocuous: it works because there are many other de-compactifiable coordinates. Problems arise when proceeding to further twisting, as we observed in Ref [15].

The result is that there are "hidden" sectors, whose origin has to be traced in the original, neglected T-duality of the theory, which are non-perturbative, and where all the breaking of supersymmetry is relegated. All this is the misleading consequence of an artificial, "illegal" flattening of an intrinsically curved space. The coordinate, or better, the curvature, which is related to the size of the flattened coordinate, works as "order parameter" for the breaking of supersymmetry. Some aspects of this phenomenon are precisely responsible for what we observe on the type II side.

As we have seen in appendix B, on the type II side we have two mirror situations, in which the breaking of supersymmetry manifests itself in a different way. However, both of them can be related to the same mechanism; they must be seen as two aspects of the same phenomenon. The key point is that non-freely acting projections can be viewed as obtained at the corner of the moduli space of freely acting constructions. In this case, at a generic point in the orbifold moduli space, projected states receive a non-vanishing mass as a consequence of a coordinate shift associated to the orbifold twist. In the decompactification limit of this coordinate, masses become infinite and the projected states disappear from the spectrum, as they usually do in ordinary non-freely acting orbifolds. This allows us to get an idea of the scale at which supersymmetry is broken: the supersymmetric partners are lifted by a shift picked along an internal coordinate X, which is also twisted. In order to describe the GSO projection process in terms of freely acting shifts, we must look for a dual configuration in which X, that we know to be fixed by minimization of symmetry to a value around the Planck scale, reaches this value as a limit at the corner of the moduli space. The map between the two descriptions, $X \to R(X)$, must therefore be some kind of logarithm, so that:

$$X \to 1 \quad \leftrightarrow \quad R \to 0/\infty \, . \tag{3.1}$$

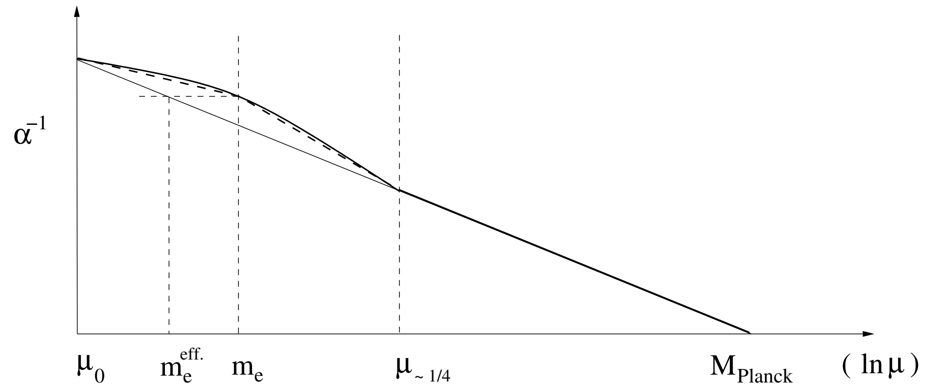
This implies that either $R \approx \ln X$ or $1/R \approx \ln X$. As discussed in ref. [15, 12], a change in sign of the $(b_i|b_j)$, projections corresponds to the inversion of some internal radius. The mirror constructions of appendix B correspond therefore precisely to the one or the other of these two possibilities, for some of the internal coordinates. According to the mechanism of freely acting projections, in the dual picture the mass of the projected states reads:

$$\tilde{m} \, (\sim \ln m) \;\sim\; R \, . \tag{3.2}$$

Pulled back to the physical picture, we obtain that, at the "twisting point", the mass is of order one in Planck units. This is the mass gap between the observed particles (and fields), with sub-Planckian mass, vanishing at first order, and their superpartners.

D Local correction to effective beta-functions

The running of the electromagnetic and weak couplings in the representation in which they are going to be compared with experimental data is logarithmic, with a slope determined by an effective beta-function coefficient. However, as discussed in section 5.4, around the scale $\sim m_e$, the volumes of the matter phase space are expanded (or, logarithmically, shifted), in such a way that for instance the electromagnetic coupling at the scale m_e (i.e. the fine structure constant) effectively corresponds to the value of the coupling *without correction* at a run-back scale, $m_e^{\mathrm{eff.}}$. The amount of running-back in the scale of the logarithmic effective coupling is equivalent to the amount of the forward shift in the logarithmic representation of the volumes of particles in the phase space. If volumes get multiplied by a factor, their logarithm gets shifted, and so gets shifted back the scale at which the coupling in its logarithmic representation is effectively evaluated. This deviation can be considered as a perturbation of the logarithmic running, that we illustrate here. In the figure, μ_0 stays for the starting scale of the running: $\mu_0 = (1/2)\, \mathcal{T}^{-1/2}$, $\mu_{\sim 1/4}$ for the upper end scale of the matter sector, the thick solid line shows the approximate expected behaviour of the inverse coupling α^{-1}, including the correction to the shape, while the thin solid line indicates the original logarithmic behaviour. The dashed segments indicate the linear approximation of the curve we considered in the footnote at page 97 in order to compute the effective weak coupling at the W-boson scale:



E A note on the string partition function

The string partition function is defined in the perturbative constructions at the one-loop (genus one) level of expansion over the string coupling, as the integral over the "modular parameter" τ of the sum over all string excitations at any energy level. In the closed superstring, it is an expression of the type [58]:

$$Z = \int d^2\tau \sum q^{\frac{1}{2}P_{\mathrm{L}}^2} \bar{q}^{\frac{1}{2}P_{\mathrm{R}}^2} \times F\left[\tau, \eta(q), \vartheta(q)\right] \,, \tag{5.1}$$

where $q = \exp \mathrm{i}\tau$, and we separated the sum over momenta (and windings) of left and right movers, from the rest of the contribution, here collected in the function $F\left[\tau, \eta(q), \vartheta(q)\right]$: The details of this expression are of no interest for our present discussion [59]; what we want here to remark is the appearance in the summation of the *square* of the momenta, i.e. of energies. This is due to the fact that these momenta come from the expansion over the modes of bosonic target space coordinates, for which the Hamiltonian, i.e. the energy operator, is quadratic in the momenta. The bosonic relation between momentum and energy is in fact of the type $E \sim P^2/2m$. In a string construction, the mass is assumed to be one (the string mass, defined as the inverse of the string length), and one obtains an identification of energy with the square of a momentum. All this is perfectly fine and legal. However, in our context it sounds a bit strange, because we are going to eventually identify the lowest level of momentum ($m \sim p_0 \sim 1/R$) with a mass, and we would like to see this to be linearly related to an energy: $E_0 \sim m$. The point is that we started with bosonic coordinates, and we and up with the energies of a physical system in which as elementary matter particles we have fermions (leptons and quarks). From a technical point of view, things are adjusted

[58] We skeep here to consider the type I string, in which the parameter τ has a slightly different definition [78].

[59] See for instance Ref. [79].

by the fact that the orbifold operation which lifts all matter states to a non-zero mass, producing the string vacuum of highest entropy, acts perturbatively as $R \to \frac{1}{2}R$, therefore as $R \to \sqrt{R}$ in the physical coordinates (see section 4.3, page 48). As a consequence, also *physical* momenta are "square-root" contracted:

$$P^2 \sim \left(\frac{1}{R}\right)^2 \quad \longrightarrow \quad P = \left(\frac{1}{\sqrt{R}}\right)^2 . \tag{5.2}$$

Owing to this shift, the string partition function bears therefore the correct dependence on the physical momenta and masses, and can be matched with the partition function of a statistical system, as implied by the combinatorial approach discussed in [1], and encoded in 1.1. From a physical point of view, this contraction can be viewed as follows: in order to produce massive matter, the shift must couple fermions two by two. Indeed, the entire perception of the physical world an observer can have is mediated by boson fields (graviton, photon), which couples to fermion pairs (not only the photon but also the graviton couples to fermions through terms of the effective action in which fermions are paired, as are also all mass terms), and so, i.e. of boson type, are also all the energy levels appearing in the partition function of the universe. Physical momenta are made out of the pairing of two momenta of the elementary description.

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