The Diophantine Equations $a^2 \pm mb^2 = c^n$, $a^3 \pm mb^3 = d^2$ and $y_1^4 \pm my_2^4 = R^2$

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Abstract

The Diophantine equations $a^2 \pm mb^2 = c^n$, and $a^3 \pm mb^3 = d^2$ have infinitely many nonzero integer solutions, Using the methods of infinite descent and infinite ascent we prove $y_1^4 \pm my_2^4 = R^2$. Using this method you prove Beal conjecture and obtain a prize of $100,000[4].

Using this method in 1978 Jiang has proved Fermat last theorem[Chun-Xuan Jiang,A general proof of Fermat last theorem,July 1978,Mimeograph papers].

The Diophantine equation

$$a^2 + b^2 = c^3, \tag{1}$$

has infinitely many nonzero integer solutions. But it is difficult to prove this [1,2]. In this paper we prove some theorems.

Theorem 1. The Diophantine equation

$$a^2 + mb^2 = c^n \tag{2}$$

has infinitely many nonzero integer solutions.

We define supercomplex number [3]

$$z = \begin{pmatrix} x & -my \\ y & x \end{pmatrix} = x + yJ, \tag{3}$$

where

$$J = \begin{pmatrix} 0 & -m \\ 1 & 0 \end{pmatrix}, \quad J^2 = -m.$$

Then from equation (3)

$$z^n = (x + yJ)^n = a + bJ. \tag{4}$$

Let n be an odd number

$$a=\sum_{k=0}^{(n-1)/2}\binom{n}{2k}(-m)^k x^{n-2k}y^{2k}, b=\sum_{k=0}^{(n-1)/2}\binom{n}{2k+1}(-m)^k x^{n-2k-1}y^{2k+1},$$

Let n be an even number

$$a=\sum_{k=0}^{n/2}\binom{n}{2k}(-m)^k x^{n-2k}y^{2k}, b=\sum_{k=0}^{n/2-1}\binom{n}{2k+1}(-m)^k x^{n-2k-1}y^{2k+1}.$$

Then from (4) the circulant matrix

$$\begin{pmatrix} x & -my \\ y & x \end{pmatrix}^n = \begin{pmatrix} a & -mb \\ b & a \end{pmatrix}, \qquad (5)$$

Then from (5) circulant determinant

$$\begin{vmatrix} x & -my \\ y & x \end{vmatrix}^n = \begin{vmatrix} a & -mb \\ b & a \end{vmatrix}, \qquad (6)$$

Then from equation (6)

$$c^n = a^2 + mb^2, \qquad (7)$$

where

$$c = x^2 + my^2 .$$

We prove that (2) has infinitely many nonzero integer solutions.

Theorem 2. The Diophantine equation

$$a^2 - mb^2 = c^n \qquad (8)$$

has infinitely nonzero integer solutions.

Define supercomplex number [3]

$$z = \begin{pmatrix} x & my \\ y & x \end{pmatrix} = x + yJ , \qquad (9)$$

where

$$J = \begin{pmatrix} 0 & m \\ 1 & 0 \end{pmatrix}, \quad J^2 = m .$$

Then from equation (9)

$$z^n = (x + yJ)^n = a + bJ . \qquad (10)$$

Let n be an odd number

$$a=\sum_{k=0}^{(n-1)/2}\binom{n}{2k}m^k x^{n-2k}y^{2k}, b=\sum_{k=0}^{(n-1)/2}\binom{n}{2k+1}m^k x^{n-2k-1}y^{2k+1}.$$

Let n be an even number

$$a=\sum_{k=0}^{n/2}\binom{n}{2k}m^k x^{n-2k}y^{2k},b=\sum_{k=0}^{n/2-1}\binom{n}{2k+1}m^k x^{n-2k-1}y^{2k+1}.$$

Then from (10) circulant matrix

$$\begin{pmatrix} x & my \\ y & x \end{pmatrix}^n=\begin{pmatrix} a & mb \\ b & a \end{pmatrix}. \qquad (11)$$

Then from (11) circulant determinant

$$\begin{vmatrix} x & my \\ y & x \end{vmatrix}^n=\begin{vmatrix} a & mb \\ b & a \end{vmatrix}. \qquad (12)$$

Then from equation (12)

$$c^n=a^2-mb^2, \qquad (13)$$

where

$$c=x^2-my^2.$$

We prove that (8) has infinitely many nonzero integer solutions.

Theorem 3. The Diophantine equation

$$a^3+mb^3+m^2c^3-3mabc=d^n \qquad (14)$$

has infinitely many nonzero integer solutions

Define supercomplex number [3]

$$w=\begin{pmatrix} x & mz & my \\ y & x & mz \\ z & y & x \end{pmatrix}=x+yJ+zJ^2, \qquad (15)$$

where

$$J=\begin{pmatrix} 0 & 0 & m \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},\quad J^2=\begin{pmatrix} 0 & m & 0 \\ 0 & 0 & m \\ 1 & 0 & 0 \end{pmatrix},\quad J^3=m.$$

Then from (15)

$$w^n=(x+yJ+zJ^2)^n=a+bJ+cJ^2 \qquad (16)$$

Then from equation (16) circulant matrix

$$\begin{pmatrix} x & mz & my \\ y & x & mz \\ z & y & x \end{pmatrix}^n=\begin{pmatrix} a & mc & mb \\ b & a & mc \\ c & b & a \end{pmatrix} \qquad (17)$$

Then from equation (17) circulant determinant

$$\begin{vmatrix} x & mz & my \\ y & x & mz \\ z & y & x \end{vmatrix}^n = \begin{vmatrix} a & mc & mb \\ b & a & mc \\ c & b & a \end{vmatrix} \quad (18)$$

Then from equation (18)

$$d^n = a^3 + mb^3 + m^2c^3 - 3mabc \quad (19)$$

where

$$d = x^3 + my^3 + m^2z^3 - 3mxyz \quad (20)$$

We prove that (14) has infinitely many nonzero integer solutions.
Suppose $n = 2$ and $c = 0$. Then from (19)

$$a^3 + mb^3 = d^2 \quad (21)$$

when $n = 2$ from (16)

$$a = x^2 + 2myz \neq 0, \quad b = 2xy + mz^2 \neq 0, \quad c = y^2 + 2xz = 0 \quad (22)$$

Then from (22) $y = -2xz$.

Let

$$z = -2, \; x = P^2, \quad y = 2P, \quad (23)$$

where $P > 1$ is an odd number.
Substituting (23) into (20) and (22)

$$d = P^6 + 20mP^3 - 8m^2, \quad a = P^4 - 80mP, \quad b = 4P^3 + 4m \quad (24)$$

Using equation (24) we prove that (21) has infinitely many nonzero integer solutions.
Theorem 4. The Diophantine equation

$$a^3 - mb^3 + m^2c^3 + 3mabc = d^n \quad (25)$$

has infinitely many nonzero integer solutions.
Define supercomplex number [3]

$$w = \begin{pmatrix} x & -mz & -my \\ y & x & -mz \\ z & y & x \end{pmatrix} = x + yJ + zJ^2, \quad (26)$$

where

$$J = \begin{pmatrix} 0 & 0 & -m \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad J^2 = \begin{pmatrix} 0 & -m & 0 \\ 0 & 0 & -m \\ 1 & 0 & 0 \end{pmatrix}, \quad J^3 = -m,$$

Then from (26)

$$w^n = (x + yJ + zJ^2)^n = a + bJ + cJ^2 \quad (27)$$

Then from (27) circulant matrix

$$\begin{pmatrix} x & -mz & -my \\ y & x & -mz \\ z & y & x \end{pmatrix}^n = \begin{pmatrix} a & -mc & -mb \\ b & a & -mc \\ c & b & a \end{pmatrix}, \qquad (28)$$

Then from equation (28) circulant determinant

$$\begin{vmatrix} x & -mz & -my \\ y & x & -mz \\ z & y & x \end{vmatrix}^n = \begin{vmatrix} a & -mc & -mb \\ b & a & -mc \\ c & b & a \end{vmatrix}. \qquad (29)$$

Then from (29)

$$d^n = a^3 - mb^3 + m^2c^3 + 3mabc \qquad (30)$$

where

$$d = x^3 - my^3 + m^2z^3 + 3mxyz. \qquad (31)$$

We prove that (25) has infinitely many nonzero integer solutions.
Suppose $n = 2$ and $c = 0$. Then from (30)

$$a^3 - mb^3 = d^2 \qquad (32)$$

When $n = 2$ from (27)

$$a = x^2 - 2myz \neq 0, \quad b = -mz^2 + 2xy \neq 0, \quad c = y^2 + 2xz = 0 \qquad (33)$$

Then from (33) $y^2 = -2xz$

Let $z = -2, \quad x = P^2, \quad y = 2P$, (34)

where $P > 1$ is an odd numer.
Substitutin (34) into (31) and (33)

$$d = P^6 - 20mP^3 - 8m^2, \quad a = P^4 + 8mP, \quad b = 4P^3 - 4m \qquad (35)$$

Using (35) we prove that (32) has infinitely many nonzero integer solutions.
Theorem 5. Define supercomplex number

$$w = \begin{pmatrix} x_1 & -mx_4 & -mx_3 & -mx_2 \\ x_2 & x_1 & -mx_4 & -mx_3 \\ x_3 & x_2 & x_1 & -mx_4 \\ x_4 & x_3 & x_2 & x_1 \end{pmatrix} = x_1 + x_2J + x_3J^2 + x_4J^3, \qquad (36)$$

where

$$J = \begin{pmatrix} 0 & 0 & 0 & -m \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad J^2 = \begin{pmatrix} 0 & 0 & -m & 0 \\ 0 & 0 & 0 & -m \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad J^3 = \begin{pmatrix} 0 & -m & 0 & 0 \\ 0 & 0 & -m & 0 \\ 0 & 0 & 0 & -m \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad J^4 = -m$$

Then from (36)

$$w^n = (x_1 + x_2 J + x_3 J^2 + x_4 J^3)^n = y_1 + y_2 J + y_3 J^2 + y_4 J^3 \qquad (37)$$

Then from (37)

$$R^n = |y_i|, \qquad (38)$$

where

$$R = x_1^4 + m(x_2^4 + 2x_1^2 x_3^2 - 4x_1 x_2^2 x_3 + 4x_2 x_1^2 x_4) + m^2(x_3^4 + 2x_2^2 x_4^2 + 4x_1 x_3 x_4^2 - 4x_2 x_4 x_3^2)$$
$$+ m^3 x_4^4,$$

$$|y_i| = y_1^4 + m(y_2^4 + 2y_1^2 y_3^2 - 4y_1 y_2^2 y_3 + 4y_2 y_1^2 y_4) + m^2(y_3^4 + 2y_2^2 y_4^2 + 4y_1 y_3 y_4^2 - 4y_2 y_4 y_3^2)$$
$$+ m^3 y_4^4.$$

(39)

We prove that (38) has infinitely many nonzero integer solutions,

Suppose $n = 2,\ y_1 \neq 0,\ y_2 \neq 0,\ y_3 = 0$ and $y_4 = 0$, from (38) and (39)

$$R^2 = y_1^4 + m y_2^4 \qquad (40)$$

When $n = 2$ from (37)

$$y_1 = x_1^2 - m x_3^2 - 2m x_2 x_4 \neq 0, \qquad (41)$$

$$y_2 = 2(x_1 x_2 - m x_3 x_4) \neq 0, \qquad (42)$$

$$y_3 = x_2^2 - m x_4^2 + 2x_1 x_3 = 0, \qquad (43)$$

$$y_4 = 2(x_1 x_4 + x_2 x_3) = 0, \qquad (44)$$

Then from (44)

$$x_3 = -\frac{x_1 x_4}{x_2}. \qquad (45)$$

Substituting (45) into (43)

$$x_4 = \frac{-x_1^2 \pm \sqrt{x_1^4 + m x_2^4}}{m x_2}. \qquad (46)$$

Then from (46)

$$R_1^2 = x_1^4 + m x_2^4. \qquad (47)$$

If (47) has no nonzero integer solutions, $R_1 < R$, using the method of infinite descent we prove (40) has no nonzero integer solutions. If (47) has one nonzero integer solution, $R_1 < R$, using the method of infinite ascent we prove (40) has infinitely many nonzero integer solutions.

Suppose $m=1$ from (47)

$$R_1^2 = x_1^4 + x_2^4 \qquad (48)$$

has no integer solutions.
Suppose $m=2$ from (47)

$$R_1^2 = x_1^4 + 2x_2^4 \qquad (49)$$

has no nonzero integer solutions.
Suppose $m=8$ [4] from (47)

$$R_1^2 = x_1^4 + 8\times x_2^4 \qquad (50)$$

We have a solution $3^2 = 1^4 + 8(1^4)$. Let $x_1 = 1, x_2 = 1$. Then from (45) and (46) $x_3 = 1/2$, $x_4 = -1/2$. Then from (41) and (42) $y_1 = 7, y_2 = 6$, $7^4 + 8\times 6^4 = 113^2$

Let $x_1 = 7, x_2 = 6, x_3 = -14/9, x_4 = 4/3$. Then from (41) and (42) $y_1 = -7967/81$, $y_2 = 9492/81$. $7967^4 + 8\times 9492^4 = 262621633^2$.

Suppose $m = 73$ [5]. From (47)

$$R_1^2 = x_1^2 + 73\times x_2^4. \qquad (51)$$

We have $6^4 + 73(1^4) = 37^2$, $1223^4 + 73\times 444^4 = 2252593^2$.

Suppose $m = 89$ [5]. From (47),

$$R_1^2 = x_1^4 + 89\times x_2^4. \qquad (52)$$

We have $2^4 + 89\times 3^4 = 85^2$, $7193^4 + 89\times 1020^4 = 52662001^2$

If $m = R_1^2 - 1$ and $m = R_1^2 - x_1^4$, then (40) has infinitely many nonzero integer solutions.

Theorem 6. The Diophantine equation

$$y_1^4 - my_2^4 = R^2, \qquad (53)$$

where

$$y_1 = x_1^2 + mx_3^2 + 2mx_2x_4 \neq 0, \qquad (54)$$

$$y_2 = 2(x_1x_2 + mx_3x_4) \neq 0, \qquad (55)$$

$$y_3 = x_2^2 + mx_4^2 + 2x_1x_3 = 0, \qquad (56)$$

$$y_4 = 2(x_1x_4 + x_2x_3) = 0. \qquad (57)$$

Then from (57)

$$x_3 = -\frac{x_1 x_4}{x_2}. \qquad (58)$$

Substituting (58) into (56)

$$x_4 = \frac{x_1^2 \pm \sqrt{x_1^4 - m x_2^4}}{m x_2}. \qquad (59)$$

Then from (59)

$$x_1^4 - m x_2^4 = R_1^2. \qquad (60)$$

If (60) has no nonzero integer solutions. $R_1 < R$, using the method of infinite descent we prove (53) has no nonzero integer solutions. If (60) has one nonzero integer solution, $R_1 < R$, using the method of infinite ascent we prove (53) has infinitely many nonzero integer solutions.

Suppose $m = 1$ from (60)

$$x_1^4 - x_2^4 = R_1^2. \qquad (61)$$

has no nonzero integer solutions.

Suppose $m = 2$, from (60)

$$x_1^4 - 2x_2^4 = R_1^2, \qquad (62)$$

has no nonzero integer solutions.

Suppose $m = 7$ from (60)

$$x_1^4 - 7x_2^4 = R_1^2. \qquad (63)$$

We have one solution

$$2^4 - 7(1^4) = 3^2.$$

Let $x_1 = 2, x_2 = 1$. Then from (58) and (59) $x_3 = -2$, $x_4 = 1$. Then from (54) and (55) $y_1 = 2 \times 23$, $y_2 = -2 \times 12$. $23^4 - 7 \times 12^4 = 367^2$. We prove (63) has infinitely many nonzero integer solutions.

If $m = x_1^4 - R_1^2$, then (53) has infinitely many nonzero integer solutions. Our method [3] is used in studies of the Diophantine equations

$$y_1^n \pm m y_2^n = R^e, n = 2,3,4,\ldots; e = 2,3,4,\ldots; m = 1,2,3,\ldots \qquad (61)$$

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21 February 2011.

2011 年 **1** 月蒋看到乐茂华论文决定研究这个问题，后来看到 **Mordell** 论文并查有关这方面资料，发现当代这方面水平不高，决定写本文并提出 **method of infinite ascent.** 这是新方法大有前途. 中国和国外都不会发表蒋论文，但国外有两网发表蒋论文，全世界都会看到，以下蒋文和上文基本思路一样. 中国到今天仍在疯狂封杀蒋成果，不允许任何单位和个人支持蒋。蒋母校北航当面对蒋说：北航不需要你的成果。中国不需要蒋划时代成果。

Automorphic Functions And Fermat's Last Theorem(1)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat

proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and P, where P is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \qquad (1)$$

where J denotes a n th root of unity, $J^n = 1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions(complex hyperbolic functions) of order n with $n-1$ variables [1-7].

$$S_i = \frac{1}{n}\left[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}\right)\right] \qquad (2)$$

where i=1,2,…,n;

$$A = \sum_{\alpha-1}^{n-1} t_\alpha, \qquad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos\frac{\alpha j\pi}{n},$$

$$\theta_j = (-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \sin\frac{\alpha j\pi}{n}, \qquad A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j = 0 \qquad (3)$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} = \frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp B_{\frac{n-1}{2}} \sin\theta_{\frac{n-1}{2}} \end{bmatrix} \qquad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} \tag{5}$$

From (5) we have

$$e^{A} = \sum_{i=1}^{n} S_i\,, \qquad e^{B_j}\cos\theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j = (-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n}, \tag{6}$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent n has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$=\frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \frac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{n}{2} \end{bmatrix}\begin{bmatrix} e^{A} \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$=\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \qquad (7)$$

where $1+\sum_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2=\frac{n}{2}$, $\sum_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2=\frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=1. \qquad (8)$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix}, \qquad (9)$$

where $(S_i)_j=\frac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \qquad (10)$$

If $S_i\neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1\neq 0$, $S_2\neq 0$, $S_i=0$ where $i=3,4,\cdots,n$. $S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^{A}=S_1+S_2,\quad e^{2B_j}=S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n}. \tag{11}$$

From (10) and (11) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=(S_1+S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n})=S_1^n+S_2^n=1 \tag{12}$$

Example[1]. Let $n=15$. From (3) we have

$$A=(t_1+t_{14})+(t_2+t_{13})+(t_3+t_{12})+(t_4+t_{11})+(t_5+t_{10})+(t_6+t_9)+(t_7+t_8)$$

$$B_1=-(t_1+t_{14})\cos\frac{\pi}{15}+(t_2+t_{13})\cos\frac{2\pi}{15}-(t_3+t_{12})\cos\frac{3\pi}{15}+(t_4+t_{11})\cos\frac{4\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{5\pi}{15}+(t_6+t_9)\cos\frac{6\pi}{15}-(t_7+t_8)\cos\frac{7\pi}{15},$$

$$B_2=(t_1+t_{14})\cos\frac{2\pi}{15}+(t_2+t_{13})\cos\frac{4\pi}{15}+(t_3+t_{12})\cos\frac{6\pi}{15}+(t_4+t_{11})\cos\frac{8\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{10\pi}{15}+(t_6+t_9)\cos\frac{12\pi}{15}+(t_7+t_8)\cos\frac{14\pi}{15},$$

$$B_3=-(t_1+t_{14})\cos\frac{3\pi}{15}+(t_2+t_{13})\cos\frac{6\pi}{15}-(t_3+t_{12})\cos\frac{9\pi}{15}+(t_4+t_{11})\cos\frac{12\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{15\pi}{15}+(t_6+t_9)\cos\frac{18\pi}{15}-(t_7+t_8)\cos\frac{21\pi}{15},$$

$$B_4=(t_1+t_{14})\cos\frac{4\pi}{15}+(t_2+t_{13})\cos\frac{8\pi}{15}+(t_3+t_{12})\cos\frac{12\pi}{15}+(t_4+t_{11})\cos\frac{16\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{20\pi}{15}+(t_6+t_9)\cos\frac{24\pi}{15}+(t_7+t_8)\cos\frac{28\pi}{15},$$

$$B_5=-(t_1+t_{14})\cos\frac{5\pi}{15}+(t_2+t_{13})\cos\frac{10\pi}{15}-(t_3+t_{12})\cos\frac{15\pi}{15}+(t_4+t_{11})\cos\frac{20\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{25\pi}{15}+(t_6+t_9)\cos\frac{30\pi}{15}-(t_7+t_8)\cos\frac{35\pi}{15},$$

$$B_6=(t_1+t_{14})\cos\frac{6\pi}{15}+(t_2+t_{13})\cos\frac{12\pi}{15}+(t_3+t_{12})\cos\frac{18\pi}{15}+(t_4+t_{11})\cos\frac{24\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{30\pi}{15}+(t_6+t_9)\cos\frac{36\pi}{15}+(t_7+t_8)\cos\frac{42\pi}{15},$$

$$B_7=-(t_1+t_{14})\cos\frac{7\pi}{15}+(t_2+t_{13})\cos\frac{14\pi}{15}-(t_3+t_{12})\cos\frac{21\pi}{15}+(t_4+t_{11})\cos\frac{28\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{35\pi}{15}+(t_6+t_9)\cos\frac{42\pi}{15}-(t_7+t_8)\cos\frac{49\pi}{15},$$

$$A+2\sum_{j=1}^{7}B_j=0,\qquad A+2B_3+2B_6=5(t_5+t_{10}). \tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{7}B_j)=S_1^{15}+S_2^{15}=(S_1^5)^3+(S_2^5)^3=1. \tag{14}$$

From (13) we have

$$\exp(A+2B_3+2B_6)=[\exp(t_5+t_{10})]^5. \tag{15}$$

From (11) we have

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5. \tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5=[\exp(t_5+t_{10})]^5. \tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

Theorem 1. [1-7]. Let $n=3P$,where $P>3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A+2\sum_{j=1}^{3P-1}B_j)=S_1^{3P}+S_2^{3P}=(S_1^P)^3+(S_2^P)^3=1. \tag{18}$$

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=[\exp(t_P+t_{2P})]^P. \tag{19}$$

From (11) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P. \tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P=[\exp(t_P+t_{2P})]^P. \tag{21}$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P>3$ [1, 3-7].

Theorem 2. In 1847 Kummer write the Fermat's equation

$$x^P+y^P=z^P \tag{22}$$

in the form

$$(x+y)(x+ry)(x+r^2y)\cdots(x+r^{P-1}y)=z^P \tag{23}$$

where P is odd prime, $r=\cos\frac{2\pi}{P}+i\sin\frac{2\pi}{P}$.

Kummer assume the divisor of each factor is a P th power. Kummer proved FLT for prime exponent p<100 [8]..

We consider the Fermat's equation

$$x^{3P}+y^{3P}=z^{3P} \tag{24}$$

we rewrite (24)

$$(x^P)^3 + (y^P)^3 = (z^P)^3 \tag{25}$$

From (24) we have

$$(x^P + y^P)(x^P + ry^P)(x^P + r^2 y^P) = z^{3P} \tag{26}$$

where $r = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

We assume the divisor of each factor is a P th power.

Let $S_1 = \frac{x}{z}$, $S_2 = \frac{y}{z}$. From (20) and (26) we have the Fermat's equation

$$x^P + y^P = [z \times \exp(t_P + t_{2P})]^P \tag{27}$$

Euler proved that (25) has no integer solutions for exponent 3[8]. Therefore we prove that (27) has no integer solutions for prime exponent P.

Fermat Theorem. It suffices to prove FLT for exponent 4. We rewrite (24)

$$(x^3)^P + (y^3)^P = (z^3)^P \tag{28}$$

Euler proved that（25）has no integer solutions for exponent 3 [8]. Therefore we prove that (28) has no integer solutions for all prime exponent P [1-7].

We consider Fermat equation

$$x^{4P} + y^{4P} = z^{4P} \tag{29}$$

We rewrite (29)

$$(x^P)^4 + ((y^P)^4 = (z^P)^4 \tag{30}$$

$$(x^4)^P + (y^4)^P = (z^4)^P \tag{31}$$

Fermat proved that (30) has no integer solutions for exponent 4 [8]. Therefore we prove that (31) has no integer solutions for all prime exponent P [2,5,7].This is the proof that Fermat thought to have had.

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7].Using modular elliptic curves Wiles and Taylor prove FLT[9,10].This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformations. Automorphic functions are generalization of the trigonometric,hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

Acknowledgments.We thank Chenny and Moshe Klein for their help and suggestion.

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