Energy transfer and differential entropy of two charged systems that show potential difference

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Energy transfer and differential entropy of two charged systems that show potential difference

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Abstract

In the process to combine two charged systems, potential-different systems, we testified that energy would transfer from the high-potential system to the low-potential one, during which the entropy of the two systems show corresponding changes.

Key words: energy transfer, differential entropy.

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1 introduction

Each of the two charged-particle-consisted systems $A$ and $B$ would acquire its own probability density $\rho_A$ and $\rho_B$. In the process to combine the two systems into one by starting from their probabilistic average values, the author found the probabilistic average values showed changes. By using the Gibbs
inequality formula, we testified energy transfer from the high-potential sys-
tem to the low-potential one, during which the entropy of the two systems
showed corresponding changes. If the aforesaid process were repeated for
many times for the same system, it would be possible to materialize the en-
ergy transfer $\epsilon \to 0$ for the charged system, and see concurrent weakening of
the system’s electric field $E$.

2 Energy shift of charged particles in the po-
tential difference

We now make the probability of systems $A$ and $B$ in the phase space re-
spectively probability density $\rho_A$ and $\rho_B$, merge systems $A$ and $B$, and then
assume these are energy $\mathcal{E}$ and their potential $\Phi_A > \Phi_B$ so as to set them
out of balance. In this system by normalized condition of probability density
$\rho$, we have [1]

\begin{align}
1 &= \int_0^\infty \rho \, d\Omega, \quad \langle \mathcal{E} \rangle = \int_0^\infty \mathcal{E} \rho \, d\Omega, \quad \langle \sigma \rangle = \langle \ln \rho \rangle = \int_0^\infty \rho \ln \rho \, d\Omega.
\end{align}

By calculating the variation of (1) and multiplying the Lagrangian multi-
pliers [2], $\alpha$, $\xi$, and take variational we have

\begin{align}
\delta \int (\rho \ln \rho + \alpha \mathcal{E} \rho - \xi \rho) \, d\Omega &= 0, \\
\text{hence } \sigma &= \ln \rho = K - \alpha \mathcal{E}, \text{ set } K = \xi - 1, \text{ we have}
\end{align}

\begin{align}
\sigma + \alpha \mathcal{E} &= K,
\end{align}
become $\sigma = K - \alpha \mathcal{E}$

$$\rho = \exp(K - \alpha \mathcal{E}) = \exp \sigma.$$ \hspace{1cm} (4)

After merging systems $A$ and $B$ into a system $AB$, both $A$ and $B$ will be in an unstable state. At time $t$, the $\sigma'$ and energy $\mathcal{E}'$ will take mean value $\langle \sigma'_A \rangle$, $\langle \mathcal{E}'_A \rangle$, $\langle \sigma'_B \rangle$ and $\langle \mathcal{E}'_B \rangle$.

At the initial time $t = 0$ have $\langle \sigma_A(0) \rangle$, $\langle \sigma_B(0) \rangle$. For $A$ and $B$ stay in balance and takes its $\rho$ maximum value. hence we have

$$\langle \sigma_A(0) \rangle + \langle \sigma_B(0) \rangle \geq \langle \sigma'_A \rangle + \langle \sigma'_B \rangle.$$ \hspace{1cm} (5)

However, $A$ and $B$ have not yet reached the equilibrium state at time $t$, and we assume a little shift $\Delta \sigma$ exists before the system reaches its equilibrium state, hence we change (3) into

$$\sigma' = \ln \rho' = K - \alpha \mathcal{E} + \Delta \sigma = \sigma + \Delta \sigma,$$ \hspace{1cm} (6)

therefore, from (3) we have

$$\ln \rho' = \sigma + \Delta \sigma.$$ \hspace{1cm} (7)

Now, we calculate difference of the mean value of (3), (6) and (7), and set (4)
\[\langle \sigma' + \alpha \mathcal{E} \rangle - \langle \sigma + \alpha \mathcal{E} \rangle = \int_0^\infty [(K + \Delta \sigma) \exp(\ln \rho') - K \exp(\ln \rho)]d\Omega\]
\[= \int_0^\infty \Delta \sigma \exp(\ln \rho')d\Omega\]
\[= \int_0^\infty [\Delta \sigma \exp(\Delta \sigma + \sigma) - \rho' + \rho]d\Omega\]
\[= \int_0^\infty [\Delta \sigma \exp(\Delta \sigma) \exp(\sigma) - \exp(\Delta \sigma) \exp(\sigma) + \exp(\sigma)]d\Omega\]
\[= \int_0^\infty [\Delta \sigma \exp(\Delta \sigma) - \exp(\Delta \sigma) + 1] \exp(\sigma)d\Omega,\]

(8)

from the Gibbs inequality \(x \exp x - \exp x + 1 \geq 0\) [1], above formula (8), we have \(\int_0^\infty [\Delta \sigma \exp(\Delta \sigma) - \exp(\Delta \sigma) + 1] \exp(\sigma)d\Omega \geq 0\), namely,

\[\langle \sigma' + \alpha \mathcal{E} \rangle \geq \langle \sigma + \alpha \mathcal{E} \rangle.\]

(9)

For systems \(A\) and \(B\), from (9) and \(\mathcal{E}' = \mathcal{E} + \Delta \mathcal{E} \geq \mathcal{E}\), we get the following equation,

\[\langle \sigma'_A \rangle + \langle \alpha_A (\mathcal{E}_A + \Delta \mathcal{E}) \rangle \geq \langle \sigma_A(0) \rangle + \langle \alpha_A \mathcal{E}_A(0) \rangle\]
\[\langle \sigma'_B \rangle + \langle \alpha_B (\mathcal{E}_B + \Delta \mathcal{E}) \rangle \geq \langle \sigma_B(0) \rangle + \langle \alpha_B \mathcal{E}_B(0) \rangle,\]

(10)

by sorting up (10), we get

\[\begin{align*}
[\langle \sigma'_A \rangle + \langle \alpha_A \mathcal{E}'_A \rangle + \langle \sigma'_B \rangle + \langle \alpha_B \mathcal{E}'_B \rangle] & \geq [\langle \sigma_A(0) \rangle + \langle \alpha_A \mathcal{E}_A(0) \rangle + \langle \alpha_B \mathcal{E}_B(0) \rangle] + [\langle \alpha_A \mathcal{E}_A(0) \rangle + \langle \alpha_B \mathcal{E}_B(0) \rangle],
\end{align*}\]

(11)

from (5), we make (11)

\[\langle \alpha_A \mathcal{E}'_A \rangle + \langle \alpha_B \mathcal{E}'_B \rangle - [\langle \alpha_A \mathcal{E}_A(0) \rangle + \langle \alpha_B \mathcal{E}_B(0) \rangle] \geq 0,\]

(12)
or

\[ \alpha_A [\langle E_A' \rangle - \langle E_A(0) \rangle] + \alpha_B [\langle E_B' \rangle - \langle E_B(0) \rangle] \geq 0. \] (13)

According to energy conservation, the energy shift of systems A and B should be equal

\[ \langle \Delta E_A \rangle = \langle E_A' \rangle - \langle E_A(0) \rangle = -\langle \Delta E_B \rangle = -\langle \langle E_B' \rangle - \langle E_B(0) \rangle \rangle. \] (14)

By this time, from \( \alpha = 1/(q\Phi) \)[3], (13) and (14) would become

\[ \langle \Delta E_A \rangle / (q\Phi_A) - \langle \Delta E_A \rangle / (q\Phi_B) \geq 0. \] (15)

Considering the initial assumption of \( \Phi_A \geq \Phi_B \), (14) would be

\[ \langle \Delta E_A \rangle \leq 0. \] (16)

By this time, we may say that under the influence of potential difference, energy of the system’s charged particles has shifted from the high-potential level to the low-potential ones, and that the process of energy transmission would not terminate till the system’s potential difference reaches zero. Such a state is identical with what we already experienced.

3 The energy state of particles in entropy \( \Delta S \)

In \( \text{ith} \) particle potential \( \varphi_i = \int_{a_i}^{b_i} E_{k_i} \cdot dk \) [4], the coupling a particle \( q \) produces both potential difference \( \Delta \varphi_i \) and energy difference \( \Delta \epsilon_i \)

\[ \Delta \varphi_i = \int_{a_i}^{b_i} E_{k_i} \cdot dk = \varphi(b_i) - \varphi(a_i), \quad \Delta \epsilon_i = q \Delta \varphi_i, \] (17)

and a total number of \( N \) particles would produce energy in the system
\[ \Delta \mathcal{E} = \sum_i q \Delta \varphi_i. \quad (18) \]

In a system, there are the energy of large number charge particles in the potential function \( \Phi \) and entropy function \( S \), we have \([5, 6]\) \[ \delta \mathcal{E} = \sum_i \frac{1}{C} M(F_i) \, dF_i \, C \, \exp(L(\tau)) \, d\tau = \sum_i \Phi \, dS_i, \]  hence, in the system, the potential \( \Phi \) and the entropy \( S \) \([6]\).

\[ \Phi = \int_{\infty}^\infty C \, \exp(L(\tau)) \, d\tau, \quad (19) \]

\[ S = \frac{1}{C} \int_{\infty}^\infty M(F) \, dF, \quad (20) \]

the entropy difference would be

\[ \Delta S = \Delta \mathcal{E} / \Phi. \quad (21) \]

If each of the particles in the system has field \( E_k \to 0 \) then (17) would have \( \Delta \varphi_i \to 0 \), and hence (18) and (19) also have \( \Delta \mathcal{E} \to 0 \), and \( \Delta S \to 0 \). If we sequentially reduce the system’s \( \Delta S \) and \( \Delta \Phi \), and repeat the process many times till the particles gradually lose their fields, and thus bring the particles’ energy to their ground state, it would then be possible to set the system’s ground in the ground state. Under the assumption that the system is made up exclusively by free particles, we could use the above-mentioned process to gradually cut the particle’s electric field \( E \) till \( E \to 0 \). Eventually, the free charges will approach the state of bare charges. On the contrary, when we continuously increase the system’s \( \Delta S \) and \( \Delta \Phi \), the particles would continuously increase their \( E \) and the energy state of the particles would
rise from the low-energy state to high-energy state, and the system’s energy would also accordingly increase.

References


