What the Null Energy Condition tells us about Gravitational wave frequencies in / for relic Cosmology?

A. Beckwith

1) abeckwith@uh.edu, Chongqing University department of physics; Institute of Theoretical Physics; Beckwith@iibep.org, American Institute of Beamed Energy Propulsion (iibep.org), Seculine Consulting, USA

Abstract

We introduce a criterion as to the range of HFGW generated by early universe conditions. The 1 to 10 Giga Hertz range is constructed initially starting with what Grupen writes as far as what to expect of GW frequencies which can be detected assuming a sensitivity of $h \sim 10^{-27}$. From there we examine the implications of an earlier Hubble parameter at the start of inflation, and a phase transition treatment of pre to post Planckian inflaton physics. We close with an analysis of how gravitational constant $G$ may vary with time, the tie in with the NEC condition and how to select a range of relic GW frequencies.

Introduction

We begin looking at what to expect via the ratio of the energy of relic gravitational waves, over a fixed energy density as a way to quantify the allowed frequency range, and sensitivity allowed, ie. 1 GHz. This permits, if we do it right at looking at a phenomenological treatment of acquisition of the data needed to understand the Hubble parameter, via experimental temperature inputs. Next, if that same Hubble parameter is proportional to the square root of the inflaton potential, the regime of potential change from $\phi^a$ to $\phi^{-a}$ as given by Beckwith’s [1] adaptation of Weinberg’s [2] discussion of scale factor and potentials may signify a phase transition. This assuming that $\phi$ as an input keeps growing. The choice of $\phi^{-a}$ is tantamount to the de facto decrease in the scalar field contribution to the scalar potential.

We refer to the concept of the null energy condition [4] and how we can then look at if it is kept, or violated as part of the confirmation of if conditions exist for the Null energy condition. Fidelity to the null energy condition as we assume is combined with $G \sim G(t)$, ie. Gravitational ‘constant’ parameter having a slight time variation, and a cosmological vacuum energy parameter changing with background temperature as part of what helps give a range of values as to the relic GW frequency.

Vacuum energy, sources and commentary

Begin first with looking at different value of the cosmological vacuum energy parameters, in four and five dimensions [7]

$$|\Lambda_{5-dim}| \approx c_1 \cdot \left(\frac{1}{T^a}\right)$$  \hspace{1cm} (1)

. Part of this document will be in, by use of an article by Finelli, Cerioni, and Gruppuso, [5] [6] that there is a way to test for inputs as to the spectral index, $n_S$. A case by case analysis of what can be ascertained via such inputs will be presented, with recommendations as to how to get these inputs set up experimentally.
in contrast with the more traditional four-dimensional version of the same, minus the minus sign of the brane world theory version. The five-dimensional version is actually connected with Brane theory and higher dimensions, whereas the four-dimensional version is linked to more traditional De Sitter space-time geometry, as given by Park [8] as cited by the author gives additional refinements [7]

\[ \Lambda_{4-\text{dim}} \approx c_2 \cdot T^\beta \]  \hspace{1cm} (2)

Right after the gravitons are released, one still sees a drop-off of temperature contributions to the cosmological constant. Then one can write, for small time values \( t \approx \delta^1 \cdot t_p \), \( 0 < \delta^1 \leq 1 \) and for temperatures sharply lower than \( T \approx 10^{12} \text{Kelvin} \), Beckwith, where for a positive integer \( n \) [7]

\[ \frac{\Lambda_{4-\text{dim}}}{\Lambda_{5-\text{dim}}} \approx 1 \approx \frac{1}{n} \]  \hspace{1cm} (3)

If there is order of magnitude equivalence between such representations, there is a quantum regime of gravity that is consistent with fluctuations in energy and growth of entropy. An order-of-magnitude estimate will be used to present what the value of the vacuum energy should be in the neighborhood of Planck time in the advent of nucleation of a new universe. The significance of Eq. (3) is that at very high temperatures, it reenforces what the author brought up with Tigran Tchrakian, in Bremen, [9] August 29th, 2008. I.e., one would like to have a uniform value of the cosmological constant in the gravitating Yang-Mills fields in quantum gravity in order to keep the gauges associated with instantons from changing. When one has, especially for times \( t_1, t_2 < \text{Planck time} \ t_p \) and \( t_1 \neq t_2 \), with temperature \( T(t_1) \neq T(t_2) \) , then \( \Lambda_4(t_1) \neq \Lambda_4(t_2) \). I.e., in the regime of high temperatures, one has \( T(t_1) \neq T(t_2) \) for times \( t_1, t_2 < \text{Planck time} \ t_p \) and \( t_1 \neq t_2 \), such that gauge invariance necessary for soliton (instanton) stability is broken [10].

That breaking of instanton stability due to changes of \( \Lambda_4(t_1) \neq \Lambda_4(t_2) \) will be our point of where we move from an embedding of quantum mechanics in an analog reality, to the quantum regime. I.e. as one reaches to high temperature, analog reality mimics digital quantum mechanics. Let us now look at different characterizations of the discontinuity, which is the boundary between analog reality, and Octonian gravity [10] [11]. Figure 1 below is also using material from Barvinsky [12], and will be useful

**Figure 1**

<table>
<thead>
<tr>
<th>Time ( 0 \leq t &lt;&lt; t_p )</th>
<th>Time ( 0 \leq t &lt; t_p )</th>
<th>Time ( t \geq t_p )</th>
<th>Time ( t &gt; t_p \rightarrow \text{today} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_5 ) undefined,</td>
<td>( \Lambda_5 \approx \varepsilon^+ ),</td>
<td>( \Lambda_5 \approx \Lambda_{4-\text{dim}} ),</td>
<td>( \Lambda_5 \approx \text{huge} ),</td>
</tr>
<tr>
<td>( T \approx \varepsilon^+ \rightarrow T \approx 10^{12} K )</td>
<td>( \Lambda_{4-\text{dim}} \approx \text{extremely large} )</td>
<td>( T \approx 10^{12} K )</td>
<td>( \Lambda_{4-\text{dim}} \approx \text{constant} ),</td>
</tr>
<tr>
<td>( \Lambda_{4-\text{dim}} \approx \text{almost } \infty )</td>
<td>( 10^{12} K &gt; T &gt; 10^{12} K )</td>
<td>( T \approx 3.2 K )</td>
<td></td>
</tr>
</tbody>
</table>

For times \( t > t_p \rightarrow \text{today} \), a stable instanton is assumed, along the lines brought up by t’Hooft [13], due to the stable \( \Lambda_{4-\text{dim}} \approx \text{constant} \sim \text{very small value} \), roughly at the value given today. This assumes a radical drop-off of the cosmological constant for, say right after the electroweak transition. This would be in line with Kolb’s assertion of the net degrees of freedom in space-time drop from about 1000 to less than two, especially if \( t > t_p \rightarrow \text{today} \) in terms of the value of time after the big bang. The supposition we are making here is that the value of \( N \) so obtained is actually proportional to a numerical graviton density we will refer to as \( <n>_H \), provided that there is a bias toward HFGW, which would mandate a very small value for \( V \approx R_H^3 \approx \lambda^3 \). Furthermore, structure formation arguments, as given by Perkins [14] give ample
evidence that if we use an energy scale, \( m \), over a Planck mass value \( M_{\text{Planck}} \), as well as contributions from field amplitude \( \phi \), and using the contribution of scale factor behavior \( \dot{a} / a = H \approx -m \cdot \frac{\phi}{3 \cdot \phi} \), where we assume \( \dot{\phi} \equiv 0 \) due to inflation

\[
\frac{\Delta \rho}{\rho} \sim H \Delta t \sim \frac{H^2}{\phi} \sim \left( \frac{m}{M_{\text{Planck}}} \right) \times \left( \frac{\phi}{M_{\text{Planck}}} \right) \sim 10^{-5}
\]

At the very onset of inflation, \( \phi \ll M_{\text{Planck}} \), and if \( m \) (assuming \( \hbar = c = 1 \)) is due to inputs from a prior universe, we have a wide range of parameter space as to ascertain where \( \Delta S \approx \Delta N_{\text{gravitons}} \neq 10^8 \) [12] comes from and plays a role as to the development of entropy in cosmological evolution. In the next Chapter, we will discuss if or not it is feasible / reasonable to have data compression of prior universe ‘information’. It suffices to say that if \( S_{\text{initial}} \sim 10^5 \) is transferred from a prior universe to our own universe at the onset of inflation., at times less than Planck time \( t_p \sim 10^{-44} \) seconds, that enough information MAY exit for the preservation of the prior universe’s cosmological constants, i.e. \( \hbar, G, \alpha \) (fine structure constant) and the like. We do not have a reference for this and this supposition is being presented for the first time. Times after \( t = 10^{-44} \) are not less important. Issues raised in [11], [12], [13], [14], [15], [16], [17] are important as to the research protocols.

Consider now what could happen with a phenomenological model bases upon the following inflection point i.e. split regime of different potential behavior

\[
V(\phi) = g \cdot \phi^\alpha
\]

De facto, what we come up with pre, and post Planckian space time regimes, when looking at consistency of the emergent structure is the following. Namely by adjusting what is done by Weinberg [2] we have [1],

\[
V(\phi) \propto \phi^{|r|} \quad \text{For} \quad t < t_{\text{Planck}}
\]

Also, we would have

\[
V(\phi) \propto 1/\phi^{|r|} \quad \text{For} \quad t >> t_{\text{Planck}}
\]

Eq. (12) and Eq. (13) are predicated on the idea that \( \phi \) increases, as given by Eq. (16) below.

The switch between Eq. (12) and Eq. (13) is not justified analytically. I.e. it breaks down. Beckwith et al (2011) designated this as the boundary of a causal discontinuity. Now according to Weinberg [2], if

\[
eq -\frac{\lambda^2}{16\pi G}, H = 1/\epsilon t \quad \text{so that one has a scale factor behaving as [2]}
\]

\[
a(t) \propto t^{1/\epsilon}
\]

Then, if [14]

\[
|V(\phi)| << (4\pi G)^{-2}
\]

there are no quantum gravity effects worth speaking of. I.e., if one uses an exponential potential a scalar field could take the value of, when there is a drop in a field from \( \phi_1 \) to \( \phi_2 \) for flat space geometry and times \( t_1 \) to \( t_2 \) [2]

\[
\phi(t) = \frac{1}{\lambda} \ln \left[ \frac{8\pi G \epsilon^2 t^2}{3} \right]
\]

Then the scale factors, from Planckian time scale as [2]
The more \( \frac{a(t_2)}{a(t_1)} \gg 1 \), then the less likely there is a tie in with quantum gravity. Note that the way this potential is defined is for a flat, Roberson-Walker geometry, and that if and when \( t_1 < t_{\text{Planck}} \) then what is done in Eq. (11) no longer applies, and that one is no longer having any connection with even an Octonionic Gravity regime. The details as to what may be expected via Octonionic gravity and its violation are given in Beckwith [1] as an adaptation of the above, and linked to the next section which is that there is a way to link the phase shift involved in Eq. (5) to Eq. (11) with a degrees of freedom mapping as given in the next section.

**Increase in degrees of freedom in the sub Planckian regime.**

Starting with [18], [19]

\[ E_{\text{thermal}} \approx \frac{1}{2} k_B T_{\text{temperature}} \propto \left[ \Omega_0 \tilde{T} \right] \sim \tilde{\beta} \]  

The assumption is that there would be an initial fixed entropy arising, with \( \bar{N} \) as a nucleated structure arising in a short time interval as a temperature \( T_{\text{temperature}} e^{\left(0^+,10^9 GeV\right)} \) arrives. One then obtains, dimensionally speaking [18], [19]

\[ \frac{\Delta \tilde{\beta}}{\text{dist}} \approx (5k_B \Delta T_{\text{temp}}/2) \cdot \frac{\bar{N}}{\text{dist}} \sim q E_{\text{net-electric-field}} \sim \left[ T \Delta S / \text{dist} \right] \]  

The parameter, as given by \( \Delta \tilde{\beta} \) will be one of the parameters used to define chaotic Gaussian mappings. Candidates as to the inflation potential would be in powers of the inflation, i.e. in terms of \( \phi^N \), with \( N=4 \) effectively ruled out, and perhaps \( N=2 \) an admissible candidate (chaotic inflation). For \( N=2 \), one gets [11], [18]

\[ [\Delta S] = \left[ \frac{\hbar}{T} \right] \cdot \left[ \frac{2k^2 - \frac{1}{\eta^2} \left[ M_{\text{Planck}}^2 \left[ \frac{6}{4\pi} - \frac{12}{4\pi} \left( \frac{1}{\phi} \right)^2 - \frac{6}{4\pi} \left( \frac{1}{\phi^2} \right)^2 \right] \right]^{1/2} \right] \sim n_{\text{Particle-Count}} \]  

If the inputs into the inflation, as given by \( \phi^2 \) becomes a random influx of thermal energy from temperature, we will see the particle count on the right hand side of Eq. (14) above a partly random creation of \( n_{\text{Particle-Count}} \) which we claim has its counterpart in the following treatment of an increase in degrees of freedom. The way to introduce the expansion of the degrees of freedom from nearly zero, at the maximum point of contraction to having \( N(T) \sim 10^3 \) is to first define the classical and quantum regimes of gravity in such a way as to minimize the point of the bifurcation diagram affected by quantum processes. [18]. The diagram, in a bifurcation sense would look like an application of the Gauss mapping of [11], [18]

\[ x_{i+1} = \exp \left[ -\tilde{\alpha} \cdot x_i^2 \right] + \tilde{\beta} \]  

In dynamical systems type parlance, one would achieve a diagram, with tree structure looking like what was given by Binous [19], using material written up by Lynch [20]. Now that we have a model as to what could be a change in space time geometry, let us consider what may happen during the Higgs mechanism break down, as given by Beckwith [1] and in very early universe geometry

**The role Critical density plays in analyzing the frequency produced in relic GW production**

We are now going to bring up what Grupen [21] brings up about the role of energy density, GW, and also of \( \Omega_{GW} \) in terms of setting up frequency changes due to phase shifts in early universe cosmology. To do this, note first that
\[ \rho_{GW} = \left[ \frac{\hbar^2 \omega_{GW}^2}{32\pi \cdot G} \right] \]  

(16)

'This expression leads to

\[ \Omega_{GW} = \left[ \frac{\hbar^2 \omega_{GW}^2}{12 \cdot H^2} \right] \]  

(17)

As stated before in the introduction as well as given in \([21]\) having \( h \sim 10^{-27} \Leftrightarrow \omega_{GW} \sim 1000 \cdot Hz \).

We can now seriously consider candidates as to the Hubble frequency, as far as phenomenology and to use that to be part of an estimate as far as a permitted range of GW frequencies generated by relic early universe phase transitions. The current idea by \([22]\) is that the Electro weak regime, as designated by Duerr et al. \([22]\) is by far a greater contributor to GW production, and it is now time to revisit that assumption in detail.

As stated by Sarkar \([23]\), page 481 of his reference, a good temperature based phenomenological treatment of a Hubble parameter would look like

\[ H \equiv 1.66\sqrt{g^*(T)} \cdot \left[ \frac{T^2}{M_{PL}} \right] \]  

(18)

As stated by Beckwith \([1]\) and re duplicated in Eq. (15), the argument given is that there would be, if certain conditions are met, a starting low temperature, rapidly rising, with at about the Planck regime of space time a top degrees of freedom expression of about \( g^*(T) \bigg|_{\text{Maximum}} \sim 1000 \), for the temperature reaching \( T|_{\text{Maximum}} \sim T_{PL} \) in value from an initially much lower value. This is also a datum, which if we reach \( g^*(T) \bigg|_{\text{Maximum}} \sim 1000 \) would be in sync with Sarkar’s \([23]\)

\[ H \sim \sqrt{8\pi V(\phi)/3M_{PL}^2} \]  

(19)

The matter to consider, would be, frankly, that looking at the following expression of energy flux being re formulated for each universe. I.e. start with the Alcubierre’s \([24]\) formalism about energy flux, assuming that there is a solid angle for energy distribution \( \Omega \) for the energy flux to travel through. \([24]\), \([25]\) looking at a change of energy if

\[ \frac{dE}{dt} = \lim_{r \to \infty} \left[ \frac{r^2}{16\pi} \right] \int_{-\infty}^{t} \int_{-\infty}^{\infty} \Psi_4 dt \right] \cdot d\Omega \]  

(20)

The expression \( \Psi_4 \) is a Weyl scalar which we will write in the form of \([24]\), \([25]\)

\[ \Psi_4 = -\frac{1}{4} \left[ \partial_i^2 h^+ - 2\partial_i \partial_j h^+ + \partial_i^2 h^+ \right] + \frac{i}{4} \left[ \partial_i^2 h^+ - 2\partial_i \partial_j h^+ + \partial_i^2 h^+ \right] \]  

(21)

Our assumptions are simple, that if the energy flux expression is to be evaluated properly, before the electro weak phase transition, that time dependence of both \( h^+ \) and \( h^\tau \) is miniscule and that initially \( h^+ \approx h^\tau \), so as to initiate a re write of Eq. (21) above as \([24]\)
\[ \Psi_4 \approx -\frac{1}{4} \left[ + \partial^2_+ h^+ \right] \cdot (-1 + i) \]  

(22)

The upshot, is that the initial energy flux about the inflationary regime would lead to looking at [25]

\[ \left| \int_{-\infty}^{\Psi_4 dt} \approx \frac{1}{2} \left[ + \partial^2_+ h^+ \right] \cdot (\vec{n} \cdot t_{Planck}) \right| \]  

(23)

This will lead to an initial energy flux at the onset of inflation which will be presented as [25]

\[ \frac{dE}{dt} = \left[ \frac{r^2}{64\pi} \right] \cdot [\partial^2_+ h^+]^2 \cdot [\vec{n} \cdot t_{Planck}]^2 \cdot \Omega \]  

(24)

If we are talking about an initial energy flux, we then can approximate the above as [25]

\[ E_{initial-flux} \approx \left[ \frac{r^2}{64\pi} \right] \cdot [\partial^2_+ h^+]^2 \cdot [\vec{n} \cdot t_{Planck}]^2 \cdot \Omega_{effective} \]  

(25)

Inputs into both the expression \[ \partial^2_+ h^+ \], as well as \( \Omega_{effective} \) will comprise the rest of this document, plus our conclusions. The derived value of \( \Omega_{effective} \) as well as \( E_{initial-flux} \) will be tied into a way to present energy per graviton, as a way of obtaining \( n_f \). The \( n_f \) value so obtained, will be used to make a relationship , using Y. J. Ng’s entropy [15] counting algorithm of roughly \( S_{entropy} \sim n_f \). We assert that in order to obtain \( S_{entropy} \sim n_f \) from initial graviton production, as a way to quantify \( n_f \), that a small mass of the graviton can be assumed.

For the sake of convenience, one can write [24], [25],[28],[29]

\[ \left| \partial^2_+ h^+ \right| \sim k^2 h^+ \]  

(26)

So, then [25]

\[ E_{initial-flux} \sim \left[ \frac{r^2}{64\pi} \right] \cdot k^4 \cdot \left[ h^+ \right]^2 \cdot [\vec{n} \cdot t_{Planck}]^2 \cdot \Omega_{effective} \]  

(27)

For our purposes, we shall call \( r \sim l_{Planck} \propto 10^{-34} cm \), \( t_{Planck} \sim 10^{-44} sec \), \( \Omega_{effective} \) an effective cross sectional area as to the emission of gravitons, and \( k \) defined as a physical wave vector. L. Crowell stated that GW would undergo massive red shifting , [26], [27] Needless to state, the value of \( k \) to consider would be for the GHz band of GW [28],[29]

\[ \left( k \approx k_{GW} \right)^2 \gg \left| \frac{1}{a} \frac{d^2 a}{d\eta^2} \right| \]  

(28)

Also, for the frequencies of [29],[30] \( 10^9 \sim 10^{10} \) Hz, then

\[ h \sim h_{rms} \sim 10^{-30} - 10^{-34} \]  

(29)
Namely, if a net acceleration is such that \( a_{\text{accel}} = 2\pi k_B c T / \hbar \) as mentioned by Verlinde [31],[32] as an Unruh result, and that the number of ‘bits’ is

\[
n_{\text{Bit}} = \frac{\Delta S}{\Delta x} \cdot \frac{c^2}{\pi \cdot k_B T} \approx \frac{3 \cdot (1.66)^2 g^*}{\Delta x \approx l_p} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2} \approx \frac{3 \cdot (1.66)^2 \sqrt{g^*}}{\Delta x \approx l_p} \cdot T^3
\]  

(30)

This Eq. (30) has a \( T^2 \) temperature dependence for information bits, as opposed to \[15\], [23],[32], [33]

\[
S \sim 3 \cdot \left[ 1.66 \cdot \sqrt{g^*} \right] T^3 \sim n_f
\]  

(31)

Should the \( \Delta x \approx l_p \) order of magnitude minimum grid size hold, then when \( T \sim 10^{19} \text{ GeV}[31],[32] \)

\[
n_{\text{Bit}} \approx \frac{3 \cdot (1.66)^2 g^*}{\Delta x \approx l_p} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2} \sim 3 \cdot \left[ 1.66 \cdot \sqrt{g^*} \right] T^3
\]  

(32)

The situation for which one has [31], [32] \( \Delta x \approx l^{1/3} l_{\text{Planck}}^{2/3} \) with \( l \sim l_{\text{Planck}} \) corresponds to \( n_{\text{Bit}} \propto T^3 \) whereas \( n_{\text{Bit}} \propto T^2 \) if \( \Delta x \approx l^{1/3} l_{\text{Planck}}^{2/3} >> l_{\text{Planck}} \).

Here, we make the assumption that either \( \vec{n} \sim n_{\text{Bit}} \propto T^2 \) or \( \vec{n} \sim n_{\text{Bit}} \propto T^3 \) per unit volume of phase space, with the temperature \( T \) varying from a low value to up to \( 10^{34} \) Kelvin (Planck temperature scale).

We will next reference as to conditions permitting release of \( \vec{n} \sim n_{\text{Bit}} \propto T^2 \) or \( \vec{n} \sim n_{\text{Bit}} \propto T^3 \) per unit volume of phase space, while also noting a way to also identify dimensional contributions to relic particle conditions. Taking into account, as given by U. Sarkar [23] for relic Graviton production, as a function of extra dimensions we can denote by

\[
n_{\text{relic–graviton–production}}(T) \sim T^2 \cdot \left[ T / M \right]^{d+2}
\]  

(33)

We can though, if we wish to reconcile Eq. (39) with release of \( \vec{n} \sim n_{\text{Bit}} \propto T^2 \) or \( \vec{n} \sim n_{\text{Bit}} \propto T^3 \) look at temperature dependence of the scaled mass value, \( M \), Furthermore, if a phase transition exists, as mentioned by Subir Sarkar the following change is revealing

Recall Subir Sarkar’s [34] 2001 investigation of a simple choice of variant of the standard chaotic inflationary potential given by

\[
V \equiv V_0 - c_3 \phi^3 + \frac{1}{2} \lambda \phi^2 \rho^2 + \ldots
\]  

(34)

Sarkar treated the inflaton as having a varying effective mass, with an initial value of effective mass of

\[
m^2 = \frac{d^2 V}{d \phi^2}
\]

given a before and after phase transition value of [34]

\[
m^2 = - 6 c_3 \cdot \langle \phi \rangle \quad \text{Before–phase–transition} \rightarrow \quad 6 c_3 \cdot \langle \phi \rangle + \lambda \cdot \Sigma^2 \quad \text{after–phase–transition}
\]  

(35)

This is, when Hunt and Sarkar [34] did it, with \( \lambda = \kappa \cdot m^2 / M_p^2 \) as a coupling term. This would also affect the spectral index value, and it also would be a way to consider an increase in inflation based entropy. The value of \( M \) so given in Eq. (33) we believe is connected with an appropriate choice of the details of the phase transition alluded to in Eq. (35) above.
There are two ways to reconcile information from Eq. (35) as far as a temperature dependence affecting $M$, as I see it, and connecting it with Eq (35) and release of $\tilde{n} \sim n_{Bit} \propto T^2$ or $\tilde{n} \sim n_{Bit} \propto T^3$

First [23]

$$T^2 \cdot \left[ T/M \right]^{d+2} \sim T^2 \iff M \propto T$$

(36)

This will as we will present below apparently implying $\Delta x \cong \frac{1}{l_{Planck}^{1/2}} \gg l_{Planck}$

It so happens that Eq. (36) with a direct temperature dependence of a net mass $M$ is equivalent to the production of gravitons/ relic particles as dictated by an initially fixed starting temperature, i.e. making the $\Delta x \cong \frac{1}{l_{Planck}^{1/2}}$ with $l \sim l_{Planck}$ corresponds to $n_{Bit} \propto T^3$ whereas $n_{Bit} \propto T^2$ if $\Delta x \cong l_{Planck}^{1/2} \gg l_{Planck}$. The minimum grid size being possibly of the form $\Delta x \cong \frac{1}{l_{Planck}^{1/2}} \gg l_{Planck}$. The minimum grid size being possibly of the form $\Delta x \cong \frac{1}{l_{Planck}^{1/2}} \gg l_{Planck}$ implies a fixed set of initial space parameters, with temperature not affected by extra dimensions.

Secondly, having [23]

$$T^2 \cdot \left[ T/M \right]^{d+2} \sim T^3 \iff M \propto T^n, |\alpha| < 1$$

(37)

This may correspond to implying $\Delta x \cong \frac{1}{l_{Planck}^{1/2}} \gg l_{Planck}$

Summary as to what is known, and not known about the Null Energy Condition in Cosmology. And information exchange between Prior to Present Universes.

As stated in [4], the NEC is linked to the following, i.e. look at the general null energy condition first

The null energy condition stipulates that for every future-pointing null vector field (for all of the GR) $\bar{k}$

$$\rho = T_{ab} k^a k^b \geq 0.$$  

(38)

With respect to a frame aligned with the motion of the matter particles, the components of the matter tensor take the diagonal form, in Euclidian space that

$$T^{\hat{a}\hat{b}} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}.$$  

(39)

The simplest statement of the Null energy condition is that he null energy condition stipulates that $\rho + p \geq 0$.

(40)

I.e. the equation of state to consider is, if $w \leq -1$, then if what [1] suggests is true, then there will be a reason to consider the relative import of Eq. (38), Eq. (39), and Eq. (40) in terms of contributions. I.e. we do have problems with the idea of variance of the cosmological constant, $G$, and will reference it. We also will build upon the consequences of $w \leq -1$. We can generalize this idea to initial domain wall physics. Spherical geometry as we know it does not violate the NEC. Further domain wall physics may lead to a
break down of the NEC [4]. We also refer to a treatment of the NEC, and its possible consequences if we look at an effective Friedman equation as given by [2], as seen by

$$H_a^2 = \left[ \frac{d}{a} \right]^2 = \frac{8 \pi G}{3} G \cdot \rho + \left( \frac{\kappa^2}{24 \cdot (3 + n)} \right) \cdot [2 + n \cdot [1 - 3 \cdot (v - w)] \cdot \rho^2 - e$$

(41)

The scaling done in this situation has [2], especially if $e$ is a constant in Eq. (46)

$$\rho = a^{-3[1+w]}$$

(42)

As stated in [16]. We expect that there will be flat space geometry almost in the beginning of the early big bang. I.e. this will lead to Eq. (47), if $w < -1$ implying that $\rho = a^{-3[1+w]} \sim a^{-\epsilon} \rightarrow 0^+$ if there is a violation of the NEC. As quoted from [18]. I.e. as seen in a colloquium presentation done by Dr. Smoot in Paris [35] (2007); he alluded to information theory constructions which bear consideration as to how much is transferred between a prior to the present universe in terms of information ‘bits’.

0) Physically observable bits of information possibly in present Universe - $10^{180}$
1) Holographic principle allowed states in the evolution/development of the Universe - $10^{120}$
2) Initially available states given to us to work with at the onset of the inflationary era - $10^{10}$
3) Observable bits of information present due to quantum/statistical fluctuations - $10^{8}$

Our guess is as follows. That the thermal flux accounts for perhaps $10^{10}$ bits of information. These could be transferred from a prior universe to our present, and that there could be, perhaps $10^{120}$ minus $10^{10}$ bytes of information temporarily suppressed during the initial bozonification phase of matter right at the onset of the big bang itself. Beckwith [37] stated criteria as far as graviton production, and a toy model of the universe. If one has Eq. (42) shut off due to $w < -1$, so then that

$\rho = a^{-3[1+w]} \sim a^{-\epsilon} \rightarrow 0^+$ occurs, then the causal discontinuity so references in [18] by Beckwith et al, will have major consequences as far as a away to determine if gravitons have a small mass, and if there is a way to determine if a prior universe has contribution as to the information transferred as to the present universe. We will now assume, that the catastrophe given as stated by $\rho = a^{-3[1+w]} \sim a^{-\epsilon} \rightarrow 0^+$ does not occur.

. Determination if the NEC is valid is essential as establishing a necessary condition for transfer of information from a prior universe to Today’s Cosmos;

How to do this? I.e. how to determine if, as an example there is a thermal, flux from a prior universe carrying prior universe information? We will briefly revisit a first principle introduction as to inflaton fluctuations in the beginning which may be part of how to obtain experimental falsifiable criterion. From Weinberg [2], we can write, from page 192-93, if an inflaton potential $V(\phi) \sim M^{4+\sigma} \phi^{-\sigma}$ then, the inflaton potential has the fluctuation behavior given by [1]

$$\delta \phi \sim t^2$$

(43)

Then, this assumes
\[
\gamma = -.25 \pm \sqrt{\frac{1}{16} - \frac{(6 + \alpha) \cdot (1 + \alpha)}{(2 + \alpha)^2}}
\]  
(44)

The resulting contributions to the CMBR, if worked out, and also connections to gravitational wave astronomy as can be gleaned eventually can be used to pin point an eventual CMBR physics behavior as referred to by Beckwith [1] may after time start giving us ideas if the NEC holds, or does not hold.

**How to calculate the Spectral index \( n_s \) for a dissipative regime of the inflaton?**

We are largely borrowing in this introduction from work done by Finelli, Cerion, and Gruppuso [2], [3] and we will introduce the motivation behind their work as well as the actual Spectral index \( n_s \). To begin with look at what Finelli at al [5], [6] postulate as to the case of warm inflation. I.e. as given by [1],[5], [6], if the equation of state \( \omega_F = p_F / \rho_F \) is linked to \( [3H \rho_F \cdot (1 + \omega_F)] \) \( \cong \Gamma \phi^2 \) so then we get the statement of

\[
\ddot{\phi} + [3H + \Gamma] \dot{\phi} + \left( \frac{\partial V}{\partial \phi} \right) = 0
\]

(45)

We can count the term given as \( [3H + \Gamma] \dot{\phi} \) as a damping term, as well as consider the slow roll value of

\[
[3H + \Gamma] \dot{\phi} + \left( \frac{\partial V}{\partial \phi} \right) \cong 0
\]

(46)

The above dynamics, if \( V_{\phi \phi} = \frac{d^2 V}{d \phi^2} \), and \( \Gamma = \Gamma_0 \cdot \left[ \frac{\phi}{\phi_0} \right]^b \cdot \left[ \sqrt{\frac{\rho_F}{M}} \right]^c \), and

\[
\gamma = \frac{\Gamma}{3H^3} \cdot \epsilon = -\frac{\dot{H}}{H^2} \cdot \eta_{\phi \phi} = \frac{V_{\phi \phi}}{3H^2}
\]

(47)

For the sake of convenience, we can use \( V_{\phi \phi} \sim \) constant, i.e. the quadratic scalar potential. But this is a special case of what we will refer to later. If so, then the equations for perturbations, inflaton perturbations, \( Q_\phi, Q_F \) as respectively the inflaton, and the fluid fluctuations leads to initial conditions of

\[
Q_\phi |_{k} \cong \exp \left[ -i k \cdot (\tau - \tau_i) \right] \left[ a^{\frac{3}{2}} \sqrt{2k} \right], \quad (48)
\]

\[
Q_F |_{k} \cong \exp \left[ -i k \cdot (\tau - \tau_i) \right] \left[ a^{\frac{3}{2}(1+\omega_F)(1+\gamma)} \left( 4k^2 \omega_F \right) \right]
\]

The upshot is that one gets the following as far as a running index [1], [5], [6]

\[
n_s - 1 = -3\gamma^* + \frac{2}{1 + \gamma^*} \eta^* - 6 \cdot \frac{1 + \frac{4}{3} \gamma^* + \frac{1}{2} [\gamma^*]^3}{1 + \gamma^*} \cdot \epsilon^*
\]

(49)

Here, the * factor is for values of the parameters when the cosmological evolution crosses a radius defined by \( (k = a_0 H_0) \). In [2] there are two tables as far as inputs/ out puts into running index, which have to take into account several constraints. I.e. when one has, as was stated a situation for which [1]
\[ \Gamma = \Gamma_0 \cdot \left[ \frac{\phi}{\phi_0} \right]^b \cdot \left[ \sqrt[4]{\frac{\rho_F}{M}} \right]^c = \text{const} \]  

(50)

Either \( b = C = 0 \), which is possible, or one could have, if \( b \neq 0, C \neq 0 \), a situation for which one can have

\[ \left[ \frac{\phi}{\phi_0} \right]^b \cdot \left[ \sqrt[4]{\frac{\rho_F}{M}} \right]^c = \text{const} \]  

(51)

What if one had, \( \phi_0 \) being a present day, very small value of a scalar field \([1]\)

\[ \phi = \text{const} \cdot \phi_0 \cdot \left[ \frac{M}{\sqrt[4]{\rho_F}} \right]^{C/b} \]  

(52)

We can probably assume in all of this that \( M \) as a mass scale is fixed. When the author looks at Eq. (52), it appears to be implying the relative value of density, i.e. \( \rho_F \) varies with time. I.e. if one looked at the Octonion gravity formation regime we could look at variation of looking maybe like \( \rho_F \sim H^2 \text{_{observed}} / G \) The term about the relationship of \([37]\), where a is a constant, and \( g^*(T) \) is the number of degrees of freedom,

\[ \rho_F \sim H^2 \text{_{observed}} / G \approx 4\pi \cdot aT^4 g^*(T) / 3c^2 \]  

(53)

There are two different scenarios as far as temperature build up and how it affects \( g^*(T) \), and also initial temperatures.

**1st, version of classical/ standard cosmology treatment of the start of inflation. I.e. the ultra high temperature regime to cooler temperatures**

Here, as given by Kolb and Turner \([38]\), \( g^*(T) \) has a peak of about 100-120 during the electro weak regime, and that there is allegedly little sense in terms of modeling of talking about \( g^*(T) \) before the electro weak regime. What it means? In so many words, we would then have \( \rho_F \) undefined before the electro weak regime. \( \phi \) would then be undefined before the electro weak regime. It does mean that at the start of the electro weak regime, we would see an increasing \( \phi \). Which is the opposite of what we see. I.e. we need \( \phi \) decreasing. Meaning that either \( g^*(T) \) is defined before the electro weak phase transition, or Eq. (53) no longer holds.

How to tie in the entropy with the growth of the scale function? Racetrack models of inflation, assuming far more detail than what is given in this simplistic treatment provide a power spectrum for the scalar field given by \([39], [40]\)

\[ P \sim \frac{1}{150\pi^2} \cdot \frac{V(\phi)}{\epsilon} \]  

(54)

This is very close to what Giovannini puts in, \([40]\), and \( n_s \) being the spectral index

\[ P(k) = \frac{8}{3M_p^4} \cdot \left( \frac{V(\phi)}{\epsilon} \right)_{k=aH} \approx k^{n_s - 1} \]  

(55)

This is assuming a slow roll parameter treatment with \( \epsilon << 1 \), and for \( t > t_p \). An increase in scalar power, is then proportional to an increase in entropy via
\[
\frac{\Delta E}{l_p^3} \sim \frac{\Delta P \in 150\pi^2}{l_p^3} \approx |\Delta S|
\]

(56)

This presumes that we have a well defined \( V(\phi) \) before the start of the Planck time interval. That is, if we want to make the equivalent statement \( |\Delta S| \sim \Delta n \) for [15] a numerical relic count, as done by Ng [12] does not tell us where the relic particles came from. As we also note in [20] we can employ Sherrer k essence arguments [3] as to how to form relic particles without using a potential explicitly for times less than Planck time interval.

**1st , new treatment of the start of inflation. I.e. first low temperature, then ultra high temperature regime to cooler temperatures (low to high then low temperature evolution)**

This involves using the initial analysis, except that one has \( g^*(T) \) defined initially as of about 2 in pre Planckian space time, rising to about 1000, as of Planck time, [18] and then from there declining. The initial temperature would be low, which would rise to a peak temperature, i.e. Planck temperature value, and then subsequently moving to values seen today. This scenario is outlined in [1], and has the advantage of explaining at least before to about the Planck time interval, how Eq. (54) could resort to a rising temperature. Now, having said, that, what is the advantage toward having

\[
\Gamma = \Gamma_0 \cdot \left[ \frac{\phi}{\phi_0} \right]^b \cdot \left[ \frac{\sqrt{P_F}}{M} \right]^C
\]

constant with rising inflaton value, \( \phi \) and with \( b \neq 0, C \neq 0 \).

**Comparing the re acceleration of the universe, via deceleration parameter, initially and finally speaking**

The use of Eq. (79) below to have re-acceleration in this formulation is dependent upon ‘heavy gravity’ as the rest mass of gravitons in four dimensions has a small mass term. This equation below is developed by Beckwith [41], [42], and [43]

\[
q = -\frac{\ddot{a}a}{\dot{a}^2}
\]

(56)

We wish next to consider what happens not a billion years ago, but at the onset of creation itself. If a correct understanding of initial graviton conditions is presented, it may add more credence to the idea of a small graviton mass, in a rest frame,. Here, we are making use of refining the following estimates. In what follows, we will have even stricter bounds upon the energy value (as well as the mass) of the graviton based upon the geometry of the quantum bounce, with a radii of the quantum bounce on the order of

\[
l_{\text{Planck}} \sim 10^{-35} \text{ meters}
\]

(44) [45].

\[
m_{\text{graviton}}|_{\text{RELATIVISTIC}} < 4.4 \times 10^{-22} h^{-1} eV / c^2
\]

\[
\Leftrightarrow \lambda_{\text{graviton}} = \frac{\hbar}{m_{\text{graviton}} c} < 2.8 \times 10^{-13} \text{ meters}
\]

(57)

For looking at the onset of creation, with a LQG bounce; if we look at \( \rho_{\text{max}} \propto 2.07 \cdot \rho_{\text{planck}} \) for the LQG quantum bounce with a value put in for when \( \rho_{\text{planck}} \approx 5.1 \times 10^{99} \text{ grams/ meter}^3 \), where

\[
E_{\text{eff}} \propto 2.07 \cdot l_{\text{Planck}}^3 \cdot \rho_{\text{planck}} \approx 5 \times 10^{24} \text{ GeV}
\]

(58)
Then, taking note of this, one is obtaining having scaled entropy of \( S \equiv E/T \sim 10^5 \) when one has an initial Planck temperature \( T \approx T_{\text{Planck}} \sim 10^{19} \text{GeV} \). One then needs to consider, if the energy per given graviton is, if a frequency \( \nu \propto 10^{10} \text{Hz} \) and \( E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV} \), then

\[
S \equiv \frac{E_{\text{eff}}}{T} \sim \left[ 10^{38} \times E_{\text{graviton-effective}} \left\{ \nu \approx 10^{10} \text{Hz} \right\} \right] / \left[ T \sim 10^{19} \text{GeV} \right] \approx 10^5
\]  

(59)

Having said that, the \( E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV} \) is \( 10^{22} \) greater than the rest mass energy of a graviton if \( E \sim m_{\text{graviton}} \left[ \text{red-shift} \sim .55 \right] \sim \left( 10^{-27} \text{eV} \right) \) grams is taken.

**Now, for permitted frequency ranges for the relic graviton**

As given by Hambler [ ] for the effective Friedman equation, on pages 318-319. In this procedure, we can set \( \xi \approx a_0 \) with \( a_0 \) as defined by what is known as the running gravitational coupling in the vicinity of the ultra violet fixed point as given in equation 9.1 of Hambler [46] (page 305)

\[
G(t) = G \cdot \left[ 1 + C_{\xi} \cdot \left( \frac{t}{\xi} \right)^{\nu} \right]
\]  

(60)

This does lead to an effective density as given by

\[
\rho_{\text{effective}}(t) = \frac{G(t)}{G} \cdot \rho(t)
\]  

(61)

If one is making an analysis of the effective energy, as given by an analysis in part given by Ng[ ] and Beckwith [ ]

\[
\frac{\Delta E}{E_p^3} \approx \Delta n_{\text{relav-graviton}} \propto h \Delta \omega_{\text{relav-graviton}} \approx h \cdot \left[ 1 + C_{\xi} \cdot \left( \frac{t}{\xi} \right)^{\nu} \right] \cdot \omega_{\text{relav-graviton}}
\]  

(62)

The relic graviton frequency so described would be from \( E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV} \) which is \( 10^{22} \) greater than the rest mass energy of a graviton, taking \( \omega_{\text{relav-graviton}} \sim E_{\text{graviton-effective}} / h \), with \( E_{\text{graviton-effective}} \propto 2 \cdot h \nu \approx 5 \times 10^{-5} \text{eV} \) is \( 10^{22} \) greater than the rest mass energy of a graviton.

The spread in the frequencies would be given by the factor \( 1 + C_{\xi} \cdot \left( \frac{t}{\xi} \right)^{\nu} \). Let us for the sake of completeness analyze where this came from. the Friedman equation, as given by Hambler[ 46 ] with \( k \) the curvature factor, and

\[
\frac{k}{a^2(t)} + \left[ \frac{\dot{a}(t)}{a(t)} \right]^2 = \frac{8\pi G}{3} \cdot \left[ 1 + C_{\xi} \cdot \left( \frac{t}{\xi} \right)^{\nu} \right] \cdot \rho(t) + \frac{1}{3\xi^2}
\]  

(63)

In short, we would get, then a variance in the Friedman equation, and what we think is that this is in part also linkable to the following, namely
\[
\Delta \rho_F \sim 4\pi \cdot a(\Delta T)^4 \frac{g^* (\Delta T)}{3c^2}
\]

As mentioned earlier, we have, in Eq. (63) and also in Eq. (64) a duration of time for which there is a build up of temperature, of the magnitude \(\Delta T\) just before the inflationary era, and that the time factor is tied into \(1 + C_\xi \left( \frac{t}{\xi} \right)^\gamma \) of Eq.(63) It means that in the context of relic graviton production, that the frequency range as of GW production is, indeed nearly a delta function. Why is this significant? If one looks at a frequency for relic GW in terms of an upper bound as to GW frequency, i.e. what was predicted by Buonanno, namely, if frequency \(f > f_* = 4.4 \times 10^{-9}\) Hz

\[
h_0^2 \Omega_{GW} \leq 4.8 \times 10^{-9} \cdot (f/f_*)^2
\]

One gets a bias toward low frequencies, and this is accentuated by an estimate which needs to be looked at and questioned, namely, if there is, according to Buonanno, purely adiabatic evolution of the universe, Here, we have that \(a_0\) is today’s value of the cosmological constant, whereas \(a_*, f_*\) are initial scale factor and frequency values for the production of GWs

\[
f = f_* \cdot (a_*/a_0)
\]

If the bias toward low frequency values is removed and we look at generation of say having \(a_* \sim 10^{-25} a_0\) for nearly relic conditions, one gets astonishingly high initial values for \(f_*\), i.e.

\[
f_* = f \cdot (a_0/a_*) \sim 10^{25} \cdot f_{\text{Today}} \propto 10^{33}\text{ Hz}
\]

Note that next to Eq. (59) we calculated de facto \(10^{22}\) times greater than the rest mass energy of a graviton for relativistic graviton energy. I.e. what was being predicted by the adiabatic approximation has a value of, already about \(10^{25}\) times larger for the frequency.

Of course, though, an adiabatic approximation is nonsense for the initial phases of the universe, but it is still indicative as to what could be the starting point to a legitimate inquiry

We should note that researchers as of China and the United States have project work on answering the feasibility of this sort of measurement. [47] Should there be a way to make such a measurement, some of the issues so referred to, i.e. the feasibility of semi adiabatic approximations can be considered. Secondly, and most importantly, if the genesis of initial GW production is within the Planck regime as so mentioned above, for the initial \(10^{35}\text{ Hz}\) value for frequency will be congruent with extremely tiny starting geometries

**Conclusion. What to make of Pre – Planckian physics**

We will initially quote part of the conclusion as of [1] here, and add more to it.

Finelli et al [6] claims that \(\gamma^* \geq 0.01\) does not match observations, with \(\gamma = \frac{\Gamma}{3H}\). We gave arguments in the prior session as to the feasibility of having \(\Gamma\) as a constant, which often appears to create serious difficulties. If one has \(\Gamma\) as a constant, with rising inflaton value, \(\dot{\phi}\) up to Planck value, we have a natural reason for \(\Lambda_{4-\text{dim}}\) varying, and also \(\rho_F \neq \text{const}\), assuming that with rising inflaton value, \(\dot{\phi}\) up to Planck time interval?
1st we have a natural reason for $\Lambda_{4\text{-dim}}$ varying, and also $\rho_F \neq \text{const}$ varies with $g^*(T)$ varying from 2 to 1000 before the electro weak era, and $\rho_F \neq \text{const}$ having $S \sim 3 \cdot \left(1.66 \cdot \sqrt{g_s}\right)^3 T^3$ increasing in a net temperature increase up to at least $10^5$ from nearly zero, initially.

Having said that, we should also revisit what the author brought up in [18] namely in how likely we are to be able to get such measurements. Doing so, asks the question of if gravitons have a small rest mass, and that leads to the second real issue to consider. From [18] we wrote how to isolate the effects of a 4 dimensional graviton with rest mass.

If one looks at if a four dimensional graviton with a very small rest mass included [18] we can write how a graviton would interact with a magnetic field within a GW detector.

$$\frac{1}{\sqrt{-g}} \partial_{\alpha} \left( \sqrt{-g} g^{\mu \nu} g^{\rho \sigma} F_{\mu \nu} \right) = \mu_0 J^\mu + J_{\text{effective}}$$

(68)

where for $\varepsilon^+ \neq 0$ but very small

$$F_{[\mu \nu, \alpha]} \sim \varepsilon^+$$

(69)

The claim which A. Beckwith made [18] is that

$$J_{\text{effective}} \approx n_{\text{count}} \cdot m_{4-D-\text{Graviton}}$$

(70)

As stated by Beckwith, in [18], $m_{4-D-\text{Graviton}} \sim 10^{-65}$ grams, while $n_{\text{count}}$ is the number of gravitons which may be in the detector sample. What would be needed to do would be to try to isolate out an appropriate stress energy tensor contribution due to the interaction of gravitons with a static magnetic field $T^{\mu \nu}$ assuming a non zero graviton rest mass.

The details of the $n_{\text{count}}$ would be affected by the degree of the graviton mass, the frequency range and a whole lot of other parameters, but the key point would be in finding a specified frequency range, which the author claims for relic gravitons is almost a spike, as well as their energy level.

From there, using some of the details brought up in this document would be relevant, in a program of action as to how to get necessary experimental confirmation. We hope to do so, as soon as circumstances permit. We also seek to find ways to confirm what t’Hooft brought up in [13].

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Bibliography


[21] H. Binous 2010 Bifurcation Diagram for the Gauss Map from the Wolfram Demonstrations Project


[27] L. Crowell, private communication with the author
[34] P. Hunt. and S. Sakar., “Multiple Inflation and the WMAP “glitches” “, Phys Rev D Volume 70, 103518