The Nature's Selection of Cubic Roots

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Abstract The English naturalist Charles Darwin established that all species of life have descended over time from common ancestry, and proposed the scientific theory that this branching pattern of evolution resulted from a process that he called natural selection. In fact, Darwin theory dealt with the evolutional phenomena of the biosphere, not its origins. Further more, there exist the natural worlds other than our beloved one. Compared to the large-scale structure of galaxies, the biosphere is "microscopic". The electromagnetic and nuclear forces which rule the world disappear in the formation of large-scale galaxy structure. Similarly they disappear in the formation of the solar system. My previous papers showed that large-scale galaxy structure originates rationally from an algebraic cubic equation. This paper presents the nature's selection of the cubic roots and its application to the galaxy NGC 3275.

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1 Darwin Theory and its Limitation

Since the English naturalist Charles Robert Darwin FRS proposed his theory of natural selection [1], there has been a long battle of creationism vs. evolutionism. The following is the partial notes from Wikipedia [2]. The creation-evolution controversy (also termed the creation vs. evolution debate or the origins debate) is a recurring cultural, political, and theological dispute about the origins of the Earth, humanity, life, and the universe. The dispute is between those who support a creationist view based upon their religious beliefs, versus those who accept evolution, as supported by scientific consensus. The dispute particularly involves the field of evolutionary biology, but also the fields of geology, palaeontology, thermodynamics, nuclear physics and cosmology. Though also present in Europe and elsewhere, and often portrayed as part of the culture wars, this debate is most prevalent in the United States. While the controversy has a long history, today it is mainly over what constitutes good science, with the politics of creationism primarily focusing on the teaching of creation and evolution in public education.

However, it is a fact that Darwin theory of natural selection dealt mainly with the evolutionary aspect of the biosphere on Earth, not its origin. If we consider that the origin or creation of the biosphere came from a universal power, then the power must still govern the biosphere. Further more, there are natural worlds other than our beloved biological one. Compared to the large-scale structure of galaxies, human bodies and their immediate environment consist of the "microscopic" world. The electromagnetic and nuclear forces which rule the world, however, disappear in the formation of large-scale galaxy structures. Similarly they disappear in the formation of the solar system. My previous papers found many evidences that galaxy structure is rational and determined by an algebraic cubic equation. This paper presents the nature's selection of the cubic roots and its application to the galaxy NGC 3275. The computational method is extremely simple, and the model can be deeply investigated and generalized.

2 My Model of Rational Galaxy Structure

My model of galaxy structure is very simple: galaxies are rational. I proposed the concept of rational structure. The density of material distribution on a plane is not arbitrary. For a special net of orthogonal curves on the plane, if the ratio of the material densities on both sides of any curve from the net is constant along the curve, the curve is called a proportion one. The net of curves is called an orthogonal net of proportion curves. Such a structure of material distribution is called a rational structure. Exponential disks of spiral galaxies are all rational. They have many nets of orthogonal proportion curves; the curves are golden spirals, i.e., logarithmic spirals (or equiangular spirals). My research showed that the bar of a barred galaxy is composed of two or three dual-handle structures. Each dual-handle structure is also a rational structure. Its orthogonal proportion curves are all confocal ellipses and hyperbolas [3-5]. A spiral arm is the perturbation to the galaxy body structure. In order to minimize the disturbance, the perturbation waves propagate along the orthogonal proportion curves or the non-orthogonal proportion curves [6]. However, spiral arms are usually broken-shaped (not connecting end to end), composed of segments which follow orthogonal or non-orthogonal proportion curves [6].

There has not been a mathematical proof if an exponential disk combined with several dual-handle structures is a rational one. However, I found the equation that the slopes of the tangential lines to the orthogonal proportion curves satisfy. Surprisingly, it is a cubic algebraic equation with one variable, called the instinct equation [6]. In other words, although I did not find a general mathematical solution to rational structure, I found a complete solution to the slopes of the tangential lines of orthogonal proportion curves. We can use this complete solution to analyze galaxy images. Firstly, I present the formulas of cubic roots.

Any rational structure on a plane must satisfy the following cubic algebraic equation [6], i. e., the instinct equation,

$$a(x,y)G^3 + b(x,y)G^2 + c(x,y)G + d(x,y) = 0 \tag{1}$$

where, (x, y) is the Cartesian coordinates on the plane. The coefficients a, b, c, d depend on the differentiations of the stellar density $\rho(x, y)$. Assuming the azimuthal angle of the tangential lines of the orthogonal proportion curves (i. e., the angle between the tangent direction and the x-axis) is α, then we have $G = \tan(2\alpha)$. That is, assuming the slope of the tangential lines is K, we have

$$G = \frac{2K}{1 - K^2} \tag{2}$$

To any point (x, y) on the galaxy plane corresponds a cubic equation (1). The roots of the equation are slopes of the tangential lines of the orthogonal proportion curves at that point. Each of the roots corresponds to a cross (i.e., the orthogonal tangential lines of the proportion curves at the point). If the equation has three roots, there are three crosses corresponding to the same point, forming a "snowflake" shape (see the Fig. 1 of [7]). In a barred galaxy, if multiple points are selected, we get a "snowflake map" for the galaxy. Through investigating the images of different barred galaxies, we found that galaxy snowflake maps present regular patterns. There are areas of the map where no snowflake exists. Instead a cluster of single crosses are present. That means the corresponding instinct equation has single real root. The snowflake maps suggest that one and only one global net of orthogonal proportion curves exists for each barred galaxy. It would be expected to have three global nets. The following Section presents the formula of the unique net of orthogonal proportion curves. This net is called the natural selection of galaxy cubic roots.

3 The Nature's Selection of Cubic Roots

Now we come to present the well known solution of the cubic algebraic equation. The number of real roots of the equation is determined by its discriminant. The discriminant is

$$\Delta = B^2 - 4A^3 \tag{3}$$

where

$$\begin{aligned} A &= b^2 - 3ac, \\ B &= 2b^3 - 9abc + 27a^2 d. \end{aligned} \tag{4}$$

If the discriminant is greater than zero, there is only one real root,

$$x_1 = -\left(b + \sqrt[3]{0.5\left(B+\sqrt{\Delta}\right)} + \sqrt[3]{0.5\left(B-\sqrt{\Delta}\right)}\right)/3a. \tag{5}$$

If the discriminant is less than zero, there are three real roots,

$$\begin{aligned} x_m = & -\left(b + 2\sqrt{A}\cos(2\pi m/3 + \phi/3)\right)/3a, \\ & \text{for} \quad m = 0, 1, 2 \end{aligned} \tag{6}$$

where $\phi = \arccos\left(B/2\sqrt{A^3}\right)$. If the discriminant is zero and $A = 0$, we have only one real root,

$$x_1 = -b/3a. \tag{7}$$

If the discriminant is zero and $A \neq 0$, we have two real roots,

$$x_1 = (9ad - bc)/2A, \;\; x_2 = (-b^3 + 4abc - 9a^2 d)/aA. \tag{8}$$

We humans do not have any reason to prefer one real root of the cubic equation to another. But the nature do. For the above model of galaxies, the nature prefers the root x_2 to the others in the case the discriminant is negative (see the formula (6)). For this assertion I do not have any strict mathematical proof. The evidence from the image analysis of nine barred galaxies is that the choice of other roots presents chaotic pattern of crosses on the barred galaxy plane, and can not be integrated to give a global net of orthogonal curves (See the Figures 1 and 2).

In the case the discriminant is zero and there correspond two real roots (see the formula (8)), the nature's choice is unknown. Fortunately, rational structure is very smooth which connects to complex analytic functions (harmonic functions). Therefore, the points on the galaxy plane that correspond to the case of zero discriminant can not make an area. Instead they consist of a few curves, called the critical lines of barred galaxies. The area of any curve is zero and has little impact on the numerical calculation of proportion curves. From now on, the third real root x_2 is called the natural root when the galaxy instinct equation has negative discriminant. The natural root plus the single real root which corresponds to positive discriminant should be integrable to give the unique global net of orthogonal proportion curves. We study this in the following Section.

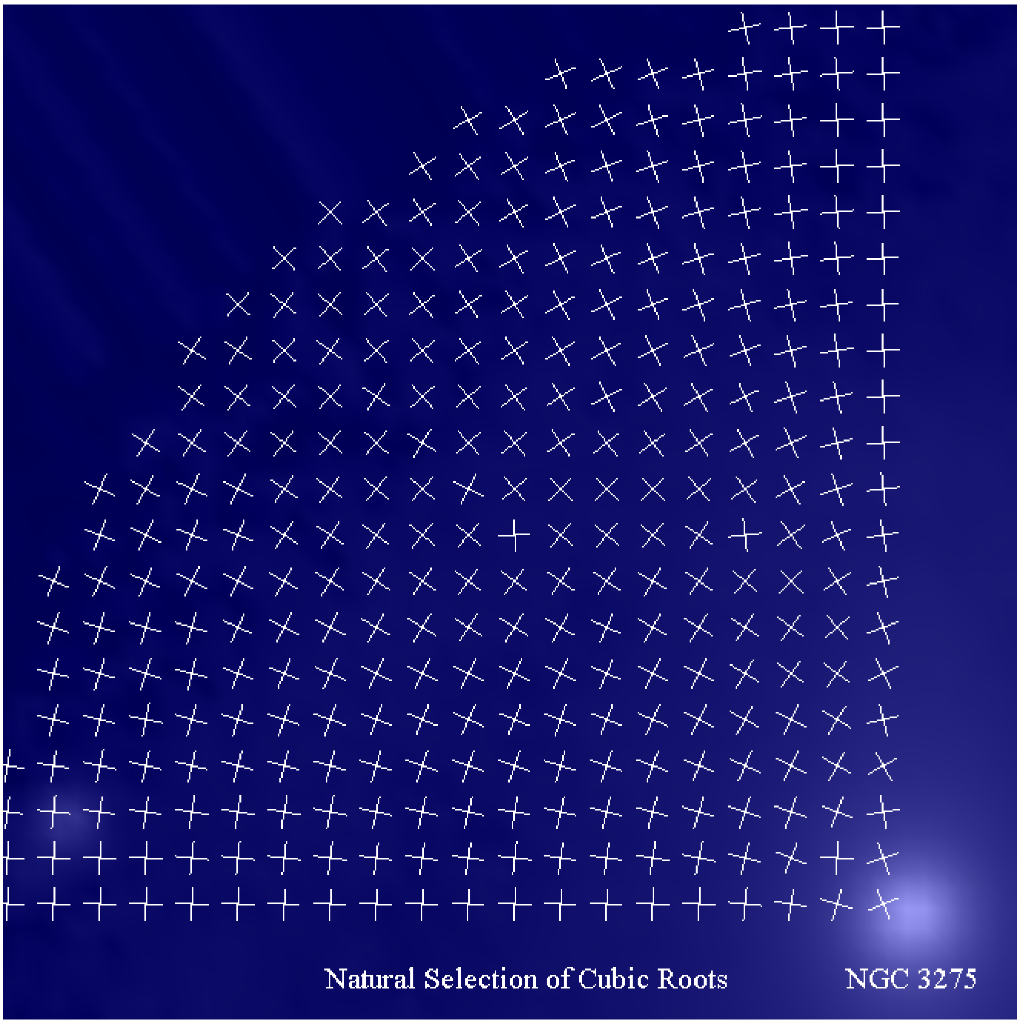

Figure 1: The natural selection of cubic roots whose cross distribution is orderly and promising to provide the unique global net of orthogonal proportion curves. The selection is $x_2 = -\left(b + 2\sqrt{A}\cos(4\pi/3 + \phi/3)\right)/3a$.

4 The Application of Natural Root

The root of instinct equation is supposed to give the slope $K(x, y)$ of the tangential lines of the orthogonal proportion curves (See the formula (2)). In numerical analysis, the Runge-Kutta method is an iterative method for the approximation of solutions of ordinary differential equation: $dy/dx = K(x, y)$. Fig. 3 is the results of the method applied to the galaxy NGC 3275. We see that the three integral curves C1, C2, C3 are not smooth. The possible reason is that the curves cross the areas where the formulas of the cubic roots are unstable (the denominators may tend to zero). If a integral curve represents the orthogonal proportion curve then the directional derivatives of the logarithmic stellar density along the perpendicular directions to the curve is constant along the curve [3-5]. Fig. 4 presents the derivatives corresponding to the curves in Fig. 3. The derivatives are very roughly constant, which suggests that the numerical calculation involved is very primitive. One reason is that the formulas of coefficients of the cubic equation are very complicated (see the formulas (19) through (22) in [6] and the computer program in the Appendix of [7]). A deep analysis of the numerical method and further reduction of the formulas are expected.

Now I give some Do-It-Yourself notes for layman. Rational structure is a simple concept and it admits a complete solution taking the form of algebraic equation, i.e., the instinct equation (1). Every layman can try a galaxy image to convince himself or herself that rational structure is true.

On the internet are many images of galaxies. Do not be confused with color images. Color is essentially the different frequencies or wavelengths of light. In fact, the shape of an object or its image is the distribution of light arriving at your eyes from the surface of the object. That is, it is the distribution of light frequency and density varying with the surface of the object. Light of longer wavelength that appears reddish has strong penetrating ability. In other words, reddish light refuses to be absorbed by dust or gas. Elliptical galaxies are very clean, with little observation of gas and dust. Therefore, it does not matter to catch which color for you to take the images of elliptical galaxies. Images of the same elliptical galaxy of different colors are very similar and smooth. They are the good demonstration of star distribution in the galaxy. But elliptical galaxies are three-dimensional while their images are two-dimensional. The image of an elliptical galaxy is the cumulative density of stars in the observing directions.

Spiral galaxies are just the opposite. They have a large amount of gas and dust. Although their structures are two-dimensional, they have a certain degree of thickness. Therefore, if we take images of spiral galaxies at the shorter wavelength (i.e., bluish light) then the light from the stars that are behind gas and dust are basically absorbed by the gas and dust. As a result, the image is more or less the distribution of gas and dust. Because the distribution of gas and dust is not smooth, the image looks ugly. Internet images of spiral galaxies are usually short-wavelength ones, therefore, people are daunted by the mysterious look of gas and dust. Therefore, to get an image of spiral galaxy which is mainly stellar density distribution, we take light of longer wavelength from the galaxy, e.g., infrared image. The resulting image is reddish. Although gas and dust have charming and bright colors, they have negligible mass.

Now you can ask for longer-wavelength images of barred galaxies from astronomers. The digital images should represent linearly the light density of galaxies. Most galaxies are inclined to the sky plane, therefore, you ask for de-projected galaxy images so that the galaxies look like face-on. What you ask for is, in fact, an array of real positive numbers which is proportional to the stellar density distribution, i.e., $\rho(x, y)$.

It is best to model galaxy bars and disks *visually* (I modeled bars firstly then the disks). Therefore, you may need some graphic softwares like Maple, Matlab, Mathematica, etc. Otherwise you need to know some computer language with graphic tools. My image analysis is made with

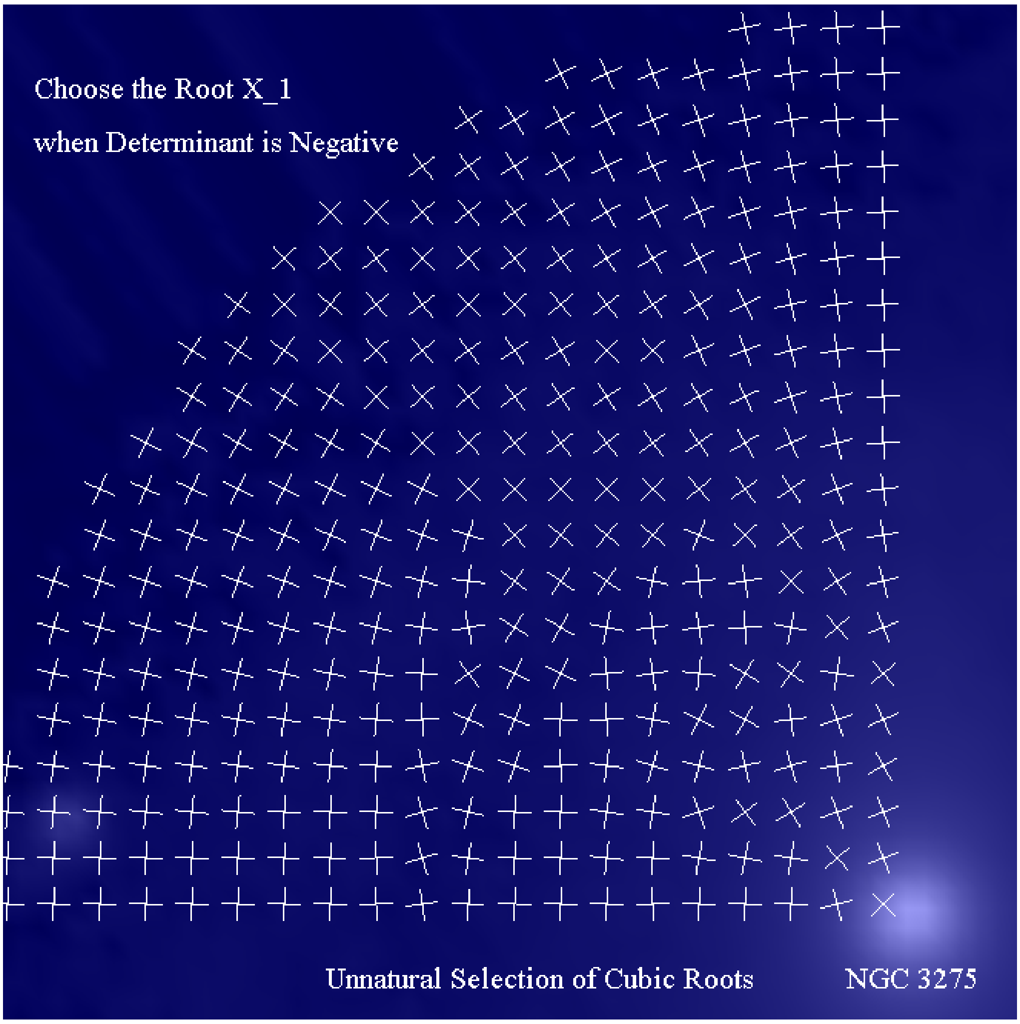

Figure 2: The unnatural selection of cubic roots whose cross distribution is chaotic and can not provide the unique global net of orthogonal proportion curves. The selection is $x_1 = -\left(b + 2\sqrt{A}\cos(2\pi/3 + \phi/3)\right)/3a$.

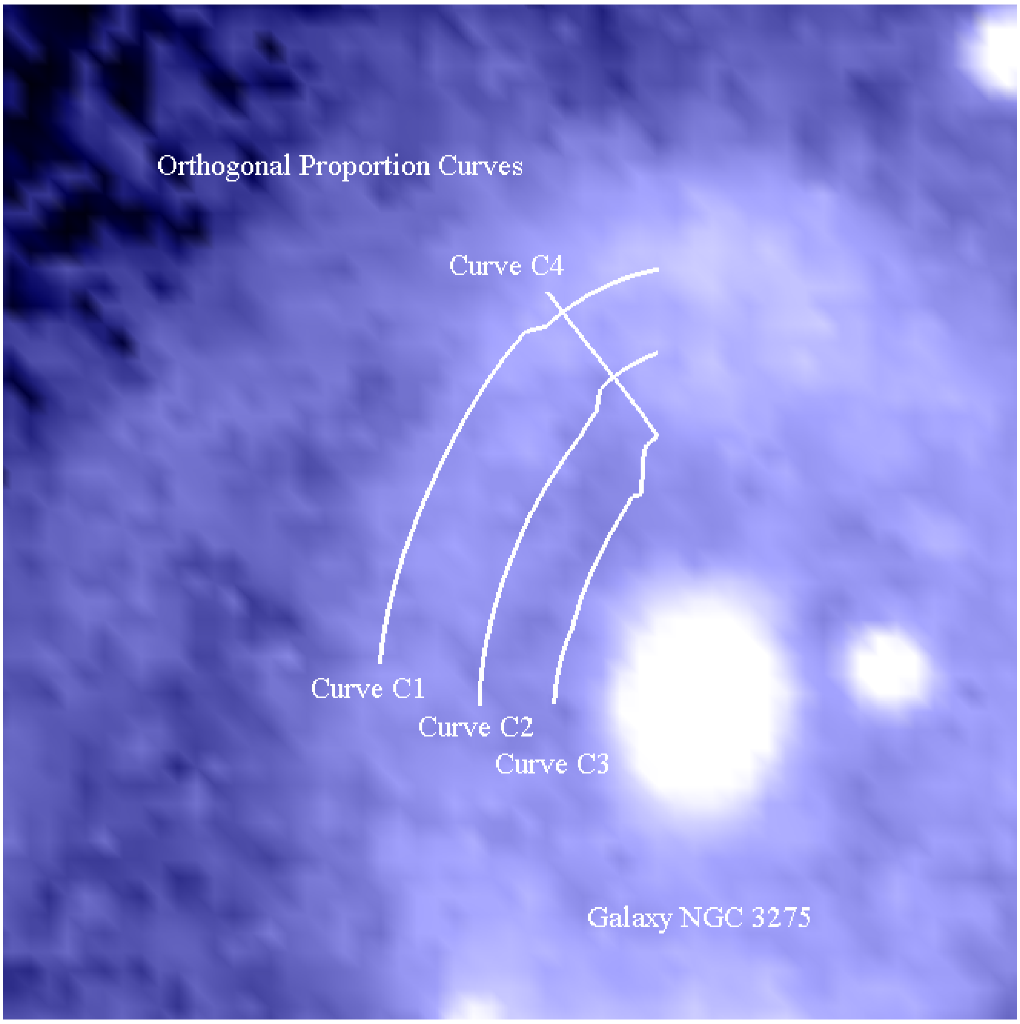

Figure 3: The integral curves of the slope $K(x, y)$ determined by the natural cubic root with the Runge-Kutta method. The curves are not smooth for the possible reason that the curves cross the areas where the formulas of the cubic roots are unstable (the denominators may tend to zero).

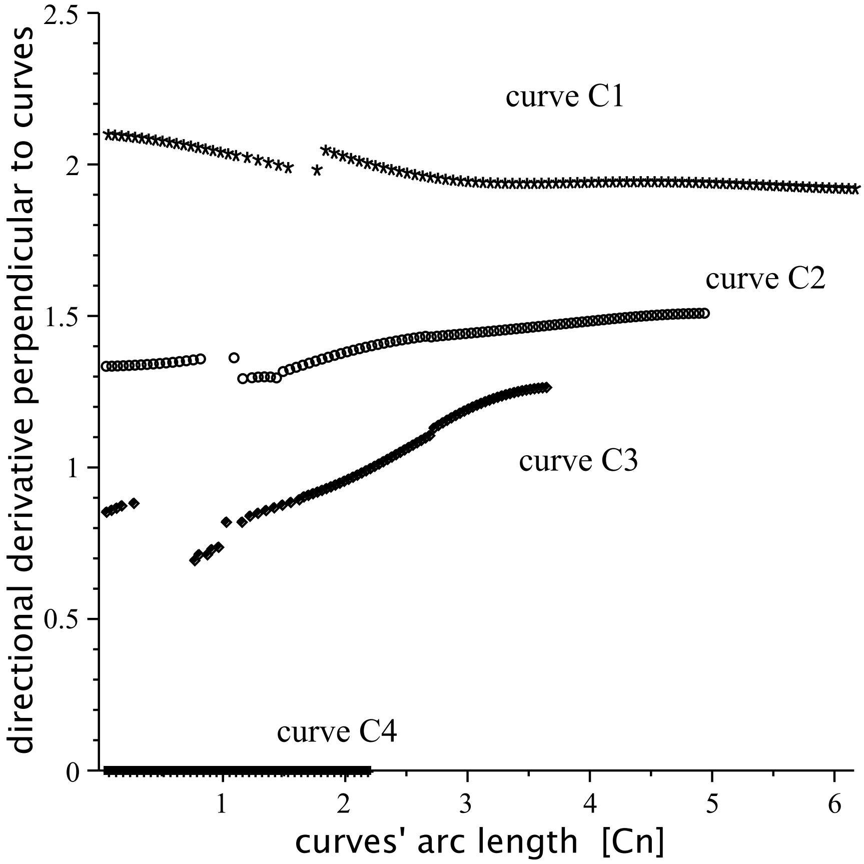

Figure 4: The directional derivatives of the logarithmic stellar density along the perpendicular directions to the curves in Fig. 3 are expected to be constant along the curves. The constants are rough, which should result from the complicated formulas of coefficients of the cubic equation (see the formulas (19) through (22) in [6] and the complicated computer program in the Appendix of [7]).

C++ language and OpenGL tools.

In summary, the first step of the numerical analysis is to find a long-wavelength image of a barred galaxy. In the second step, the body structure of the galaxy is fitted with an exponential disk plus two or three dual-handle structures. Here the body structure means the whole galaxy structure but ignoring the central bulge and the outer arms or rings. I found that the body structure can be well represented with a few parameters, which means that galaxies are rational. For galaxy NGC 3275, the fitting values for the parameters d_0, d_1 (disk), b_0, b_1, b_2 (the inner dual-handle), c_0, c_1, c_2 (the outer dual-handle) are respectively: 1500, -1.6, 134, 1.76, -0.2, 72, 3.25, -0.095. For their definition, see [3, 6]. These fitting values are very primitive and deserve strict refinement. In the third step, with the simulated (analytical) body structure of the galaxy, we can apply calculus to the analytical expression of the structure. At any point in the galaxy, the coefficients of instinct equation can be calculated by taking differentiation to the expression.

5 Future Work

In [3, 7], I listed 14 miracles of rational galaxy structure. The nature's selection of cubic roots can be considered the 15th miracle. It is noted that the selection is not directly the slope of the tangential lines of proportion curves. The relation between the cubic root G and the slope K is $G = 2K/(1 - K^2)$. With further investigation we expect more and more miracles. On the road of exploring the truth, we still have a long way to go. For example, we need to analyze images of more barred galaxies, and prove with each galaxy that galaxy rings follow orthogonal proportion curves while arms follow non-orthogonal proportion curves. We need to simplify the mathematical formulas for the roots of instinct equation. We should provide a strict mathematical proof that the summation of rational structures is still rational for spiral galaxies.

In fact, the scientific bridge between the galaxy study and the study of earthly structure is completely missing. One end of the bridge may be the scientific understanding of the connection between galaxy structure and Newton universal gravity. The other end is the true progress of astrobiology. I expect that an open study on galaxy structure can help the general public advance to the building of the bridge. By the time of its true progress, we may see a harmonic human society will come true.

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