[**pdf**] Submitted on Jan/20/2011

Particle characteristics energy and action of the free electromagnetic field

Authors: Xianzhao Zhong.

Comments: p11.

abstract: When discussing the free electromagnetic field, the author takes the electric field and magnetic field as two physical events on the time-space coordinate system. In line with the restricted theory of relativity, the author discusses the particle characteristics of the free electromagnetic field, and deems that the electromagnetic particle and the form of particle-captured energy are in perfect conformity with the Planck quantum assumption. In the ending part of the paper, the author has discussed the value of action exerted by dissimilar particles.

Category: Classical physics.

Particle characteristics energy and action of the free electromagnetic field

Xianzhao Zhong *

Meteorological college of Yunnan Province,

Kunming, 650228, China

Abstract

When discussing the free electromagnetic field, the author takes the electric field and magnetic field as two physical events on the time-space coordinate system. In line with the restricted theory of relativity, the author discusses the particle characteristics of the free electromagnetic field, and deems that the electromagnetic particle and the form of particle-captured energy are in perfect conformity with the Planck quantum assumption. In the ending part of the paper, the author has discussed the value of action exerted by dissimilar particles.

Key words: particle characteristics, field action.

PACS: 03.50.-E, 03.50.De.

* *E-mail address*: zhxzh46@163.com

1 Introduction

Starting from the Maxwell equation and restricted theory of relativity, the author takes the electric field $\mathbb{E}$ and magnetic field $\mathbb{B}$ as two independent physical events on the time-space coordinate system. Based on the theory of restricted relativity, we know that if two events absolutely coincide on a coordinate system, they should absolutely coincide even if the coordinate system has changed. For this reason, these two events might be taken as a single physical event. Therefore, it would be reasonable to deem the electromagnetic field as an electromagnetic particle that shows particle characteristics. Starting from the continuous equation of free electromagnetic energy and the covariant features of the restricted relativity, we derived action A of the electromagnetic particle, which is a constant that remains unchanged regardless of variations of the coordinate system. The energy form of particle acquired in this way would be in perfect conformity with the Planck quantum assumption, and its action $2A$ would be the Planck constant $\hbar$. A particle in the polarization state has fields that are mutually perpendicular in space, and shows $\pi/2$ phase difference in time. Hence we get the action $2A$ of a free electromagnetic particle. The action of other particles are respectively A and 0.

2 Particle Characteristics of the Free Electromagnetic Field

From free electromagnetic field Maxwell equations when $\rho = 0, \mathbb{J} = 0$ [1]

$$\nabla \times \mathbb{B} = 1/c\partial\mathbb{E}/\partial t,\ \nabla \times \mathbb{E} = -1/c\partial\mathbb{B}/\partial t. \tag{1}$$

By separating the spatial and temporal variants of the electric field $\mathbb{E}$ and the magnetic field $\mathbb{B}$, we get

$$\mathbb{E}(r,t) = \mathbb{E}(r)\mathrm{E}(t), \mathbb{B}(r,t) = \mathbb{B}(r)\mathrm{B}(t), \tag{2}$$

and based on (1), (2) may be changed into

$$\begin{aligned} &(1/\mathrm{E}(t))\ dB(t)/dt = -(c/\mathbb{B}(r)^2)\ \mathbb{B}(r) \cdot \nabla \times \mathbb{E}(r) = \Omega, \\ &(1/\mathrm{B}(t))\ dE(t)/dt = (c/\mathbb{E}(r)^2)\ \mathbb{E}(r) \cdot \nabla \times \mathbb{B}(r) = \Omega. \end{aligned} \tag{3}$$

Because Ω is constant, we let $\Omega = \omega$ for monochrome waves, then (3) many be changed into

$$\begin{aligned} d\mathrm{B}(t)/dt &= \omega\mathrm{E}(t), \\ \nabla \times \mathbb{E}(r) &= -\omega/c\ \mathbb{B}(r), \\ d\mathrm{E}(t)/dt &= \omega\mathrm{B}(t), \\ \nabla \times \mathbb{B}(r) &= \omega/c\ \mathbb{E}(r). \end{aligned} \tag{4}$$

As indicated by equation (4), the electric field $\mathbb{E}$ is perpendicular to the magnetic field $\mathbb{B}$ in a free electromagnetic field, all of which show a phase difference $\pi/2$, and have speed of light c is constant. We may now take the electric field $\mathbb{E}$ and the magnetic field $\mathbb{B}$ as two physical events in the coordinate system S. At instant t, events $\mathbb{E}_1$ and $\mathbb{B}_1$ would respectively take frame of axes (x_1, y_1, z_1, t_1) and (p_1, q_1, r_1, s_1) in the S. Based on equation (4), their spatial position would be $x_1 = p_1, y_1 = q_1$ and $z_1 = r_1$. $\mathbb{E}_1$ and $\mathbb{B}_1$ are two separate events occurred at the same spatial position. Based on equation (4), the time may take $t_1 = s_1$, and $\mathbb{E}_1$ and $\mathbb{B}_1$ are two separate events which occur simultaneously. Therefore, events $\mathbb{E}_1$ and $\mathbb{B}_1$ could be taken as an event couple which totally coincide and occur at the same time and same position in the S system. We now transform the coordinate system into S', then the event couple $\mathbb{E}_1$, $\mathbb{B}_1$ would absolutely coincide with $\mathbb{E}_1$ and $\mathbb{B}_1$, and could thus be regarded as one event in any coordinate system. Although the other event couple $\mathbb{E}_2$ and $\mathbb{B}_2$ could also be regarded as one event, yet it is an independent event comparing with event couple $\mathbb{E}_1, \mathbb{B}_1$. By now, we are fully confident to take the event couple $\mathbb{E}_1, \mathbb{B}_1$ as a free electromagnetic particle. Based on wave equation [2] $1/c^2\partial^2\mathbb{E}/\partial t^2 + 1/c^2\mathbb{C}\cdot\nabla\partial\mathbb{E}/\partial t - \nabla^2\mathbb{E} = 0$ and $1/c^2\partial^2\mathbb{B}/\partial t^2 + 1/c^2\mathbb{C}\cdot\nabla\partial\mathbb{B}/\partial t - \nabla^2\mathbb{B} = 0$, we know that a free electromagnetic field shows wave characteristics. Considering equation (4), we know that the event couple $\mathbb{E}_1$, $\mathbb{B}_1$ shows particle characteristics. The above equations illustrate that motion of a free electromagnetic field shows wave forms, suggesting that the free electromagnetic field exists in the form of particles.

3 Energy of the Electromagnetic Particles

For no source, electromagnetic field expressed by equations (1), $\mathbb{E}$ is used to scalar-multiply the first formula whereas $\mathbb{B}$ is used to scalar-multiply the second formula, hence we have

$$\partial(\mathbb{E}^2 + \mathbb{B}^2)/\partial t + \nabla\cdot C2\mathbb{E}\times\mathbb{B} = 0. \tag{5}$$

In the energy continuity equation (5) which illustrates the free electromagnetic field, $\mathbb{C}$ is used to denote the propagation velocity of the free electromagnetic field in time-space, whereas between $\mathbb{E}^2 + \mathbb{B}^2$ and $\mid 2\mathbb{E} \times \mathbb{B} \mid$ are used to denote the energy density, then we have continuity equation.

$$\partial(\mathbb{E}^2 + \mathbb{B}^2)/\partial t + \nabla \cdot \mathbb{C} \mid 2\mathbb{E} \times \mathbb{B} \mid = 0, \tag{6}$$

and energy density

$$\mathbb{E}^2 + \mathbb{B}^2 = \mid 2\mathbb{E} \times \mathbb{B} \mid . \tag{7}$$

In equation (6), the coordinate system S is transformed into system S'. Then we make frame of axes x, y, z and t respectively parallel with x', y', z' and t', and suppose the waves propagate along the x-axis. We further make $\beta = \sqrt{1 - v^2/c^2}$, and get [3] $\partial/\partial x'_\mu = \Lambda^\nu_\mu \partial/\partial x_\nu$, by using the Lorentz transformation, take unit vector i, j, k of the frame of axes S. Then equation (6) becomes

$$\begin{aligned} &\frac{1}{\beta}\frac{\partial[(\mathbb{E}^2 + \mathbb{B}^2) - v/c \mid 2\mathbb{E} \times \mathbb{B} \mid]}{\partial t'} + \frac{1}{\beta}\frac{\partial[c \mid 2\mathbb{E} \times \mathbb{B} \mid - v/c(\mathbb{E}^2 + \mathbb{B}^2)\,]}{\partial x'}\, i \\ &\qquad + \frac{\partial[c \mid 2\mathbb{E} \times \mathbb{B} \mid]}{\partial y'}\, j + \frac{\partial[c \mid 2\mathbb{E} \times \mathbb{B} \mid]}{\partial z'}\, k = 0. \end{aligned} \tag{8}$$

In the S' system, the energy continuity equation of the free electromagnetic field become

$$\partial(\mathbb{E}'^2 + \mathbb{B}'^2)/\partial t' + \nabla' \cdot \mathbb{C} \mid 2\mathbb{E}' \times \mathbb{B}' \mid = 0. \tag{9}$$

During the transformation of the coordinate system, the continuity equation shows covariant characteristics [4]. Comparing equations (7), (8) and (9) the energy density of electromagnetic field would be

$$\mathbb{E}'^2+\mathbb{B}'^2=1/\beta(1-v/c)(\mathbb{E}^2+\mathbb{B}^2). \tag{10}$$

In coordinate system S, the free electromagnetic field propagates along the x-axis, whereas $\mathbb{E}$ and $\mathbb{B}$ remain in the polarization state. Therefore, we know that field component exist without on the x-axis, and hence the free electromagnetic field would have energy $(\mathbb{E}^2+\mathbb{B}^2)dydz$, and its action is $(\mathbb{E}^2+\mathbb{B}^2)dydzdt$. In the coordinate system S', the energy $(\mathbb{E}'^2+\mathbb{B}'^2)dy'dz'$ and action are $(\mathbb{E}'^2+\mathbb{B}'^2)\mathrm{d}y'dz'dt'$. During the transformation of the coordinate system, we have

$$dt'=1/\beta(1+v/c)dt, \quad dy'=dy, \quad dz'=dz. \tag{11}$$

In equations (10) and (11), the action on the field is

$$(\mathbb{E}^2+\mathbb{B}^2)dydzdt=(\mathbb{E}'^2+\mathbb{B}'^2)dy'dz'dt', \tag{12}$$

and the total action on the field is

$$\iiint_\infty(\mathbb{E}^2+\mathbb{B}^2)dydzdt=\iiint_\infty(\mathbb{E}'^2+\mathbb{B}'^2)dy'dz'dt', \tag{13}$$

in which the action remains unchanged during the transformation of the coordinate systems, and we know that a wave would show the Doppler frequency drift during the transformation of the coordinate systems. Action (13) remains unchanged for any coordinate systems, because it has nothing to do with the selected coordinate systems and the frequencies, and thus equation (13) holds for any free electromagnetic wave. Therefore, we may take (13) as a physical constant, and make

$$A = \iiint_{\infty} (\mathbb{E}^2 + \mathbb{B}^2) dy\ dz dt. \tag{14}$$

Now, we may consider the energy of an electromagnetic particle. From electric field $\mathbb{E}$ continuity equation and magnetic field $\mathbb{B}$ continuity equation [2], $\partial \mathbb{E}/\partial t - \nabla \cdot \mathbb{C}\mathbb{E} = 0$, and $\partial \mathbb{B}/\partial t - \nabla \cdot \mathbb{C}\mathbb{B} = 0$ from (3) angular frequency ω, considering (2) separating the spatial and temporal variants of the field, we have

$$dE(t)/dt = \omega E(t),\ dB(t)/dt = \omega B(t), \tag{15}$$

$\mathbb{E}(r)^2\mathrm{E}(t)$ and $\mathbb{B}(r)^2\mathrm{B}(t)$ are respectively used to multiply equation (15), we have

$$\begin{aligned} d\mathbb{E}(r)^2\mathrm{E}(t)^2/dt &= 2\omega \mathbb{E}(r)^2\mathrm{E}(t)^2 = 2\omega \mathbb{E}^2, \\ d\mathbb{B}(r)^2\mathrm{B}(t)^2/dt &= 2\omega \mathbb{B}(r)^2\mathrm{B}(t)^2 = 2\omega \mathbb{B}^2. \end{aligned} \tag{16}$$

Therefore, the energy of a free electromagnetic particle would be

$$\mathcal{E} = \iiint_{\infty} d(\mathbb{E}^2 + \mathbb{B}^2)/dt dy dz dt = 2\omega \iiint_{\infty} (\mathbb{E}^2 + \mathbb{B}^2) dy dz dt. \tag{17}$$

Substituting (14) into the above equation, we have

$$\mathcal{E} = 2A\omega. \tag{18}$$

Thus we know that the energy of an free electromagnetic particle is in proportion with the angular frequency ω, and that the proportional constant is $2A$. If we compare (18) with the Planck quantum hypothesis [5]

$$\mathcal{E} = \hbar\omega, \tag{19}$$

then the proportional constant is $2A = \hbar$.

4 Action of the Particles

In Cartesian coordinate system S, the electromagnetic particle would move along the x-axis. The polarized field (4) $\mathbb{E}$ component would fall on the y -axis and the polarized field $\mathbb{B}$ component is on the z-axis. Field $\mathbb{E}$ would be perpendicular with field $\mathbb{B}$, and a phase difference $\pi/2$ would be detected. At time t_1, $\mathbb{E}$ is recorded as E_y along the $+y$ direction, and $\mathbb{B}$ is recorded as B_z along $+z$ direction. At time t_2, $\mathbb{B}$ takes the $+z$ direction B_z and $\mathbb{E}$ the $-y$ direction, hence we have $\mathrm{E}_{-y} = -\mathrm{E}_y$. At time t_3, $\mathbb{E}$ takes the $-y$ direction $\mathrm{E}_{-y} = -\mathrm{E}_y$ and $\mathbb{B}$ the $-z$ direction, then we have $\mathrm{B}_{-z} = -\mathrm{B}_z$. At time t_4, $\mathbb{B}$ takes the $-z$ direction $\mathrm{B}_{-z} = -\mathrm{B}_z$ and $\mathbb{E}$ restores its $+y$ direction E_y. During the next cycle, fields $\mathbb{E}$ and $\mathbb{B}$ would repeat the above processes. Now we have the following expression for changes of an electromagnetic particle on the $y-$ and $z-$ axis, take unit vector j and k. Within a single period $\mathbf{T}$

$$\begin{aligned}
t_1, \quad & \mathbb{S}_1 = (\mathrm{E}_y \ \mathrm{B}_z)\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (\mathrm{E}_y\mathrm{j} + \mathrm{B}_z\mathrm{k});\\
t_2, \quad & \mathbb{S}_2 = (\mathrm{E}_{-y} \ \mathrm{B}_z)\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (-\mathrm{E}_y\mathrm{j} + \mathrm{B}_z\mathrm{k});\\
t_3, \quad & \mathbb{S}_3 = (\mathrm{E}_{-y} \ \mathrm{B}_{-z})\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (-\mathrm{E}_y\mathrm{j} - \mathrm{B}_z\mathrm{k});\\
t_4, \quad & \mathbb{S}_4 = (\mathrm{E}_y \ \mathrm{B}_{-z})\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (\mathrm{E}_y\mathrm{j} - \mathrm{B}_z\mathrm{k}).
\end{aligned} \tag{20}$$

Or expressed in terms, we have it's the numeric value

$$
\begin{aligned}
&t_1, \quad S_1 = \mid \mathrm{E}_y\mathrm{j} \times \mathrm{B}_z\mathrm{k} \mid; \\
&t_2, \quad S_2 = \mid \mathrm{E}_{-y}\mathrm{j} \times \mathrm{B}_z\mathrm{k} \mid; \\
&t_3, \quad S_3 = \mid \mathrm{E}_{-y}\mathrm{j} \times \mathrm{B}_{-z}\mathrm{k} \mid; \\
&t_4, \quad S_4 = \mid \mathrm{E}_y\mathrm{j} \times \mathrm{B}_{-z}\mathrm{k} \mid .
\end{aligned} \tag{21}
$$

Taking a single period **T** we have

$$
S = S_1 + S_2 + S_3 + S_4. \tag{22}
$$

Based on (7) and (14), the gross action exerted by this free electromagnetic particle is

$$
\mathbf{T:} \quad 2A = \iiint_\infty 2(\mathrm{E}_y^2 + \mathrm{B}_z^2)dydzdt. \tag{23}
$$

For a charged particle with a polarization field, and suppose that such a charged particle has but only one field $\mathbb{F}$ which component is distributed on the $y-$ and $z-$ axis, we have

$$
\begin{aligned}
&t_1, \quad \mathbb{S}_1 = (\mathrm{F}_y \ \mathrm{F}_z)\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (\mathrm{F}_y\mathrm{j} + \mathrm{F}_z\mathrm{k}); \\
&t_2, \ \mathbb{S}_2 = (\mathrm{F}_{-y} \ \mathrm{F}_{-z})\begin{pmatrix}\mathrm{j}\\ \mathrm{k}\end{pmatrix} = (-\mathrm{F}_y\mathrm{j} - \mathrm{F}_z\mathrm{k}).
\end{aligned} \tag{24}
$$

Or expressed in terms, we have it's the numerical value

$$
t_1, \quad S_1 = \mid \mathrm{F}_y\mathrm{j} \times \mathrm{F}_z\mathrm{k} \mid, t_2, \quad S_2 = \mid \mathrm{F}_{-y}\mathrm{j} \times \mathrm{F}_{-z}\mathrm{k} \mid = \mid \mathrm{F}_y\mathrm{j} \times \mathrm{F}_z\mathrm{k} \mid . \tag{25}
$$

This particle would exert the action

T:

$$A = \iiint_{\infty} (\mathrm{F}_y^2 + \mathrm{F}_z^2) dydzdt. \tag{26}$$

Under the supposition that a particle has taken a but only one polarization field $\mathbb{F}$ which component falls exclusively on the only the y-axis, then we have

$$\mathbb{S} = (\mathrm{F}_y \ \mathrm{F}_{-y}) \begin{pmatrix} \mathrm{j} \\ \mathrm{j} \end{pmatrix} = (\mathrm{F}_y \mathrm{j} - \mathrm{F}_y \mathrm{j}) = 0. \tag{27}$$

Or for this particle, we have

$$S = \mid \mathrm{F}_y \times \mathrm{F}_{-y} \mid = 0. \tag{28}$$

This particle would exert the action

T:

$$A = \iint_{\infty} (\mathrm{F}_y^2 - \mathrm{F}_y^2) dydt = 0. \tag{29}$$

If a particle has taken 2 polarization field, one of which is the dextral field $\mathbb{F}_R$ and the other the sinistral field $\mathbb{F}_L$, then this particle would exert action A_R and A_L, whereas the gross action of this particle is **T:**

$$A = A_R + A_L = 0. \tag{30}$$

If a particle has taken many different polarization field $\mathbb{F}$, then this particle would exert actions $2A$, A and 0.

References

[1] E.M. Purcell. *ELECTRICITY AND MAGNETISM*. McGraw-Hill, Inc., 1965.

[2] X.Z. Zhong. *ELECTROMAGNETIC FIELD EQUATION AND FIELD WAVE EQUATION*. preprint, 2010.

[3] S. Weinberg. *GRAVITATION AND COSMOLOGY*. John Wiley, Inc., 1972.

[4] J.D. Jackson. *CLASSICAL ELECTRODYNAMICS*, 3rd ed. John-Wiley & Sons, Inc., 1999.

[5] L.I. Schiff. *QUANTUM MECHANICS*. 3rd ed. McGraw-Hill Book Co. Inc., 1958.