The k-Number Sieve and k-Inclusion-exclusion Formula,

Principle and Harvest

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Abstract: 1-Number Sieve: It is the Eratosthenes’-Number Sieve and the da Silva-Sylvester formula. 2-Number Sieve: We can obtain result of Goldbach’ conjecture and the number of solutions of Goldbach problem. 3-Number Sieve: We can obtain result $p_3$ in $N= p_3+p_i P_1$ and 3-Inclusion-exclusion formula. ($p_i < \sqrt{N}, P_1 > \sqrt{N}$.)

1 The principle.

N: Even number. This paper to discuss the $N \geq 50$. ($r \geq 4$.)

$p_i, p_r, p_{r+1}$: Prime, $2 \leq p_i \leq p_r < \sqrt{N} < p_{r+1} < N$. $i=1,2,...,r$, $r=\pi(\sqrt{N})$.

$[p], [p_1, p_2], [p_3], [p_4]$: A prime lying in the interval $[p_r+1, N-p_r-1]$.

“1+1”: $N= p_1+p_2$. (Goldbach’ conjecture.)

“1+2”: $N= p_1+p_i P_1= p_3+p_i P_1= p_4+p_i P_2$. (prime $P_1 > p_r$, prime $P_2 < p_r$.) This paper to discuss the $N= p_3+p_i P_1$.

If the $N= N_i + n_i p_i$, $N_i \neq 0$, We have $p= N_i + n_i p_i$.

If the $N= N_i + n_i p_i P_1$, $N_i \neq 0$, We have $p= N_i + n_i p_i P_1$.

Diagram 1 is the principle of k-Number Sieve. (When $N=152$.)

Natural number 1 ~ 152

1-Number Sieve: —— ↓ —— Harves: ① $p=13 \sim 139$. ② 1-Inclusion-exclusion formula.

Out: $n p_i$ ($n \geq 2$.)

2-Number Sieve: —— ↓ —— Harves: ① $p_1, p_2$. ② 2-Inclusion-exclusion formula.

Out: $N_i + n_i p_i = \text{prime}$

3-Number Sieve: —— ↓ —— Harves: ① $p_3$. ② 3-Inclusion-exclusion formula.

Out: $N_i j + n_i p_i p_j = \text{prime}$

(Analyse)

Diagram 1 the principle of k-Number Sieve

2 1-Number Sieve.

Object:

Natural number 1 ~ 152.

Out:

$n p_i$. $1 \leq i \leq r$, $n \geq 2$.

Harvest:


② 1-Inclusion-exclusion formula. (da Silva-Sylvester formula.)

Literature:

① Textbook.

2-Number Sieve.

**Object:**

**Out:**
\[ N_i + np_i = \text{prime}, \quad 1 \leq i \leq r. \]
They are:
\[ 2+3n = [17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137. ] \]
\[ 2+5n = [17, 37, 47, 67, 97, 107, 127, 137. ] \]
\[ 5+7n = [19, 47, 61, 89, 103, 131. ] \]
\[ 9+11n = [31, 53, 97. ] \]
Positioning for size: [17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 83, 89, 97, 101, 103, 107, 113, 127, 131, 137. ]

**Harvest:**
①\( N = p_1 + p_2. \) They are: 13, 43, 73, 79, 109, 139.
And: \( 152 = 13 + 139 = 43 + 109 = 73 + 79 + 73 = 109 + 43 = 139 + 13. \)
②2-Inclusion-exclusion formula.

**Literature:**
(\[ \text{(http://vixra.org/abs/1007.0045)} \])
③Goldbach’ Conjecture (4): The expression of the number of Goldbach’ Primes.
(\[ \text{(http://vixra.org/abs/1007.0046)} \])
(\[ \text{(http://vixra.org/abs/1007.0049)} \])
⑤Goldbach’ Conjecture (8): Upper Bound Estimation of Number of Goldbach’ Primes.
(\[ \text{(http://vixra.org/abs/1008.0064)} \])
⑥Goldbach’ Conjecture (9): Proved Hardy-Littlewood Conjecture (A)
(\[ \text{(http://vixra.org/abs/1008.0088)} \])
⑦Goldbach’ Conjecture (10): The Six Details in the Hardy-Littlewood Conjecture (A).
(\[ \text{(http://vixra.org/abs/1012.0004)} \])

3 3-Number Sieve.

**Object:**
\[ N_i + np_i = \text{prime}, \quad 1 \leq i \leq r. \]
They are: [17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 73, 79, 83, 89, 97, 101, 103, 107, 113, 127, 131, 137. ]

**Out:**
\[ N_j + np_j = \text{prime}, \quad 1 \leq i < j \leq r. \]
They are:
\[ 8+3 \times 3n = [17, 53, 71, 89, 107. ] \]
\[ 2+3 \times 5n = [17, 47, 107, 137. ] \]
\[ 5+3 \times 7n = [47, 89, 131. ] \]
2+5×5n=[127.]
20+3×11n=[53.]
12+5×7n=[47.]
54+7×7n=[103.]
42+5×11n=[97.]
75+7×11n=[0]
31+11×11n=[0]

Positioning for size: [17, 47, 53, 71, 89, 97, 103, 107, 131, 137.]

**Harvest:**

They are: 152=19+7×19=23+3×43=29+3×41=31+11×11=37+5×23=41+3×37=59+3×31=61+7×23=67+5×17=83+3×23=101+3×17=113+3×13. (P1>p_.)

② 3-Inclusion-exclusion formula.

**Literature:**

① When N>10, the N as the sum of a prime and the product of two primes…
(http://prep.istic.ac.cn/eprint/operat/showfile.jsp?d=FILEINFO&org=1134633007022&r=1134633007022)
② The number of solution for N=p_a+p_b p_c.
(http://prep.istic.ac.cn/eprint/Upload//2010/1268703639030.doc)
(http://www.mathchina.com/cgi-bin/topic.cgi?forum=12&topic=1430&show=550)

5 The primes in N_{ij}+n p_i.
The primes in N_{ij}+n p_i: [17, 47, 53, 71, 89, 97, 103, 107, 131, 137.]
17: N=17+3×3×5=“1+4”.
47: N=47+3×5×7=“1+3”.
53: N=53+3×3×11=“1+3”.
71: N=71+3×3×3×3=“1+4”.
89: N=89+3×3×7=“1+3”.
97: N=97+5×11=“1+2” (=p_4+p_1 P_2).
103: N=103+7×7=“1+2” (=p_4+p_1 P_2).
107: N=107+3×3×5=“1+3”.
131: N=131+3×7=“1+2” (=p_4+p_1 P_2).
137: N=137+3×5=“1+2” (=p_4+p_1 P_2).
They are: four “1+2” (=p_4+p_1 P_2, P_2<p_.), four “1+3”, two “1+4”.

6 Conclusion.
When 1≤i≤r, if (p, p_)=1, then p is a prime. The 1-Inclusion-exclusion formula is da Silva-Sylvester formula.
When 1≤i≤r, if p_1, p_2≠N_i+n p_i, then N= p_1+p_2=“1+1”, and we can obtain 2-Inclusion-exclusion formula.
When 1≤i<j≤r, if p_3=N_i+n p_i≠N_j+n p_j p_i, then N= p_3+p_1, P_1=“1+2”, and we can obtain 3-Inclusion-exclusion formula. (P_1>p_.)
When 1≤i<j<r, if p_4=N_i+n p=N_j+n p_j, then N= p_4+p_1, P_2=“1+2”, but we can not obtain 4-Inclusion-exclusion formula. (P_2<p_.)

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