BINARY PULSARS. ORBITAL PRECESSION AND DECAY

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ABSTRACT

Measurement of the precession and orbital decay of binary pulsars is said to support the General Theory of Relativity. This is true for the rate of precession but the rate of decay is said to be due to energy loss. The loss of energy alone cannot account for the decay. In some cases it is attributed to gravitational energy loss and in others it is due to tidal drag. The quoted theory for the decay is based on Newtonian dynamics but it is not applicable in these cases because the equations used are derived assuming that the energy is constant and the orbits are conical. This article gives justification to this comment, however this does not mean that gravitational radiation is not the cause of binary decay.

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INTRODUCTION

For the binary pulsar PSR 1913+16, which was discovered by Hulse and Taylor in 1974, (see reference [9]), the accepted data is that the masses of the two stars are 1.441 and 1.387 times the mass of the Sun, the semi-major axis is 1,950,100 km, the eccentricity is 0.617131 and the orbital period is 7.751939106 hr. Using the equation which was developed in reference [1] for calculating the precession of the perihelion of Mercury per orbit,

$$\boxed{\begin{array}{c}\theta_p = \dfrac{6\pi G(m_1+m_2)}{c^2 a\ (1-e^2)} \quad \ldots\ldots \\ \text{where } a \text{ is the semi-major axis}\end{array}}$$

we obtain the result 4.22 deg/yr. This is in agreement with the measured value and that predicted by General Relativity.

Clifford Will [3] says that for a binary pulsar the most important reason for orbital period decay is due to the energy loss resulting from gravitational wave emission. Paul Davies [11] states " The steady power drain will therefore cause the stars to spiral slowly together." Ignazio Ciufolini [4] says "…and the rate of decrease of the orbital period, explained by the loss of energy by gravitational radiation." Misner, Thorne and Wheeler [6] state, " As a binary system loses energy by gravitational radiation , the stars spiral in towards each other " Similar statements are made by several other authors.

Consider a spacecraft in circular orbit about the Earth. Assume that it fires its rocket for a short period, or simply ejects a mass, in the direction of motion then the speed of the spacecraft will be reduced. If after half an orbit the rocket is fired again then the spacecraft will be in a lower circular orbit with a shorter period. If the rocket fires to the rear then the size of the orbit will increase and the period becomes greater. The energy loss and the mass loss from the spacecraft will be the same in both instances

For a binary system with two equal masses in circular orbit the gravitational effect depends on the product of the two masses but the centripetal mass acceleration is proportional to the sum of the masses of the bodies. So, if the mass of each body reduces suddenly in a symmetric way, so as not to change the velocity, the gravity effect will reduce more than the centripetal effect, meaning that the size of the orbit will increase not decrease.

However, if there is a radiation pressure which is greater on the leading surface then this would cause a drag thus reducing the size of the orbit. Is there any evidence for gravitational radiation pressure similar to the electromagnetic form? Is energy loss associated with loss of mass? That gravity waves exist is a possibility but since they have yet to be measured any evidence of pressure is even less likely.

ENERGY LOSS AND SPIRAL ORBITS

Kepler's 3rd law is written $\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM}$ thus differentiating with respect to time yields $\frac{1}{P}\frac{dP}{dt} = \frac{3}{2a}\frac{da}{dt}$.

Where *P* is the orbital period, *M* is the sum of the masses and *a* is the semi-major axis. With $\mu = m_1 m_2 / (m_1 + m_2)$ being the reduced mass the energy is,

$$E = -\frac{GM\mu}{2a}$$ so differentiating with respect to time

gives $$\frac{1}{E}\frac{dE}{dt} = -\frac{1}{a}\frac{da}{dt}$$. Eliminating d*a*/d*t* leads to

$$\frac{1}{P}\frac{dP}{dt} = -\frac{3}{2E}\frac{dE}{dt} \ .$$

which is the equation quoted by C. M. Will [3]. Here the mass is considered to be constant. This equation certainly is applicable when considering two separate systems but is it true when applied to a single system changing its orbit ? Kepler's 3rd law and the expression for energy were both devised for steady elliptic orbits with constant kinetic plus potential energy.

Misner, Thorne and Wheeler [6] quote similar equations.

From a paper by Peters and Mathews [13] the rate of energy change due gravitational radiation is given by the following equation, which is based on General Relativity,

$$\frac{dE}{dt} = -\frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5} f(e) = -\frac{32G^4 \mu^2 M^3}{5c^5 a^5} f(e)$$

where $f(e) = \frac{1 + (73/24)e^2 + (37/96)e^4}{(1-e^2)^{7/2}}$.

Using Kepler's 3rd Law and the expression for energy, as shown above, we obtain

$$\frac{dP}{dt} = -\frac{192\pi G^{5/3}}{5c^5}\left(\frac{P}{2\pi}\right)^{-5/3} \mu M^{2/3} f(e)$$

which is given in the paper by Weisberg and Taylor [14].
The same result is derived by Will[3] but in a different format,

$$\frac{1}{P}\frac{dP}{dt} = -\frac{96G^3}{c^5 a^4}\mu M^2 f(e) \quad .$$

In the cases considered the use of equations derived for constant energy systems is inapplicable, even though the mathematics is correct.

MASS LOSS OR GAIN.

Consider a binary system in an elliptic orbit with semi-major axis a_o and total mass M_o . At a time when the separation is r the mass suddenly changes to M so as not to change the relative speed. It can be shown that the system will then be in an elliptic orbit with semi-major axis a related by the following equation,

$$\frac{r}{a} - \frac{r}{a_0} = \left(1 - \frac{M_0}{M}\right)\left[2 - \frac{r}{a_0}\right].$$

For an elliptic orbit the term in the square brackets is always positive because r cannot be greater than $2a_o$, therefore if $M < M_o$ then $a > a_o$.

MOMENT OF MOMENTUM AND TIDAL DRAG

E. R. Adams et al [10] report on the extra solar planet OGLE-TR-113b and discuss the assumption that the orbital decay is due to tidal energy dissipation. No theory is presented in the paper.

The moment of momentum, assumed to be constant, can readily be shown to be $L = (I_1\omega_1 + I_2\omega_2) + \mu\Omega r^2$, where Ω is angular velocity of the bodies rotating about each other and ω is the spin of the individual bodies. For elliptical orbits this equation may be expressed in terms of the semi-major axis a and the eccentricity e to give

$$L = (I_1\omega_1 + I_2\omega_2) + \mu\sqrt{GMa(1-e^2)}$$

Alternatively, let S be the separation when $\ddot{r} = 0$, then because $-\frac{GM\mu}{S^2} = -\mu\Omega^2 S$

$$L = (I_1\omega_1 + I_2\omega_2) + \mu\sqrt{GMS}$$

For a constant mass system, if the spin decreases then the separation will increase. Tidal drag will cause the spin rate to tend towards the orbital rate, so if the spin is greater than the orbital then the separation will increase. This is usually the case and is certainly true for Earth and Moon. If the spin is less, or in the opposite sense, to the orbital rate then the separation could decrease.

CONCLUSION

If we have two bodies in a stable orbit then if there is energy loss it does not follow that the orbit will increase or decrease, it depends on how the energy is lost. Further, if the energy loss is associated with mass loss then this must be taken into account. It could well be that gravitational energy loss and orbital decay are related by means of radiation pressure. It is a possibility that the binary stars are moving in a dust cloud so that they accumulate mass resulting in spiral decay.

In most cases energy loss due to tidal drag results in an outwards spiral as usually the spin and the orbital rotation are in the same sense. But if the spin is of opposite sign, or less than Ω, then the bodies could spiral inwards.

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