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**Title**

# 32 point groups of three dimensional crystal cells described by 5 bits

**Abstract**

There are 32 possible combinations of symmetry operations that define the external symmetry of crystals. These 32 possible combinations result in the 32 crystal classes.

But for a radar engineer it is inevitable to associate “32” to “5 bits”.

I submit a tentative classification of the 32 crystal classes with a 5 bit classification, obviously with a (tentative) physical meaning of each bit.

Each bit means a physical property.

## Introduction

As is well known to any mineralogist or mathematician [1], [2], there are only 32 crystal classes. These are mathematically 32 *point groups.*

For an electronic engineer is inevitable to associate "32" with "5 bit". This means making the assumption that there are only five basic properties. Their presence or absence will automatically determine which class or *point group* of origin.

## 5 bit classification

After lengthy analysis and various attempts have come to propose a possible classification given in the table below. The 32 crystal classes are identified with *Ornifold* and *Hermann Mauguin* symbols and finally with 5-bit I have provisionally indicated *Bettini* symbols.

**CRYSTAL CLASS – 5 bit - 4 3 2 m c**

| **Orbifold** | **Hermann Mauguin** | **Bettini** | | |
|---|---|---|---|---|
| 11 | 1 | 00000 | 0 | triclinic |
| 1x | $\underline{1}$ | 0000c | 1 | triclinic |
| 1* | m | 000m0 | 2 | monoclinic |
| 2* | 2/m | 000mc | 3 | monoclinic |
| | | | | |
| 22 | 2 | 00200 | 4 | monoclinic |
| 222 | 222 | 0020c | 5 | orthorhombic |
| *22 | mm | 002m0 | 6 | orthorhombic |
| *222 | mmm | 002mc | 7 | orthorhombic |
| | | | | |
| 33 | 3 | 03000 | 8 | exagonal |
| 3x | $\underline{3}$ | 0300c | 9 | exagonal |
| 3* | $\underline{6}$ | 030m0 | 10 | exagonal |
| 2*3 | $\underline{3}$m | 030mc | 11 | exagonal |
| | | | | |
| 66 | 6 | 03200 | 12 | exagonal |
| 223 | 32 | 0320c | 13 | exagonal |
| *223 | $\underline{6}$2m | 032m0 | 14 | exagonal |
| *33 | 3m | 032mc | 15 | exagonal |
| | | | | |
| 44 | 4 | 40000 | 16 | tetragonal |
| 2x | $\underline{4}$ | 4000c | 17 | tetragonal |
| 332 | 23 | 400m0 | 18 | isometric |
| 2*2 | $\underline{4}$2m | 400mc | 19 | tetragonal |
| | | | | |
| 224 | 422 | 40200 | 20 | tetragonal |
| 4* | 4/m | 4020c | 21 | tetragonal |
| *44 | 4mm | 402m0 | 22 | tetragonal |
| *224 | 4/mmm | 402mc | 23 | tetragonal |
| | | | | |
| 226 | 622 | 43000 | 24 | exagonal |
| 6* | 6/m | 4300c | 25 | exagonal |
| *66 | 6mm | 430m0 | 26 | exagonal |
| *226 | 6/mmm | 430mc | 27 | exagonal |
| | | | | |
| 432 | 432 | 43200 | 28 | isometric |
| 3*2 | m3 | 4320c | 29 | isometric |
| *332 | $\underline{4}$3m | 432m0 | 30 | isometric |
| *432 | m$\underline{3}$m | 432mc | 31 | isometric |

I add the Schoenflies notation.
The correspondence between the notations is:

| **Bettini** | **Hermann-Mauguin** | **Schoenflies** | **Orbifold** |
|---|---|---|---|
| 0 | 1 | $C_1$ | 11 |
| 1 | $\underline{1}$ | $S_2$ | 1x |
| 2 | m | $C_{1h}$ | 1* |
| 3 | 2/m | $C_{2h}$ | 2* |
| 4 | 2 | $C_2$ | 22 |
| 5 | 222 | $D_2$ | 222 |
| 6 | mm2 | $C_{2v}$ | *22 |
| 7 | mmm | $D_{2h}$ | *222 |
| 8 | 3 | $C_3$ | 33 |
| 9 | $\underline{3}$ | T | 332 |
| 10 | $\underline{6}$ | $C_{3h}$ | 3* |
| 11 | $\underline{3}$m | $D_{3d}$ | 2*3 |
| 12 | 6 | $C_6$ | 66 |
| 13 | 32 | $D_3$ | 223 |
| 14 | $\underline{6}$2m | $D_{3h}$ | *223 |
| 15 | 3m | $C_{3v}$ | *33 |
| 16 | 4 | $C_4$ | 44 |
| 17 | $\underline{2}$ | $S_4$ | 2x |
| 18 | 23 | T | 332 |
| 19 | $\underline{4}$2m | $D_{2d}$ | 2*2 |
| 20 | 422 | $D_4$ | 224 |
| 21 | 4/m | $C_{4h}$ | 4* |
| 22 | 4mm | $C_{4v}$ | *44 |
| 23 | 4/m mm | $D_{4h}$ | *224 |
| 24 | 622 | $D_6$ | 226 |
| 25 | 6/m | $C_{6h}$ | 6* |
| 26 | 6mm | $C_{6v}$ | *66 |
| 27 | 6/m mm | $D_{6h}$ | *226 |
| 28 | 432 | O | 432 |
| 29 | m$\underline{3}$ | $T_h$ | 3*2 |
| 30 | $\underline{4}$3m | $T_d$ | *332 |
| 31 | m$\underline{3}$m | $O_h$ | *432 |

**Meaning of each bit**

It is always possible to associate a name with 32 classes that give them the name of:

Class 0 (00000),

Class 1 (00001),

Class 2 (00 010),

Class 3 (00011),

and so on up to class 31 (11111).

However, this has no particular meaning and is essentially stupid.

It is however important to associate with each bit a physical property possessed by the crystal. This would mean that there are only five basic properties. Their presence or absence will automatically determine which class or *point group* of origin.

The meaning that I have tentatively identified is the following.

**Bit c**
0000c in position 00001

The presence of c-bit refers to the presence of the property usually called *center*.

**Bit m**
000m0 in position 00010

The presence of m-bit refers to the presence of the property usually called *planes.*

**Bit 2**
00200 in position 00100

The presence of bit 2 refers to the presence of the property usually called *axis 2.*

**Bit 3**
03000 in position 01000

The presence of bit 3 refers to the presence of the property usually called *axis 3.*

**Bit 4**
40000 in position 10000

The presence of bit 4 corresponds to the presence of a property that provisionally identify in a more subtle way: a symmetry that corresponds to half of a possible and / or existing symmetry.

This bit then appears to identify axis 4, 90° as half of 180°, or even an axis 6, 60° as half of 120°.

The following table shows the 32 crystal classes and their symmetries with Hermann Mouguin symbols (from [3]).

I add the proposed classification, with the symbols of Bettini.

| Crystal System | Crystal Class | Symmetry | Name of Class and Bettini symbols |
|---|---|---|---|
| Triclinic | 1 | none | Pedial<br>00000 |
| | $\bar{1}$ | i | Pinacoidal<br>0000c |
| Monoclinic | 2 | $1A_2$ | Sphenoidal<br>00200 |
| | m | 1m | Domatic<br>000m0 |
| | 2/m | i, $1A_2$, 1m | Prismatic<br>000mc |
| Orthorhombic | 222 | $3A_2$ | Rhombic-disphenoidal<br>0020c |
| | mm2 (2mm) | $1A_2$, 2m | Rhombic-pyramidal<br>002m0 |
| | 2/m2/m2/m | i, $3A_2$, 3m | Rhombic-dipyramidal<br>002mc |
| Tetragonal | 4 | $1A_4$ | Tetragonal- Pyramidal<br>40000 |
| | $\bar{4}$ | $\bar{A}_4$ | Tetragonal-disphenoidal<br>4000c |
| | 4/m | i, $1A_4$, 1m | Tetragonal-dipyramidal<br>4020c |
| | 422 | $1A_4$, $4A_2$ | Tetragonal-<br>Trapezohedral<br>40200 |
| | 4mm | $1A_4$, 4m | Ditetragonal-pyramidal<br>402m0 |
| | $\bar{4}$2m | $1\bar{A}_4$, $2A_2$, 2m | Tetragonal-scalenohedral<br>400mc |
| | 4/m2/m2/m | i, $1A_4$, $4A_2$, 5m | Ditetragonal-dipyramidal<br>402mc |
| Hexagonal | 3 | $1A_3$ | Trigonal-pyramidal<br>03000 |
| | $\bar{3}$ | $1\bar{A}_3$ | Rhombohedral<br>0300c |
| | 32 | $1A_3$, $3A_2$ | Trigonal-trapezohedral<br>0320c |

| | | | |
|---|---|---|---|
| | 3m | $1A_3$, 3m | Ditrigonal-pyramidal<br>032mc |
| | $\bar{3}$2/m | $1\bar{A}_3$, $3A_2$, 3m | Hexagonal-scalenohedral<br>030mc |
| | 6 | $1A_6$ | Hexagonal-pyramidal<br>03200 |
| | $\bar{6}$ | $1\bar{A}_6$ | Trigonal-dipyramidal<br>030m0 |
| | 6/m | i, $1A_6$, 1m | Hexagonal-dipyramidal<br>4300c |
| | 622 | $1A_6$, $6A_2$ | Hexagonal-trapezohedral<br>43000 |
| | 6mm | $1A_6$, 6m | Dihexagonal-pyramidal<br>430m0 |
| | $\bar{6}$m2 | $1\bar{A}_6$, $3A_2$, 3m | Ditrigonal-dipyramidal<br>032m0 |
| | 6/m2/m2/m | i, $1A_6$, $6A_2$, 7m | Dihexagonal-dipyramidal<br>430mc |
| | 23 | $3A_2$, $4A_3$ | Tetaroidal<br>400m0 |
| | 2/m$\bar{3}$ | $3A_2$, 3m, $4\bar{A}_3$ | Diploidal<br>4320c |
| Isometric | 432 | $3A_4$, $4A_3$, $6A_2$ | Gyroidal<br>43200 |
| | $\bar{4}$3m | $3\bar{A}_4$, $4A_3$, 6m | Hextetrahedral<br>432m0 |
| | 4/m$\bar{3}$2/m | $3A_4$, $4\bar{A}_3$, $6A_2$, 9m | Hexoctahedral<br>432mc |

**Final classification, ordered from 0 to 15 and from 16 to 31**

| | |
|---|---|
| 0-Pedial<br>00000 | 16-Tetragonal- Pyramidal<br>40000 |
| 1-Pinacoidal<br>0000c | 17-Tetragonal-disphenoidal<br>4000c |
| 2-Domatic<br>000m0 | 18-Tetaroidal<br>400m0 |
| 3-Prismatic<br>000mc | 19-Tetragonal-scalenohedral<br>400mc |
| | |
| 4-Sphenoidal<br>00200 | 20-Tetragonal-<br>Trapezohedral<br>40200 |
| 5-Rhombic-disphenoidal<br>0020c | 21-Tetragonal-dipyramidal<br>4020c |
| 6-Rhombic-pyramidal<br>002m0 | 22-Ditetragonal-pyramidal<br>402m0 |
| 7-Rhombic-dipyramidal<br>002mc | 23-Ditetragonal-dipyramidal<br>402mc |
| | |
| 8-Trigonal-pyramidal<br>03000 | 24-Hexagonal-trapezohedral<br>43000 |
| 9-Rhombohedral<br>0300c | 25-Hexagonal-dipyramidal<br>4300c |
| 10-Trigonal-dipyramidal<br>030m0 | 26-Dihexagonal-pyramidal<br>430m0 |
| 11-Hexagonal-scalenohedral<br>030mc | 27-Dihexagonal-dipyramidal<br>430mc |
| | |
| 12-Hexagonal-pyramidal<br>03200 | 28-Gyroidal<br>43200 |
| 13-Trigonal-trapezohedral<br>0320c | 29-Diploidal<br>4320c |
| 14-Ditrigonal-dipyramidal<br>032m0 | 30-Hextetrahedral<br>432m0 |
| 15-Ditrigonal-pyramidal<br>032mc | 31-Hexoctahedral<br>432mc |

I remember the meaning of bits:

0000c  means value 1     00001
000m0  means value 2    00010
00200   means value 4    00100
00300   means value 8    00100
40000   means value 16  10000

I remember the (tentative) physical meaning of bits:

c  meanings “center”.
m  meanings “planes”.
2  meanings “axis 2”.
3  meanings “axis 3”
4  meanings “axis 4” or half of a possible and / or existing symmetry as in 43000 = axis 6.

**Conclusions**

I have proposed a classification of the 32 crystal classes in 5 bits, each of which corresponds to a physical property. For almost all classes the result is entirely satisfactory but some cases seem to me still ambiguous, you may need further work. For the moment I do not know any theoretical justification for a possible identification with five basic properties, if not a *guess,* and trust that *Essentia non sunt moltiplicanda praeter necessitatem.*

**References**

[1] D. Hestenes, “Point Groups and Space Groups in Geometric Algebra “, in L. Dorst, C. Doran, J. Lasenby (eds.), Applications of Geometric Algebra in Computer Science and Engineering, Birkhaeuser, Boston, 2002, pp. 3-34.

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[3] S.A. Nelson, Earth & Environmental Sciences 2110, “MINERALOGY”, Tulane University, Department of Earth & Environmental Sciences