# A TREATY OF SYMMETRIC FUNCTION

# Sums of Power for an Arbitrary Arithmetic Progression for real power.

## Paper Part II

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*In remembrance of my beloved father who passed away on the 23rd of June 2009 and my special thanks to my brother Mohd Yunus Abd Shukor[2] for introducing me Fermat's Last Theorem when I was a teenager. Although I didn't get the proof for this theorem, it enhanced my understanding towards developing the generalized equations for Symmetric Function for Sums of Powers and expressing Riemann Zeta Function using Sum of Power. The finding also contributes to a formulation of a new conjecture of Prime Number of a Power Sum Origin. Lastly to my sister Nazirah Abd Shukor, thanks for all the supports and patience.*

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**Abstract**. Sums of Power mainly deal with positive integer power $p$ (i.e. $p \in^{+} \mathbf{Z}$). In this paper, I would like to show that the sums of power that I had formulated in paper part I [1] also can be applied to the non-integer power $p$. The sums of power for positive non-integers (i.e. SPPNI) in this paper still adopting the same general sums of power formulation. However, the value of $m$ has no bound and it is used as precision control. The larger the value of $m$ used, the more accuracy the result would be.

The general formulation is given as follows

$$\sum_{i=1}^{n} x_i^p = \sum_{m=0}^{u} \left[ O_m s^{2j} \frac{\left[ \sum_{i=1}^{n} x_i \right]^{p-2j}}{n^{p-(2j+1)}} \right] \qquad [1]$$

Where: $s = x_{i+1} - x_i$, $O_m$ is a coefficient and $O_0 = 1$ and $u \in^{+} \mathbf{Z}$. In the sums of power of $p \in^{+}$ R, the value of $m$ has no bound.

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## 1 Introduction

Since $p$ is positive non-integers, the value of $\sum_{i=1}^{n} x_i^p$ would approach the real value if $m \to \infty$. Expanding equation [1], yields:

$$\sum_{i=1}^{n} x_i^p = O_0 \frac{\left[\sum_{i=1}^{n} x_i\right]^p}{n^{p-1}} + O_1 s^2 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-2}}{n^{p-3}} + O_2 s^4 \frac{\left[\sum_{i=1}^{n} x_i\right]^{p-4}}{n^{p-5}} + \dots \qquad [2]$$

The coefficient $O_m$ is given by a generalized equation as follows:

$$O_m = \frac{1}{2^{2m}} \frac{1}{(2m+1)} \binom{p}{2m} \left[ n^{2m} - 2\sum_{t=1}^{m} \left[ (2t+1)(2^{2t-1}-1) \binom{m}{t} B_t n^{2(m-t)} \frac{\prod_{j=0}^{t-1} (1+2(m-j))}{\prod_{j=0}^{t} (1+2(t-j))} \right] \right]$$

## 2 Chapter 1

In this chapter the value of $p$ studied is in between 0 and 1 (i.e. $0 < p < 1$). Within this range (i.e. $0 < p < 1$), the first term (i.e. $O_0 \frac{\left[\sum_{i=1}^{n} x_i\right]^p}{n^{p-1}}$) is larger than the actual value. Therefore, the increment of $m$ is to reduce the value of the first term until it approximates the actual value. The analysis is given as follows:

Let $n$=2, $s$=1 and $p$=0.1 and $m = 0,1,\dots 4$.

Case 1,
$m = 0$

$$S_0 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} = 2^{0.9} \left[\sum_{i=1}^{2} x_i\right]^{0.1} \qquad [3]$$

Tabulating some data for equation [3], yields

Table 1 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficient $O_0$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_0$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.0717734625363 | 2.0827594879848 | -0.0109860254485 |
| 2 | 3 | 5 | 2.1878966365702 | 2.1919164527704 | -0.0040198162002 |
| 3 | 4 | 7 | 2.2648215290309 | 2.2669231633414 | -0.0021016343105 |
| 4 | 5 | 9 | 2.3233172980851 | 2.3246161304789 | -0.0012988323938 |
| 5 | 6 | 11 | 2.3708501419393 | 2.3717355572556 | -0.0008854153163 |
| 6 | 7 | 13 | 2.4110452428904 | 2.4116891599881 | -0.0006439170977 |
| 7 | 8 | 15 | 2.4459584573840 | 2.4464487484833 | -0.0004902910993 |
| 8 | 9 | 17 | 2.4768753529604 | 2.4772616999622 | -0.0003863470018 |
| 9 | 10 | 19 | 2.5046563514097 | 2.5049690030306 | -0.0003126516209 |
| 10 | 11 | 21 | 2.5299070270043 | 2.5301654763596 | -0.0002584493553 |
| 11 | 12 | 23 | 2.5530704691970 | 2.5532878563502 | -0.0002173871532 |
| 12 | 13 | 25 | 2.5744810747677 | 2.5746665870905 | -0.0001855123228 |
| 13 | 14 | 27 | 2.5943976750983 | 2.5945579339605 | -0.0001602588622 |
| 14 | 15 | 29 | 2.6130248773572 | 2.6131647782686 | -0.0001399009114 |
| 15 | 16 | 31 | 2.6305273338126 | 2.6306505761385 | -0.0001232423259 |

Case 2,

$m = 1$

$$S_1 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} = 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}} = 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9}(3)(0.1)(-0.9)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{8\left[\sum_{i=1}^{2} x_i\right]^{1.9}} \qquad [4]$$

Tabulating some data for equation [4], yields

Table 2 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficient $O_0$ and $O_1$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_1$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.0717734625363 | 2.0723456905449 | -0.0005722280086 |
| 2 | 3 | 5 | 2.1878966365702 | 2.1879710031555 | -0.0000743665852 |
| 3 | 4 | 7 | 2.2648215290309 | 2.2648412951302 | -0.0000197660992 |
| 4 | 5 | 9 | 2.3233172980851 | 2.3233246770730 | -0.0000073789880 |
| 5 | 6 | 11 | 2.3708501419393 | 2.3708535068418 | -0.0000033649024 |
| 6 | 7 | 13 | 2.4110452428904 | 2.4110469942354 | -0.0000017513450 |
| 7 | 8 | 15 | 2.4459584573840 | 2.4459594587336 | -0.0000010013496 |
| 8 | 9 | 17 | 2.4768753529604 | 2.4768759671716 | -0.0000006142111 |
| 9 | 10 | 19 | 2.5046563514097 | 2.5046567492768 | -0.0000003978671 |
| 10 | 11 | 21 | 2.5299070270043 | 2.5299072962089 | -0.0000002692046 |
| 11 | 12 | 23 | 2.5530704691970 | 2.5530706579503 | -0.0000001887533 |
| 12 | 13 | 25 | 2.5744810747677 | 2.5744812110962 | -0.0000001363285 |
| 13 | 14 | 27 | 2.5943976750983 | 2.5943977760633 | -0.0000001009650 |
| 14 | 15 | 29 | 2.6130248773572 | 2.6130249537561 | -0.0000000763988 |
| 15 | 16 | 31 | 2.6305273338126 | 2.6305273927088 | -0.0000000588962 |

Case 3,

$m = 2$

$$S_2 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-3.9}}{2^{-4.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \frac{2^{4.9} O_2}{\left[\sum_{i=1}^{2} x_i\right]^{3.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \frac{2^{4.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{3.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{8\left[\sum_{i=1}^{2} x_i\right]^{1.9}} - \frac{2^{4.9}(0.4959)}{384\left[\sum_{i=1}^{2} x_i\right]^{3.9}} \qquad [5]$$

Tabulating some data for equation [5], yields

Table 3 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficients $O_0$, $O_1$ and $O_2$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_2$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.0717734625363 | 2.0718143940274 | -0.0000409314911 |
| 2 | 3 | 5 | 2.1878966365702 | 2.1878985383975 | -0.0000019018273 |
| 3 | 4 | 7 | 2.2648215290309 | 2.2648217864672 | -0.0000002574362 |
| 4 | 5 | 9 | 2.3233172980851 | 2.3233173561798 | -0.0000000580947 |
| 5 | 6 | 11 | 2.3708501419393 | 2.3708501596670 | -0.0000000177276 |
| 6 | 7 | 13 | 2.4110452428904 | 2.4110452494951 | -0.0000000066047 |
| 7 | 8 | 15 | 2.4459584573840 | 2.4459584602201 | -0.0000000028361 |
| 8 | 9 | 17 | 2.4768753529604 | 2.4768753543147 | -0.0000000013542 |
| 9 | 10 | 19 | 2.5046563514097 | 2.5046563521119 | -0.0000000007022 |
| 10 | 11 | 21 | 2.5299070270043 | 2.5299070273932 | -0.0000000003889 |
| 11 | 12 | 23 | 2.5530704691970 | 2.5530704694243 | -0.0000000002273 |
| 12 | 13 | 25 | 2.5744810747677 | 2.5744810749066 | -0.0000000001390 |
| 13 | 14 | 27 | 2.5943976750983 | 2.5943976751865 | -0.0000000000882 |
| 14 | 15 | 29 | 2.6130248773572 | 2.6130248774151 | -0.0000000000579 |
| 15 | 16 | 31 | 2.6305273338126 | 2.6305273338517 | -0.0000000000390 |

Case 4,
$m = 3$

$$S_3 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-3.9}}{2^{-4.9}} + O_3 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-5.9}}{2^{-6.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \frac{2^{4.9} O_2}{\left[\sum_{i=1}^{2} x_i\right]^{3.9}} \frac{2^{6.9} O_3}{\left[\sum_{i=1}^{2} x_i\right]^{5.9}}$$

$$= \left[\begin{array}{l} 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \dfrac{2^{2.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \dfrac{2^{4.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{3.9}} + \\ \dfrac{2^{6.9}(3n^4-18n^2+31)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)}{967680\left[\sum_{i=1}^{2} x_i\right]^{5.9}} \end{array}\right]$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{8\left[\sum_{i=1}^{2} x_i\right]^{1.9}} - \frac{2^{4.9}(0.4959)}{384\left[\sum_{i=1}^{2} x_i\right]^{3.9}} - \frac{2^{6.9}(9.476649)}{46080\left[\sum_{i=1}^{2} x_i\right]^{5.9}} \qquad [6]$$

Tabulating some data for equation [6], yields

Table 4 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficients $O_0$, $O_1$, $O_2$ and $O_3$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_3$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.0717734625363 | 2.0717767900405 | -0.0000033275042 |
| 2 | 3 | 5 | 2.1878966365702 | 2.1878966919955 | -0.0000000554253 |
| 3 | 4 | 7 | 2.2648215290309 | 2.2648215328545 | -0.0000000038236 |
| 4 | 5 | 9 | 2.3233172980851 | 2.3233172986068 | -0.0000000005217 |
| 5 | 6 | 11 | 2.3708501419393 | 2.3708501420459 | -0.0000000001066 |
| 6 | 7 | 13 | 2.4110452428904 | 2.4110452429188 | -0.0000000000284 |
| 7 | 8 | 15 | 2.4459584573840 | 2.4459584573932 | -0.0000000000092 |
| 8 | 9 | 17 | 2.4768753529604 | 2.4768753529638 | -0.0000000000034 |
| 9 | 10 | 19 | 2.5046563514097 | 2.5046563514111 | -0.0000000000014 |
| 10 | 11 | 21 | 2.5299070270043 | 2.5299070270050 | -0.0000000000006 |
| 11 | 12 | 23 | 2.5530704691970 | 2.5530704691973 | -0.0000000000003 |
| 12 | 13 | 25 | 2.5744810747677 | 2.5744810747678 | -0.0000000000002 |
| 13 | 14 | 27 | 2.5943976750983 | 2.5943976750984 | -0.0000000000001 |
| 14 | 15 | 29 | 2.6130248773572 | 2.6130248773573 | -0.0000000000001 |
| 15 | 16 | 31 | 2.6305273338126 | 2.6305273338127 | 0.0000000000000 |

Case 5,
$m = 4$

$$S_4 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-3.9}}{2^{-4.9}} O_3 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-5.9}}{2^{-6.9}} + O_4 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-7.9}}{2^{-8.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9}O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \frac{2^{4.9}O_2}{\left[\sum_{i=1}^{2} x_i\right]^{3.9}} + \frac{2^{6.9}O_3}{\left[\sum_{i=1}^{2} x_i\right]^{5.9}} + \frac{2^{8.9}O_4}{\left[\sum_{i=1}^{2} x_i\right]^{7.9}}$$

$$= \left[\begin{array}{l} 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \dfrac{2^{2.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \dfrac{2^{4.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{3.9}} + \\ \dfrac{2^{6.9}(3n^4-18n^2+31)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)}{967680\left[\sum_{i=1}^{2} x_i\right]^{5.9}} + \\ \dfrac{2^{8.9}(5n^6-55n^4+239n^2-381)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)(p-6)(p-7)}{464486400\left[\sum_{i=1}^{2} x_i\right]^{7.9}} \end{array}\right]$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{8\left[\sum_{i=1}^{2} x_i\right]^{1.9}} - \frac{2^{4.9}(0.4959)}{384\left[\sum_{i=1}^{2} x_i\right]^{3.9}} - \frac{2^{6.9}(9.476649)}{46080\left[\sum_{i=1}^{2} x_i\right]^{5.9}} - \frac{2^{8.9}(9.476649)(40.71)}{10321920\left[\sum_{i=1}^{2} x_i\right]^{7.9}} \quad [7]$$

Tabulating some data for equation [4], yields

Table 5 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficients $O_0, O_1, O_2, O_3$ and $O_4$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_4$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.0717734625363 | 2.0717737526232 | -0.0000002900869 |
| 2 | 3 | 5 | 2.1878966365702 | 2.1878966383048 | -0.0000000017346 |
| 3 | 4 | 7 | 2.2648215290309 | 2.2648215290920 | -0.0000000000610 |
| 4 | 5 | 9 | 2.3233172980851 | 2.3233172980901 | -0.0000000000050 |
| 5 | 6 | 11 | 2.3708501419393 | 2.3708501419400 | -0.0000000000007 |
| 6 | 7 | 13 | 2.4110452428904 | 2.4110452428905 | -0.0000000000001 |
| 7 | 8 | 15 | 2.4459584573840 | 2.4459584573840 | 0.0000000000000 |
| 8 | 9 | 17 | 2.4768753529604 | 2.4768753529604 | 0.0000000000000 |
| 9 | 10 | 19 | 2.5046563514097 | 2.5046563514097 | 0.0000000000000 |
| 10 | 11 | 21 | 2.5299070270043 | 2.5299070270043 | 0.0000000000000 |
| 11 | 12 | 23 | 2.5530704691970 | 2.5530704691970 | 0.0000000000000 |
| 12 | 13 | 25 | 2.5744810747677 | 2.5744810747677 | 0.0000000000000 |
| 13 | 14 | 27 | 2.5943976750983 | 2.5943976750983 | 0.0000000000000 |
| 14 | 15 | 29 | 2.6130248773572 | 2.6130248773572 | 0.0000000000000 |
| 15 | 16 | 31 | 2.6305273338126 | 2.6305273338126 | 0.0000000000000 |

**Discussion**

It can be seen on the data tabulated in the tables [1]-[4], that the error reduces as the value of *m* increases. On the other hands, it is also can be seen that, as the symmetric function $\sum_{i=2}^{2} x_i$ is getting larger the error is getting smaller and approaching zero. In this research, the calculation is done using Microsoft Excel with precision up to 15 digits.

Now consider $n$=2, $s$=2 and $p$=0.1 and $m = 1,4$.

Case 1,
$m = 1$

$$S_1 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 s^2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} = 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{(4)2^{2.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{(4)2^{2.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}} = 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{2^{2.9}(12)(0.1)(-0.9)}{24\left[\sum_{i=1}^{2} x_i\right]^{1.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{2\left[\sum_{i=1}^{2} x_i\right]^{1.9}}$$

Tabulating some data for equation [4], yields

Table 6 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficients $s$=2, $O_0$ and $O_1$,

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_1$ | Error |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 2.1161231740339 | 2.1194320221655 | -0.0033088481316 |
| 2 | 4 | 6 | 2.2204718175333 | 2.2210851163275 | -0.0006132987941 |
| 3 | 5 | 8 | 2.2907421171219 | 2.2909352817472 | -0.0001931646253 |
| 4 | 6 | 10 | 2.3449295538484 | 2.3450092579809 | -0.0000797041326 |
| 5 | 7 | 12 | 2.3894329871271 | 2.3894718197055 | -0.0000388325784 |
| 6 | 8 | 14 | 2.4273756121962 | 2.4273967969768 | -0.0000211847806 |
| 7 | 9 | 16 | 2.4605449836546 | 2.4605575298586 | -0.0000125462040 |
| 8 | 10 | 18 | 2.4900698251391 | 2.4900777337426 | -0.0000079086035 |
| 9 | 11 | 20 | 2.5167125548257 | 2.5167177907177 | -0.0000052358921 |
| 10 | 12 | 22 | 2.5410142657810 | 2.5410178721941 | -0.0000036064131 |
| 11 | 13 | 24 | 2.5633738359910 | 2.5633764024399 | -0.0000025664489 |
| 12 | 14 | 26 | 2.5840943083043 | 2.5840961853494 | -0.0000018770451 |
| 13 | 15 | 28 | 2.6034116438206 | 2.6034130489875 | -0.0000014051670 |
| 14 | 16 | 30 | 2.6215133650904 | 2.6215144383103 | -0.0000010732199 |
| 15 | 17 | 32 | 2.6385510979283 | 2.6385519320459 | -0.0000008341176 |

Case 2,
$m = 4$

$$S_4 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{0.1}}{2^{0.1-1}} + O_1 s^2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-1.9}}{2^{-2.9}} + O_2 s^4 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-3.9}}{2^{-4.9}} + O_3 s^6 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-5.9}}{2^{-6.9}} + O_4 s^8 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-7.9}}{2^{-8.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} + \frac{(4)2^{2.9}O_1}{\left[\sum_{i=1}^{2} x_i\right]^{1.9}} + \frac{(16)2^{4.9}O_2}{\left[\sum_{i=1}^{2} x_i\right]^{3.9}} + \frac{(64)2^{6.9}O_3}{\left[\sum_{i=1}^{2} x_i\right]^{5.9}} + \frac{(256)2^{8.9}O_4}{\left[\sum_{i=1}^{2} x_i\right]^{7.9}}$$

$$= 2^{0.9}\left[\sum_{i=1}^{2} x_i\right]^{0.1} - \frac{2^{2.9}(0.09)}{2\left[\sum_{i=1}^{2} x_i\right]^{1.9}} - \frac{2^{4.9}(0.4959)}{24\left[\sum_{i=1}^{2} x_i\right]^{3.9}} - \frac{2^{6.9}(9.476649)}{720\left[\sum_{i=1}^{2} x_i\right]^{5.9}} - \frac{2^{8.9}(9.476649)(40.71)}{40320\left[\sum_{i=1}^{2} x_i\right]^{7.9}} \qquad [8]$$

Tabulating some data for equation [4], yields

Table 7 The value of $\sum_{i=2}^{2} x_i^{0.1}$ using coefficients $s$=2, $O_0$, $O_1$, $O_2$, $O_3$ and $O_4$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{0.1}$ | $S_4$ | Error |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 2.1161231740339 | 2.1161428803411 | -0.0000197063072 |
| 2 | 4 | 6 | 2.2204718175333 | 2.2204721284408 | -0.0000003109075 |
| 3 | 5 | 8 | 2.2907421171219 | 2.2907421343828 | -0.0000000172609 |
| 4 | 6 | 10 | 2.3449295538484 | 2.3449295557074 | -0.0000000018591 |
| 5 | 7 | 12 | 2.3894329871271 | 2.3894329874297 | -0.0000000003026 |
| 6 | 8 | 14 | 2.4273756121962 | 2.4273756122616 | -0.0000000000654 |
| 7 | 9 | 16 | 2.4605449836546 | 2.4605449836720 | -0.0000000000174 |
| 8 | 10 | 18 | 2.4900698251391 | 2.4900698251445 | -0.0000000000054 |
| 9 | 11 | 20 | 2.5167125548257 | 2.5167125548276 | -0.0000000000019 |
| 10 | 12 | 22 | 2.5410142657810 | 2.5410142657817 | -0.0000000000007 |
| 11 | 13 | 24 | 2.5633738359910 | 2.5633738359913 | -0.0000000000003 |
| 12 | 14 | 26 | 2.5840943083043 | 2.5840943083044 | -0.0000000000001 |
| 13 | 15 | 28 | 2.6034116438206 | 2.6034116438207 | -0.0000000000001 |
| 14 | 16 | 30 | 2.6215133650904 | 2.6215133650904 | 0.0000000000000 |
| 15 | 17 | 32 | 2.6385510979283 | 2.6385510979283 | 0.0000000000000 |

**Discussion**

The result shows that the formulation following the same form as in the sums of power with different $s$. Comparing the errors between $s$=1 and $s$=2 for the same $m$=1, suggesting better accuracy for larger $s$ could be achieved by making the value of $m$ larger. However, the accuracy for $s$=2 and $m$=4 has no difference up to 13 decimal places compared to $s$=1 and $m$=4 for larger $\sum_{i=2}^{2} x_i$ , suggesting the function is sensitive for larger symmetric function $\sum_{i=2}^{2} x_i$ even at small $m$ (i.e. $m$=4).

**2 Chapter 2**

In this chapter the value of $p$ studied is in between 0 and 1 (i.e. $p > 1$). Within this range (i.e. $0 < p < 1$), the first term (i.e. $O_0 \frac{\left[\sum_{i=1}^{n} x_i\right]^p}{n^{p-1}}$) is smaller than the actual value. Therefore, the increment of $m$ is to increase the value of the first term until it approximates the actual value. The analysis is given as follows:

Let $n$=2, $s$=1 and $p$=1.1 and $m = 0,1,\ldots 4$.

Case 1,
$m = 0$

$$S_0 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}}$$

Tabulating some data for equation [4], yields

Table 8 The value of $\sum_{i=2}^{2} x_i^{1.1}$ using coefficients $s$=1 and $O_0$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{1.1}$ | $S_0$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 3.1435469250726 | 3.1241392319772 | 0.0194076930954 |
| 2 | 3 | 5 | 5.4919164471743 | 5.4797911319261 | 0.0121253152482 |
| 3 | 4 | 7 | 7.9431629420899 | 7.9342310716949 | 0.0089318703950 |
| 4 | 5 | 9 | 10.4678881354282 | 10.4607725871548 | 0.0071155482734 |
| 5 | 6 | 11 | 13.0504819085480 | 13.0445455649058 | 0.0059363436422 |
| 6 | 7 | 13 | 15.6810855013814 | 15.6759795399225 | 0.0051059614589 |
| 7 | 8 | 15 | 18.3528536150328 | 18.3483656136246 | 0.0044880014082 |
| 8 | 9 | 17 | 21.0607337632990 | 21.0567244496788 | 0.0040093136202 |
| 9 | 10 | 19 | 23.8008325744813 | 23.7972055287909 | 0.0036270456905 |
| 10 | 11 | 21 | 26.5700518852532 | 26.5667375017756 | 0.0033143834777 |
| 11 | 12 | 23 | 29.3658640151533 | 29.3628103480271 | 0.0030536671262 |
| 12 | 13 | 25 | 32.1861651179926 | 32.1833323386306 | 0.0028327793620 |
| 13 | 14 | 27 | 35.0291752305954 | 35.0265321084663 | 0.0026431221291 |
| 14 | 15 | 29 | 37.8933677060408 | 37.8908892848943 | 0.0024784211465 |
| 15 | 16 | 31 | 40.7774179179626 | 40.7750839301471 | 0.0023339878154 |

Case 2,
$m = 1$

$$S_1 = \sum_{i=1}^{2} x_i^{1.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-0.9}}{2^{-1.9}} = \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{0.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{0.9}} = \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(3)(1.1)(0.1)}{24\left[\sum_{i=1}^{2} x_i\right]^{0.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(0.11)}{8\left[\sum_{i=1}^{2} x_i\right]^{0.9}}$$

Tabulating some data for equation [4], yields

Table 9 The value of $\sum_{i=2}^{2} x_i^{1.1}$ using coefficients $s$=1, $O_0$ and $O_1$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{1.1}$ | $S_1$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 3.1435469250726 | 3.1432311939504 | 0.0003157311222 |
| 2 | 3 | 5 | 5.4919164471743 | 5.4918466724163 | 0.0000697747580 |
| 3 | 4 | 7 | 7.9431629420899 | 7.9431368412652 | 0.0000261008247 |
| 4 | 5 | 9 | 10.4678881354282 | 10.4678755808868 | 0.0000125545414 |
| 5 | 6 | 11 | 13.0504819085480 | 13.0504749037990 | 0.0000070047490 |
| 6 | 7 | 13 | 15.6810855013814 | 15.6810811900686 | 0.0000043113127 |
| 7 | 8 | 15 | 18.3528536150328 | 18.3528507696634 | 0.0000028453694 |
| 8 | 9 | 17 | 21.0607337632990 | 21.0607317847816 | 0.0000019785173 |
| 9 | 10 | 19 | 23.8008325744813 | 23.8008311418216 | 0.0000014326598 |
| 10 | 11 | 21 | 26.5700518852532 | 26.5700508137089 | 0.0000010715443 |
| 11 | 12 | 23 | 29.3658640151533 | 29.3658631922032 | 0.0000008229501 |
| 12 | 13 | 25 | 32.1861651179926 | 32.1861644718764 | 0.0000006461162 |
| 13 | 14 | 27 | 35.0291752305954 | 35.0291747137694 | 0.0000005168260 |
| 14 | 15 | 29 | 37.8933677060408 | 37.8933672859771 | 0.0000004200636 |
| 15 | 16 | 31 | 40.7774179179626 | 40.7774175717873 | 0.0000003461753 |

Case 3,
$m = 2$

$$S_2 = \sum_{i=1}^{2} x_i^{1.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-0.9}}{2^{-1.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-2.9}}{2^{-3.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9} O_2}{\left[\sum_{i=1}^{2} x_i\right]^{2.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{2.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(011)}{8\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9}(0.1881)}{384\left[\sum_{i=1}^{2} x_i\right]^{2.9}} \qquad [9]$$

Tabulating some data for equation [4], yields

Table 10 The value of $\sum_{i=2}^{2} x_i^{1.1}$ using coefficients $s$=1, $O_0$, $O_1$ and $O_2$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{1.1}$ | $S_2$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 3.1435469250726 | 3.1435334833483 | 0.0000134417243 |
| 2 | 3 | 5 | 5.4919164471743 | 5.4919153889971 | 0.0000010581772 |
| 3 | 4 | 7 | 7.9431629420899 | 7.9431627406971 | 0.0000002013928 |
| 4 | 5 | 9 | 10.4678881354282 | 10.4678880768943 | 0.0000000585339 |
| 5 | 6 | 11 | 13.0504819085480 | 13.0504818866981 | 0.0000000218499 |
| 6 | 7 | 13 | 15.6810855013814 | 15.6810854917559 | 0.0000000096255 |
| 7 | 8 | 15 | 18.3528536150328 | 18.3528536102623 | 0.0000000047705 |
| 8 | 9 | 17 | 21.0607337632990 | 21.0607337607168 | 0.0000000025822 |
| 9 | 10 | 19 | 23.8008325744813 | 23.8008325729846 | 0.0000000014967 |
| 10 | 11 | 21 | 26.5700518852532 | 26.5700518843369 | 0.0000000009163 |
| 11 | 12 | 23 | 29.3658640151533 | 29.3658640145667 | 0.0000000005866 |
| 12 | 13 | 25 | 32.1861651179926 | 32.1861651176028 | 0.0000000003898 |
| 13 | 14 | 27 | 35.0291752305954 | 35.0291752303281 | 0.0000000002673 |
| 14 | 15 | 29 | 37.8933677060408 | 37.8933677058525 | 0.0000000001883 |
| 15 | 16 | 31 | 40.7774179179626 | 40.7774179178267 | 0.0000000001358 |

Case 4,
$m = 3$

$$S_3 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-0.9}}{2^{-1.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-2.9}}{2^{-3.9}} + O_3 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-4.9}}{2^{-5.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9} O_2}{\left[\sum_{i=1}^{2} x_i\right]^{2.9}} \frac{2^{5.9} O_3}{\left[\sum_{i=1}^{2} x_i\right]^{4.9}}$$

$$= \left[ \begin{array}{l} 2\dfrac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \dfrac{2^{1.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \dfrac{2^{3.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{2.9}} + \\ \dfrac{2^{5.9}(3n^4-18n^2+31)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)}{967680\left[\sum_{i=1}^{2} x_i\right]^{4.9}} \end{array} \right]$$

$$= 2\frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(0.11)}{8\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9}(0.1881)}{384\left[\sum_{i=1}^{2} x_i\right]^{2.9}} + \frac{2^{5.9}(2.127411)}{46080\left[\sum_{i=1}^{2} x_i\right]^{4.9}} \qquad [10]$$

Tabulating some data for equation [4], yields

Table 11 The value of $\sum_{i=2}^{2} x_i^{1.1}$ using coefficients $s$=1, $O_0$, $O_1$ and $O_2$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{1.1}$ | $S_3$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 3.1435469250726 | 3.1435461459153 | 0.0000007791572 |
| 2 | 3 | 5 | 5.4919164471743 | 5.4919164252432 | 0.0000000219311 |
| 3 | 4 | 7 | 7.9431629420899 | 7.9431629399641 | 0.0000000021257 |
| 4 | 5 | 9 | 10.4678881354282 | 10.4678881350548 | 0.0000000003735 |
| 5 | 6 | 11 | 13.0504819085480 | 13.0504819084547 | 0.0000000000933 |
| 6 | 7 | 13 | 15.6810855013814 | 15.6810855013519 | 0.0000000000294 |
| 7 | 8 | 15 | 18.3528536150328 | 18.3528536150219 | 0.0000000000109 |
| 8 | 9 | 17 | 21.0607337632990 | 21.0607337632944 | 0.0000000000046 |
| 9 | 10 | 19 | 23.8008325744813 | 23.8008325744792 | 0.0000000000021 |
| 10 | 11 | 21 | 26.5700518852532 | 26.5700518852522 | 0.0000000000011 |
| 11 | 12 | 23 | 29.3658640151533 | 29.3658640151528 | 0.0000000000006 |
| 12 | 13 | 25 | 32.1861651179926 | 32.1861651179923 | 0.0000000000003 |
| 13 | 14 | 27 | 35.0291752305954 | 35.0291752305952 | 0.0000000000002 |
| 14 | 15 | 29 | 37.8933677060408 | 37.8933677060407 | 0.0000000000001 |
| 15 | 16 | 31 | 40.7774179179626 | 40.7774179179625 | 0.0000000000001 |

Case 5,
$m = 4$

$$S_4 = \sum_{i=1}^{2} x_i^{0.1} = O_0 \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + O_1 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-0.9}}{2^{-1.9}} + O_2 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-2.9}}{2^{-3.9}} O_3 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-4.9}}{2^{-5.9}} + O_4 \frac{\left[\sum_{i=1}^{2} x_i\right]^{-6.9}}{2^{-7.9}}$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9} O_1}{\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9} O_2}{\left[\sum_{i=1}^{2} x_i\right]^{2.9}} + \frac{2^{5.9} O_3}{\left[\sum_{i=1}^{2} x_i\right]^{4.9}} + \frac{2^{7.9} O_4}{\left[\sum_{i=1}^{2} x_i\right]^{6.9}}$$

$$= \left[ \begin{array}{l} \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(n^2-1)p(p-1)}{24\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9}(3n^2-7)(n^2-1)p(p-1)(p-2)(p-3)}{5760\left[\sum_{i=1}^{2} x_i\right]^{2.9}} + \\ \frac{2^{5.9}(3n^4-18n^2+31)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)}{967680\left[\sum_{i=1}^{2} x_i\right]^{4.9}} + \\ \frac{2^{7.9}(5n^6-55n^4+239n^2-381)(n^2-1)p(p-1)(p-2)(p-3)(p-4)(p-5)(p-6)(p-7)}{464486400\left[\sum_{i=1}^{2} x_i\right]^{6.9}} \end{array} \right]$$

$$= \frac{\left[\sum_{i=1}^{2} x_i\right]^{1.1}}{2^{0.1}} + \frac{2^{1.9}(0.11)}{8\left[\sum_{i=1}^{2} x_i\right]^{0.9}} + \frac{2^{3.9}(0.1881)}{384\left[\sum_{i=1}^{2} x_i\right]^{2.9}} + \frac{2^{5.9}(2.127411)}{46080\left[\sum_{i=1}^{2} x_i\right]^{4.9}} + \frac{2^{7.9}(2.127411)(28.91)}{10321920\left[\sum_{i=1}^{2} x_i\right]^{6.9}} \qquad [11]$$

Tabulating some data for equation [4], yields

Table 12 The value of $\sum_{i=2}^{2} x_i^{1.1}$ using coefficients $O_0, O_1, O_2, O_3$ and $O_4$

| $x_1$ | $x_2$ | $\sum_{i=2}^{2} x_i$ | $\sum_{i=2}^{2} x_i^{1.1}$ | $S_4$ | Error |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 3.1435469250726 | 3.1435468722543 | 0.0000000528183 |
| 2 | 3 | 5 | 5.4919164471743 | 5.4919164466416 | 0.0000000005327 |
| 3 | 4 | 7 | 7.9431629420899 | 7.9431629420636 | 0.0000000000263 |
| 4 | 5 | 9 | 10.4678881354282 | 10.4678881354254 | 0.0000000000028 |
| 5 | 6 | 11 | 13.0504819085480 | 13.0504819085475 | 0.0000000000005 |
| 6 | 7 | 13 | 15.6810855013814 | 15.6810855013813 | 0.0000000000001 |
| 7 | 8 | 15 | 18.3528536150328 | 18.3528536150328 | 0.0000000000000 |
| 8 | 9 | 17 | 21.0607337632990 | 21.0607337632990 | 0.0000000000000 |
| 9 | 10 | 19 | 23.8008325744813 | 23.8008325744813 | 0.0000000000000 |
| 10 | 11 | 21 | 26.5700518852532 | 26.5700518852532 | 0.0000000000000 |
| 11 | 12 | 23 | 29.3658640151533 | 29.3658640151533 | 0.0000000000000 |
| 12 | 13 | 25 | 32.1861651179926 | 32.1861651179926 | 0.0000000000000 |
| 13 | 14 | 27 | 35.0291752305954 | 35.0291752305954 | 0.0000000000000 |
| 14 | 15 | 29 | 37.8933677060408 | 37.8933677060408 | 0.0000000000000 |
| 15 | 16 | 31 | 40.7774179179626 | 40.7774179179626 | 0.0000000000000 |

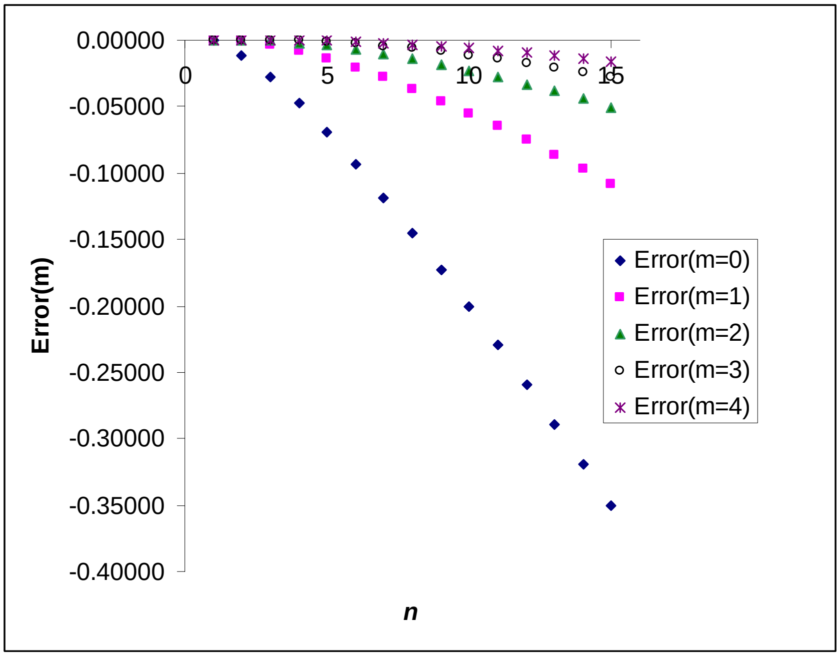
**Discussion**

The result shows the same pattern like in the power $0<p<1$. It shows that the accuracy increases with the increment of *m*.

## 2.0 The study of variables that govern the accuracy of the calculation of the sums of power.

From the results before, we can see that the accuracy of the calculation depending on several variables. The main variable is the coefficient of $O_j$, by analyzing some of the error data yields figure [1]

Figure [1] The curve of *n* versus error due to ***m***.




In this plot, the other data were made fix (i.e. $s$=1 and $a$=1). The figure [1] shows that the error decreases as the value of $m$ increases. As $m$ getting larger, the error approaches the zero line. It is also can be seen that the error increases as the term $n$ increases. Therefore, to reduce the error the value of $m$ should also be increased.

The others factor that my influence the accuracy are $s$ and $a$ variables. The plots of these variables against $n$ are given as in the figure [2] and figure [3].

Figure [1] The curve of $n$ versus error due to $s$.

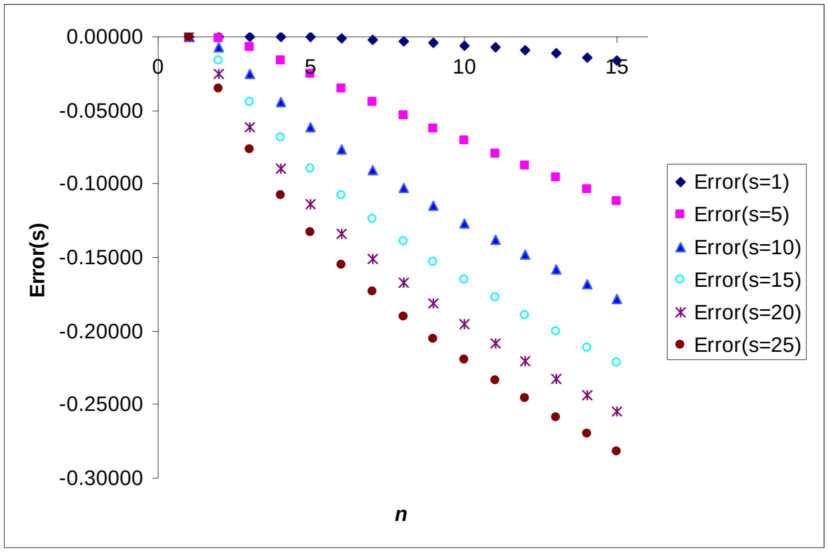
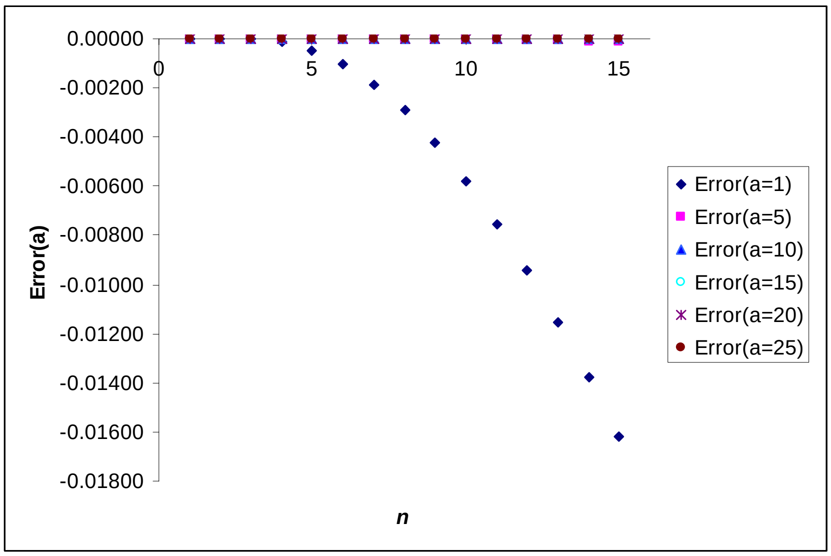



Figure [1] The curve of *n* versus error due to *a*.




**Discussion.**

Figure [2] shows that, as *s* increases while *a* remaining the same, the error increases with larger *s* and increment of *n*. While in figure [3], it shows that if the value *a* is made increased the error decreases rapidly to zero. Therefore, for larger symmetric function due to larger first term *a* or $x_1$, it is enough to have few coefficients of $O_j$.

**Conclusion.**

It has been shown that the general equation for Sum of Power presented in this paper works perfectly with the positive real power *p*. This offers a new way of calculating the approximation of the sums of power when the power *p* is a non-integer which is not possible using other method derived by Faulhaber [2] and its extension by William et al [3]. It is suggested that the generalized sums of power formulation is working well with the negative and complex *p*, thus offering a new study in the Riemann's Zeta function and reciprocal numerical power series.

References:

[1] Abd Shukor, M.S., 2010, “A Treaty of Symmetric Function”, An Approach in Deriving General Formulation for Sums of Power for an Arbitrary Arithmetic Progression and Applying the Method Formulated for Expressing Fermat’s Last theorem and Riemann Zeta Function into Symmetric Function. The Generalize equation also leads to the formulation of a new set of Prime Numbers in which Mersenne and Wagstaff numbers fall under it.

[2] Johann Faulhaber, Academia Algebræ, Darinnen die miraculosische Inventiones zu den hochsten Cossen weiters continuirt und profitiert werden. Augspurg, bey Johann Ul-rich Schonigs,1631. (Call number QA154.8 F3 1631a f MATH at Stanford University Libraries.)

[3] William Y.C. Chen, Amy M. Fu and Iris F. Zhang**,** Faulhaber’s theorem on power sums, Discrete Mathematics Volume 309, Issue 10, 28 May 2009, Pages 2974-2981.