Implementing proper length on a graph in **R**3 via polymerization produces negligible entropy when accounting for the distinctness of scattering probability distributions

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Abstract

A graph model of general relativity is discussed. The relationship between entropy and the incompressibility of information is highlighted.

1 Graphs

In [1], a graph model of the Universe called "Quantum Graphity" is discussed. The Quantum Graphity model considers the Universe to consist of a set of vertices, as well as edges that form links between them. The propagation of light from one vertex to another along an edge is considered to occur at some constant rate, which defines both a fundamental length and time.

At the moment of the Big Bang, the vertices formed a high-temperature complete graph where each and every vertex was linked to all other vertices. Because every location (vertex) in the Universe was *directly* path-connected to all other locations, the concept of long-distance did not apply at the moment of the Big Bang.

A short while after the Big Bang however, the Universe cooled and most of the edges vanished. This "phase change" left behind a low-temperature incomplete graph that forms the very structure of today's Universe. In today's Universe, light generally must now pass through many intermediate locations as it traverses the path from one specific location to another, thus taking many units of fundamental time to do so. This emergent sense of long-distance is known in the Quantum Graphity model as geometric length and time.

In this paper it is assumed that $\hbar = c = G = 1$, that one unit of coordinate length (e.g., the fundamental length) is the Planck length $\ell_p = 1$, and that one unit of coordinate time is the Planck time $t_p = 1$.

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2 Proper length via polymerization

In Einstein's general theory of relativity, the length of space and time are relative measures that depend on light's coordinate distance from sources of gravitation.

In the vicinity of a gravitational source (e.g., a black hole), space is contracted (e.g., compressed) and time is dilated, and measurements are made using what is known as proper length and time. For instance, light passing through the space near a gravitational source is delayed (e.g., the light takes a longer amount of coordinate time to reach its destination) because the light's path has a proper length that is longer than its coordinate length. Oppositely, in the absence of all gravitational sources, the light's path has a proper length that is equal to its coordinate length, and no such delay occurs. It is precisely this long-distance path of light through space that gives rise to geometric length (in both the proper and coordinate senses).

Because the contraction of space increases *gradually* as coordinate distance from the gravitational source decreases, light's path through space also bends in toward the gravitational source. This is somewhat similar to how the path of Earth through space is bent so strongly by the presence of the Sun that the path practically forms an ellipse. In turn, it is the gradient of this gradient that ultimately results in gravitational tidal forces, such as those regularly undergone by our oceans on Earth due to the presence of the Sun and the Moon.

One popular analogy by Sir Arthur Eddington compares gravitational acceleration to common optical refraction. In both cases, a "gradient of density" exists, and this causes light to 1) slow down, and 2) turn in toward the denser region (e.g., light always takes the path of least resistance).

In [2], the Schwarzschild black hole is modelled as a partially complete graph in $\mathbf{R}3$. First, a set of vertices V_0 are roughly uniformly distributed along the black hole's event horizon at a coordinate distance of $R_0 = 2V_0$ from the black hole's centre. V_0 are then used to form a complete graph's worth of edges E_C. A second set of vertices $V_{\geq 1}$ are then distributed along the black hole's exterior region. The union of V_0 and $V_{\geq 1}$ forms a final vertex set V_{BH}, which is then used to form a Delaunay tetrahedralization's worth of edges E_D. The union of E_C and E_D forms a final edge set E_{BH}, which in conjunction with V_{BH} ultimately defines the shape of the entire Schwarzschild black hole, interior, event horizon, and exterior alike.

This Schwarzschild black hole model produces an average edge coordinate length of

$$L = \frac{1}{\sqrt{1 - R_0/r}}, \tag{1}$$

where r represents the coordinate distance of an edge's midpoint from the black hole's centre. At $r \gg R_0$, edge coordinate length is roughly equal to the fundamental length (e.g., $L \approx 1$).

Fig. 1 illustrates a portion of an incomplete graph in $\mathbf{R}2$ based on this model. If one adopts the idea from Quantum Graphity that light propagates from one vertex to another at a constant rate, then this Schwarzschild black hole model immediately breaks down. Because L increases as r decreases, light would speed up as it approaches the black hole's centre, not slow down as required. An obvious solution to this problem is to perform a second-stage discretization, where new intermediary vertices are added along the edges E_{BH} as required. Ultimately, any edge in E_{BH} that has a proper length L^2 not roughly equivalent to its coordinate length L is transformed by the second-stage discretization into a polymer (e.g., a one-dimensional chain of vertices). Fig. 2 illustrates the incomplete graph from Fig. 1 after it has undergone second-stage discretization.

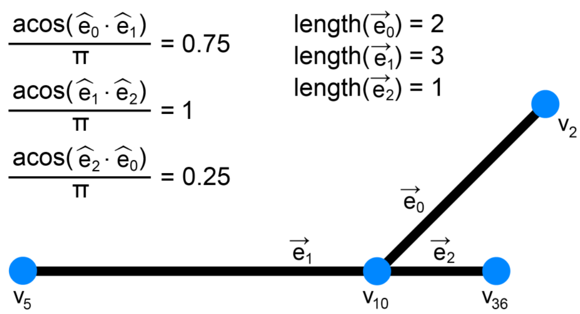

Figure 1: A portion of an incomplete graph in **R**2, with edges of varying coordinate length.

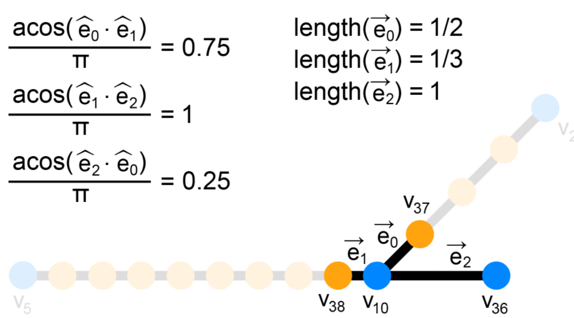

Figure 2: The incomplete graph from Fig. 1, after undergoing second-stage discretization.

In practice, edge coordinate length is usually not an integer value, and so there is some uncertainty as to how many new intermediary vertices are required to polymerize an edge. The probability that the required number of new intermediary vertices v_{poly} will be $\lfloor L^2 \rfloor$ is

$$P_0 = L^2 \bmod \lfloor L^2 \rfloor, \tag{2}$$

and the probability that the number v_{poly} will instead be $\lfloor L^2 \rfloor - 1$ is

$$P_1 = 1 - P_0. \tag{3}$$

With regard to the incomplete graph illustrated in Fig. 1, the edges for vertex v_{10} are described by a set of neighbouring vertex identifiers

$$\text{neighbours} = \begin{bmatrix} 2 & 5 & 36 \end{bmatrix}, \tag{4}$$

and a set of the corresponding edge coordinate lengths

$$\mathrm{L} = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}. \tag{5}$$

After undergoing second-stage discretization, as in Fig. 2, the edges for vertex v_{10} are described by

$$\text{neighbours}' = \begin{bmatrix} 37 & 38 & 36 \end{bmatrix}, \tag{6}$$

and

$$\mathrm{L}' = \frac{\mathrm{L}}{1 + \mathrm{v}_{\mathrm{poly}}} = \begin{bmatrix} 1/2 & 1/3 & 1 \end{bmatrix}. \tag{7}$$

This Schwarzschild black hole model now causes light to slow down as it approaches the black hole's centre when adopting the idea that light propagates from one vertex to another at a constant rate.

During the course of the propagation of light, a decision must be made at each vertex as to which edge the light will propagate along next. One tool for making such a decision is a scattering matrix. With regard to Fig. 2, a toy scattering matrix $\widehat{\mathbf{F}}$ for vertex v_{10} is

$$\begin{aligned} \mathbf{F}_{ij} &= \frac{1}{\mathrm{L}'_j} \frac{\operatorname{acos}(\widehat{\mathrm{e}}_i \cdot \widehat{\mathrm{e}}_j)}{\pi} \\ &= \begin{bmatrix} 2\times 0 & 3\times 0.75 & 1\times 0.25 \\ 2\times 0.75 & 3\times 0 & 1\times 1 \\ 2\times 0.25 & 3\times 1 & 1\times 0 \end{bmatrix} = \begin{bmatrix} 0 & 2.25 & 0.25 \\ 1.5 & 0 & 1 \\ 0.5 & 3 & 0 \end{bmatrix}, \end{aligned} \tag{8}$$

$$\sum_j \mathbf{F}_{ij} = \begin{bmatrix} 2.5 \\ 2.5 \\ 3.5 \end{bmatrix}, \tag{9}$$

$$\widehat{\mathbf{F}}_{ij} = \frac{\mathbf{F}_{ij}}{\sum_j \mathbf{F}_{ij}} = \begin{bmatrix} 0 & 2.25/2.5 & 0.25/2.5 \\ 1.5/2.5 & 0 & 1/2.5 \\ 0.5/3.5 & 3/3.5 & 0 \end{bmatrix}, \tag{10}$$

$$\sum_j \widehat{\mathbf{F}}_{ij} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{11}$$

For example, when light arrives at vertex v_{10} from v_{37} along edge e_0 (e.g., row $i = 0$ in Eq. 10), the probability that the light will continue propagation by heading toward v_{38} along edge e_1 (e.g., column $j = 1$) is $2.25/2.5$. Otherwise, the light will continue propagation by heading toward v_{36} along edge e_2 (e.g., column $j = 2$). Because the trace of $\widehat{\mathbf{F}}$ is zero, back-propagation is entirely disallowed.

As is with all 2-valent vertices, the toy scattering matrix for both vertices v_{37} and v_{38} is simply

$$\widehat{\mathbf{F}}_{ij} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{12}$$

Via $\widehat{\mathbf{F}}$, this Schwarzschild black hole model now causes the path of light to most probably bend in toward the black hole's centre.

3 On entropy and the distinctness of scattering probability distributions

Entropy is a measure of the *distinctness* of information. In this model, information is assumed to refer to the individual scattering probability distributions $\widehat{\mathbf{F}}_i$ given by each row of a vertex set's scattering matrices.

Consider a model universe consisting of $n = 4$ vertices at the moment of a big bang, where back-propagation is disallowed. All $n = 4$ scattering matrices are the same simple $(n-1) \times (n-1)$ matrix

$$\widehat{\mathbf{F}}_{ij} = \begin{bmatrix} 0 & 1/(n-2) & 1/(n-2) \\ 1/(n-2) & 0 & 1/(n-2) \\ 1/(n-2) & 1/(n-2) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}. \quad (13)$$

Since this scattering matrix is the same for all $n = 4$ vertices, the number of distinct scattering matrix rows (e.g., entropy) is $n-1$. However, if back-propagation is allowed

$$\widehat{\mathbf{F}}_{ij} = \begin{bmatrix} 1/(n-1) & 1/(n-1) & 1/(n-1) \\ 1/(n-1) & 1/(n-1) & 1/(n-1) \\ 1/(n-1) & 1/(n-1) & 1/(n-1) \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}, \quad (14)$$

then the number of distinct scattering matrix rows is only 1.

With regard to a model Schwarzschild black hole, the distribution of the n vertices V_0 along the event horizon is usually not quite perfectly uniform in practice, and so while all of the n scattering matrices are indeed similar to a very large degree, there are always small deviations in the individual elements of each of the $n^2 - n$ rows that ultimately ensure that all $n^2 - n$ rows are distinct. As such, the model Schwarzschild black hole's entropy is $n^2 - n + 2$. The final constant term in this entropy calculation reflects that entropy is roughly independent of the number of 2-valent vertices that were added along E_C during the process of second-stage discretization. This is because all 2-valent vertices share the same scattering matrix (e.g., see Eq. 12).

The aforementioned small deviations in the individual elements of each of the scattering matrix rows of a model Schwarzschild black hole are indicative of the fate of a model universe at the moment of a big freeze, where entropy is maximal. That is, for a model universe consisting of $n = 4$ vertices at the moment of a big freeze, all n scattering matrices should be similar to a very large degree, but there should always be small deviations in the individual elements of each of the $n^2 - n$ rows that ultimately ensure that all $n^2 - n$ rows are distinct.

Since entropy is a measure of the distinctness of information, it is also a measure of the *incompressibility* of information. For example, when using a simple lossless dictionary coder algorithm [3] to store the scattering matrix rows in offline storage (e.g., in a file on a hard disk drive), a certain level of compression can be achieved because only the distinct rows are actually stored. Afterward, during retrieval from offline storage, the set of all rows is fully reconstructed from the set of distinct rows. The incompressibility of the information is

$$\mathrm{I} = \frac{\text{Number of distinct scattering matrix rows}}{\text{Number of all scattering matrix rows}} = \frac{\text{Entropy}}{\text{Maximal entropy}}. \quad (15)$$

For example, a model universe consisting of $n = 4$ vertices at the moment of a big bang gives an incompressibility of

$$\mathrm{I} = \frac{1}{n^2 - n} = 0.08 \tag{16}$$

if back-propagation is allowed. However, if back-propagation is disallowed, then the incompressibility is

$$\mathrm{I} = \frac{n - 1}{n^2 - n} = 0.25. \tag{17}$$

To compare, a model universe consisting of $n = 4$ vertices at the moment of a big freeze gives an incompressibility of

$$\mathrm{I} = \frac{n^2 - n}{n^2 - n} = 1. \tag{18}$$

In conclusion, this model finds that a polymer possesses a constant amount of entropy, regardless of its length. This is contrary to the findings of string theory, where a polymer (e.g., a one-dimensional string) possesses an entropy that is roughly proportional to its length [4]. This apparent discrepancy shall be the focus of future work.

References

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