On the Accelerating Universal Expansion

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Abstract

Repulsive gravity at large distances has been included in the universal solution of Einstein's equations by introducing a cosmological constant, which excludes the dark energy interpretation. For an external-coordinate-observer cosmological model, the big-bang singularity has been replaced by a granular primeval particle, and expansion is controlled by the velocity of light. Then problems inherent in the standard model do not arise, and no inflation phase is necessary. It is advantageous to truncate the graviton field at a maximum radius, which is related to proton dimensions through the ratio (e^2/Gm^2). This governs the onset of universal repulsion at around 7Gyr, in rough agreement with observations of Type Ia supernovae.

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1. Introduction

Various observations indicate that the expansion of the Universe is not slowing with time as previously expected, but is accelerating: see Riess et al. [1]; Perlmutter et al., [2]; Riess et al.[3]; Tonry et al., [4]; Kirshner, [5]; Kirshner et al. [6]. Thus, gravity has apparently become anti-gravity at very large distances, yet remains normal within observed clusters of galaxies. This new phenomenon may be incorporated into the universal solution of Einstein's equations of general relativity, wherein gravity is attributed to physically real energetic graviton fields rather than the hypothesis of space-time curvature; see Wayte, [7], Paper 1. In the literature, dark energy of negative pressure has been added into Einstein's gravitation theory to satisfy the requirements of the space-time curvature hypothesis. We will introduce a cosmological constant Λ as a measure of gravitational repulsion, which is inherent in graviton-graviton interactions at large distances, due entirely to their own nature.

Before setting-up a cosmological model, some ways of introducing repulsion into general gravity will be investigated, to find the most realistic way.

2. Exterior field of a spherically-symmetric static body

Einstein's equations describing the spherically-symmetric static field in polar coordinates will be used, (see Tolman, [8], for clear notation). For the line element:

$$ds^2 = -e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 + e^{\nu}dt^2, \tag{2.1}$$

the surviving components of the energy-momentum tensor are:

$$8\pi\left(G/c^4\right)T_1^1 = -e^{-\lambda}\left(\nu'/r + 1/r^2\right) + 1/r^2, \tag{2.2a}$$

$$8\pi\left(G/c^4\right)T_2^2 = 8\pi\left(G/c^4\right)T_3^3 = -e^{-\lambda}\left\{\nu''/2 - \lambda'\nu'/4 + \nu'^2/4 + \left(\nu' - \lambda'\right)/2r\right\}, \tag{2.2b}$$

$$8\pi\left(G/c^4\right)T_4^4 = e^{-\lambda}\left(\lambda'/r - 1/r^2\right) + 1/r^2 \quad . \tag{2.2c}$$

From Paper 1, we have $(e^{-\lambda} = e^{\nu} = \gamma^2)$, $(T_1^1 = -T_2^2 = -T_3^3 = T_4^4 = GM^2/8\pi r^4)$, that is gravitational field properties analogous to electromagnetic theory. Then normal gravitational potential is described by the metric tensor component:

$$\gamma = \left(1 - GM/c^2 r\right) = \left(1 - r_0/r\right) \quad , \tag{2.3a}$$

which ranges from zero at gravitational radius (r_o = GM/c^2), to unity at $(r = \infty)$. The gravitational field is then:

$$F = -GM/r^2 = -c^2\left(d\gamma/dr\right) \quad . \tag{2.3b}$$

Evidently, anti-gravity could be produced beyond some arbitrary radius, simply by adding a term which would change the negative sign in Eq.(2.3a) to positive. Then the observations would require γ to increase from zero at r_o , pass through unity and continue to increase for a while before falling towards unity as r carries on to infinity. However, it is impossible to accommodate this particular type of variation in γ without incurring un-physical negative energy, for the following reason. From Eqs.(2.2a,b,c) the energy momentum tensor components may be reduced to:

$$8\pi\left(\frac{G}{c^4}\right)T_1^1 = 8\pi\left(\frac{G}{c^4}\right)T_4^4 = -\frac{2\gamma}{r}\left(\frac{d\gamma}{dr}\right) + \left(\frac{1-\gamma^2}{r^2}\right) \quad , \tag{2.4a}$$

$$8\pi\left(\frac{G}{c^4}\right)T_2^2 = 8\pi\left(\frac{G}{c^4}\right)T_3^3 = -\frac{1}{2r^2}\frac{d}{dr}\left\{r^2\frac{d}{dr}\left(\gamma^2\right)\right\} \quad . \tag{2.4b}$$

Substitution of Eq.(2.3a), as it is, gives the gravitational field energy density as positive for all radii:

$$T_4^4 = +GM^2/8\pi r^4 \quad . \tag{2.4c}$$

But for any transition from gravity to anti-gravity in which γ must increase and pass through unity, T_4^4 in Eq.(2.4a) becomes negative around $\gamma = 1$. This is not physically realistic, so an alternative form of γ is necessary, involving the cosmological constant as follows.

The cosmological constant **Λ** is introduced into Einstein's equations in the usual way (see [8], p 242), by adding it to the right side of Eqs.(2.4a,b):

$$8\pi\left(\frac{G}{c^4}\right)T_1^1 = 8\pi\left(\frac{G}{c^4}\right)T_4^4 = -\frac{1}{r}\left(\frac{d\gamma^2}{dr}\right) + \left(\frac{1-\gamma^2}{r^2}\right) - \Lambda \quad , \tag{2.5a}$$

$$8\pi\left(\frac{G}{c^4}\right)T_2^2 = 8\pi\left(\frac{G}{c^4}\right)T_3^3 = -\frac{1}{2r^2}\frac{d}{dr}\left\{r^2\frac{d}{dr}\left(\gamma^2\right)\right\} - \Lambda \quad . \tag{2.5b}$$

Then T_4^4 will keep the same positive value given in Eq.(2.4c), and γ will remain less than unity, if it has a new form:

$$\gamma^2 = \left[\left(1-\frac{r_0}{r}\right)^2 - \left(1-\frac{r_0{}^3}{r^3}\right)\frac{\Lambda r^2}{3}\right] \quad . \tag{2.6}$$

That is, by putting this γ^2 into Eq.(2.5a), field energy density remains at exactly the previous value of Eq.(2.4c) by choice, independent of Λ. Similarly, substitution into Eq.(2.5b), produces the same momentum density as would be found by putting Eq.(2.3a) in Eq.(2.4b). Consequently, the natural graviton field has the *inherent* capacity for repulsive gravity at large radii, without adding foreign dark energy. Coefficient Λ is the size of this effect and represents a *modification of the graviton field behaviour* without changing the field energy/momentum density. It is remarkable that Einstein's equations should include long-range repulsion so efficiently; but of course this choice of γ^2 has to satisfy observations. One dubious interpretation of Λ is that all space-time is occupied by ethereal dark energy/momentum, which exerts ever increasing repulsion to infinity in an open universe.

Herein, the field strength in the weak case is derivable from Eq.(2.6) as:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) \approx -c^2\left[\frac{r_0}{r^2} - \frac{\Lambda r}{3}\right] \quad . \tag{2.7}$$

Clearly, this field changes from attractive to repulsive at a particular radius,

$$r_a \approx \left(3r_0 / \Lambda\right)^{(1/3)} \quad . \tag{2.8}$$

However, as radius r increases to infinity, there is no theoretical limit to the repulsive force in Eq.(2.7) even though the field energy density T_4^4 falls rapidly towards zero. A precise reach of gravitons is more realistic than assuming they go to infinity, therefore the graviton field needs to be limited to a maximum radius r_m . For overall consistency, this will be chosen so as to set the total gravitational field energy at (½Mc^2), as in Paper 1 but now by only integrating T_4^4 from r_0 to r_m . Such field conservation and limitation could be achieved in a physical sense by steadily strengthening each graviton prior to r_m. To implement this, γ^2 will be modified to:

$$\gamma^2 = \left\{\left(1 - \frac{r_0}{r}\right)^2 - \left(1 - \frac{r_0^{\;3}}{r^3}\right)\left[\frac{\Lambda r^2}{3} + \frac{r_0^{\;2} r}{(r_m^{\;3} - r_0^{\;3})}\right]\right\} \quad . \tag{2.9}$$

Intuitively, r_a in Eq.(2.8) should be related to maximum radius r_m . Upon introducing this latest expression for γ^2 into Eq.(2.5a), we find that the field *energy density* is more complicated than the simple form of Eq.(2.4c), but remains independent of Λ, namely:

$$8\pi\left(\frac{G}{c^4}\right)T_4^4 = \frac{r_0^{\;2}}{r^4} + \frac{r_0^{\;2}}{(r_m^{\;3} - r_0^{\;3})}\left(\frac{2}{r} + \frac{r_0^{\;3}}{r^4}\right) \quad , \tag{2.10a}$$

At ($r = r_m$) this will approximate to:

$$8\pi\left(\frac{G}{c^4}\right)T_4^4 \approx \frac{r_0^{\;2}}{r_m^{\;4}} + \frac{2r_0^{\;2}}{r_m^{\;4}} \quad . \tag{2.10b}$$

The exact field strength is derived from Eq.(2.9) as:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) = -\frac{c^2}{\gamma}\left[\frac{r_0}{r^2} - \frac{r_0^{\;2}}{r^3} - \frac{\Lambda r}{3}\left(1 + \frac{r_0^{\;3}}{2r^3}\right) - \frac{r_0^{\;2}}{2(r_m^{\;3} - r_0^{\;3})}\left(1 + \frac{2r_0^{\;3}}{r^3}\right)\right], \tag{2.11}$$

which will approximate to Eq.(2.7).

The first field energy term on the right side of Eq.(2.10b) could be proposed as the usual attractive component of gravity, and the second term to represent repulsion; but this would be questionable since Λ is not present in Eq.(2.10a),. Consequently, at this stage we can only say that Λ describes how the graviton field inherently changes its force character smoothly from attractive to repulsive at large radii; and the form of this repulsion can be revealed by introducing Eq.(2.8) into Eq.(2.7), with ($r_o = GM/c^2$):

$$F \approx -\frac{GM}{r^2}\left[1-\left(\frac{r}{r_a}\right)^3\right] \quad . \tag{2.12}$$

The first term on the right represents the usual characteristic areal flux density of gravitons through a spherical surface. Then the negative cubic term suggests there is an internal mechanism for each individual graviton, which determines the strength of repulsion. Graviton propagation velocity is maintained at the velocity of light throughout, since $T_1^1 = T_4^4$ always. And for compatibility, we will presume that Λ is always proportional to r_o, then r_a is constant and the gravitational force is proportional to mass. If this were not so, then the force in Eq.(2.7) could change sign simply by making r_o very small. We will find shortly that ($\Lambda = 3r_o/r_a^3$) from Eq.(2.8) is the only viable choice.

3. Interior field of a static spherically-symmetric body

Although the aim of this paper is to present a model for the accelerating universe, it is informative to calculate some properties of a static universe.

3.1 Solid static spherical body

Einstein's equations (2.5) may be solved to get the interior gravitational field for a solid sphere of uniform material density ρ and zero pressure. Given the essential requirement of *compatibility with Newtonian gravitation*, then Eq.(2.5a) has to yield the metric tensor component:

$$\gamma^2 = 1 + \frac{8\pi G}{3c^2}\rho\frac{r^2}{2} - \frac{\Lambda}{3}r^2 \ , \tag{3.1}$$

so that field strength is given by:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) = -\frac{1}{\gamma}\left(\frac{4\pi G}{3}\rho - \frac{\Lambda c^2}{3}\right)r \quad . \tag{3.2}$$

These require ($T_4^4 = -\rho c^2/2$), which represents the *energy density* of an attractive field. According to Eq.(3.1), gravitational potential increases outwards from the centre, and density ρ could apparently be decreased to make the field repulsive; (even for a very small body). However, the constant Λ represents a repulsive modification to the existing attractive field and will be proposed to depend on the total body mass M_x, thus:

$$\left(\frac{\Lambda}{3}\right)=\frac{r_o}{r_a{}^3}=\frac{GM_x}{r_a{}^3c^2} \quad , \tag{3.3}$$

using ($r_0 = GM_x/c^2$). Then, for $(M_x = (4\pi/3)\rho r_x{}^3)$ in general, we have from Eq.(3.2) the field at any radius r, within maximum radius r_x :

$$F \approx -\left(\frac{GM_x}{r_x{}^3}-\frac{GM_x}{r_a{}^3}\right)r \quad , \tag{3.4}$$

which is compatible with Eq.(2.12) for ($r = r_x$). Zero field occurs *everywhere* in the bulk when the sphere radius r_x is increased to r_a . If r_x is increased further, the whole field becomes repulsive, with strength dependent on position r within the body.

Now, compatibility with Newtonian gravitation only resulted from the use of line element Eq.(2.1) in Eq.(2.5) by putting T^μ_μ in terms of the field energy/momentum density ($-\rho c^2/2$). Had we used the mechanical density and pressure expressions, ($T^1_1 = T^2_2 = T^3_3 = -p_o$, and $T^4_4 = \rho_{oo}c^2$) with Eq.(2.1) in Einstein's equations, then we would have had the incompatible result ([8], p246):

$$e^{-\lambda} = 1-\frac{8\pi G}{3c^2}\rho_{oo}r^2-\frac{\Lambda}{3}r^2 \quad , \tag{3.5}$$

which describes *increase in potential* upon climbing towards the centre, that is anti-gravity. A cosmological model cannot be built upon this foundation.

3.2 Fluid static spherical body

When pressure is not negligible, the material needs to be considered as a "perfect fluid". Then in view of the isotropic nature of hydrostatic pressure, the line element for a spherically-symmetric body is expressed in isotropic form ([8], p 244):

$$ds^2 = -e^\mu\left(dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\right)+e^\nu dt^2 . \tag{3.6}$$

The previous line element Eq.(2.1) will not lead to sensible physical results compatible with Newtonian theory, nor to Eq.(3.1) in the weak field. For the energy-momentum tensor components we take the local hydrostatic pressure and constant local mass density,

$$T^1_1 = T^2_2 = T^3_3 = -p_o \ , \quad \text{and} \quad T^4_4 = \rho_{oo}c^2 \ .$$

Then Einstein's equations yield the surviving components:

$$8\pi\left(\frac{G}{c^4}\right)p_o = e^{-\mu}\left(\frac{\mu'^2}{4}+\frac{\mu'\nu'}{2}+\frac{\mu'+\nu'}{r}\right)+\Lambda \tag{3.7a}$$

$$8\pi\left(\frac{G}{c^4}\right)p_o = e^{-\mu}\left(\frac{\mu''}{2}+\frac{\nu''}{2}+\frac{\nu'^2}{4}+\frac{\mu'+\nu'}{2r}\right)+\Lambda \tag{3.7b}$$

$$8\pi\left(\frac{G}{c^4}\right)\rho_{oo}c^2 = -e^{-\mu}\left(\mu''+\frac{\mu'^2}{4}+\frac{2\mu'}{r}\right)-\Lambda \quad . \tag{3.7c}$$

Solution of Eq.(3.7c) produces the metric tensor component:

$$e^{-\mu} = \left(1+\frac{8\pi G}{3c^2}\rho_{oo}\frac{r^2}{4}+\frac{\Lambda}{3}\frac{r^2}{4}\right)^2 , \tag{3.8}$$

which is compatible with Eq.(3.1) in the weak field, when the arbitrary Λ-term is defined as negative for repulsion. The field strength is also compatible with Eq.(3.2), when given by:

$$F = -c^2\frac{d}{dr}\left(e^{-\mu/2}\right) = -\left(\frac{4\pi G}{3}\rho_{oo}+\frac{\Lambda c^2}{6}\right)r \ . \tag{3.9}$$

Therefore, this isotropic form of solution is most probably suitable for describing an isotropic universe with effective pressure.

4. Standard cosmology model

Now that the phenomenon of gravitational repulsion has been explained as an inherent property of the gravitons from all mass particles, it is possible to quantify the observed universal acceleration. We shall first consider the Standard Model in order to identify its numerous failings, prior to developing an improved model in Section 5. Thus, it is normal to employ the Robertson-Walker metric:

$$ds^2 = -R^2(t)\left\{\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\right\}+dt^2 \quad , \tag{4.1}$$

yet according to the analysis above leading to Eq.(3.5), problems could arise when compatibility with Newtonian gravitation is required. We shall therefore use the metric proposed by Tolman [8], p377, explicitly for isotropic coordinates. In more practical units this can be written:

$$ds^2 = -\frac{a^2(t)}{(1+kr^2/4)^2}\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\} + dt^2 \quad , \tag{4.2}$$

where *a(t)* is a universal scale factor, and *t* is local/cosmic time. Coefficient *k* covers expansion kinetic energy, and may be negative, positive, or zero for an open, closed, or critical universe, respectively. The components of the energy-momentum tensor are to be in terms of local pressure and density:

$$T_1^1 = T_2^2 = T_3^3 = -p \ , \ \text{and} \quad T_4^4 = \rho c^2 \ . \tag{4.3}$$

Upon applying these expressions to Einstein's field equations, we obtain:

$$-\frac{8\pi G}{c^2} p = \frac{kc^2}{a^2} + 2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \Lambda c^2 \tag{4.4}$$

$$\frac{8\pi G}{3} \rho = \left(\frac{kc^2}{a^2}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \frac{\Lambda c^2}{3} \quad . \tag{4.5}$$

These results may be manipulated to get the following Friedmann-Lemaitre equations; the same as would have been found for the Robertson-Walker metric:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{4.6}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{4.7}$$

$$\dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)\left(\rho + \frac{p}{c^2}\right) \quad , \tag{4.8}$$

where H is the Hubble parameter. This means that the universal scale factor is the same for metric Eq.(4.2) as for Eq.(4.1); but nevertheless, it is important to employ the correct physical description of every mathematical expression.

In order to realise these expressions in physical terms, we will now let a(t) in Eq.(4.2) take units of length, and leave *r* dimensionless. Then, given a nominal mass M_U for the whole universe and the observed values in Eq.(4.10), Figure 1 depicts the expansion radius, velocity and acceleration as a function of time; (here 'a' is explicitly taken to represent the radius of the material universe, and Λ is governed by M_U as in Eqs.(3.3) and (4.16)). Clearly, *superluminal* expansion velocities can exist in this model universe, albeit Einstein's equations are valid up to the velocity of light; see Davis & Lineweaver [9].

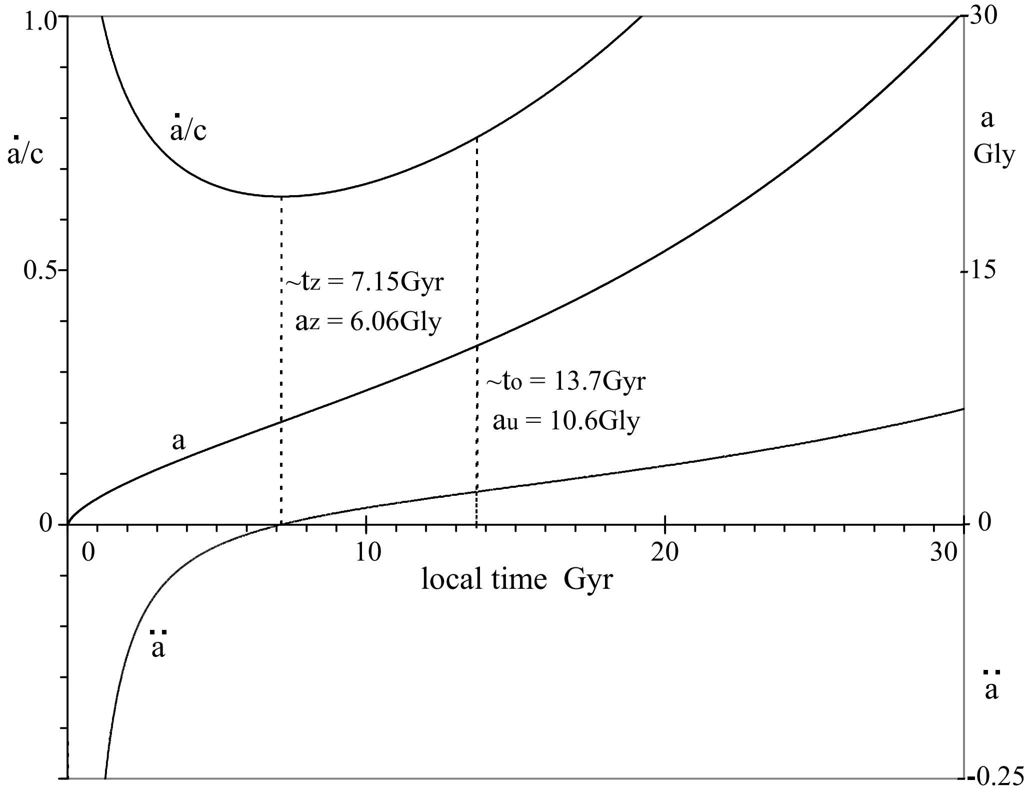

Figure 1. Friedmann-Lemaitre model: variation of expansion velocity relative to the velocity of light $(\dot{a}/c)$, radius (a, Gly), and acceleration (ä) with universal time (t, Gyr). Universal mass has been set at $M_U = (4/3)\pi\rho a^3 = 1.073\text{x}10^{52}$ kg, with the change from deceleration to accelerated expansion occurring at radius 6.06Gly corresponding to epoch 7.15Gyr from the big-bang. The present age of the universe is 13.7Gyr and its radius is 10.6Gly.

In a review article by Coles [10], it is shown how Eq.(4.6) can be conveniently expressed as:

$$1 \approx \Omega_m + \Omega_k + \Omega_\Lambda \quad , \tag{4.9}$$

where experimentally, these components take the latest (WMAP + BAO + SN Mean) observed values, from Komatsu et al [11]:

$$\Omega_m = \left(\frac{8\pi G\rho_0}{3H_0{}^2}\right) \approx 0.274\,, \quad \Omega_k = \left(\frac{-kc^2}{a^2H_0{}^2}\right) \approx 0\,, \quad \Omega_\Lambda = \left(\frac{\Lambda c^2}{3H_0{}^2}\right) \approx 0.726\ . \tag{4.10}$$

Consequently, we can evaluate Λ and ρ_o , using the latest Hubble constant value ($H_0 \sim 70.5 kms^{-1}Mpc^{-1}$):

$$\Lambda \approx 3(0.726){H_0}^2 / c^2 = 1.265 \times 10^{-52} m^{-2} \quad , \tag{4.11}$$

$$\rho_0 \approx \frac{3}{8\pi G}(0.274){H_0}^2 = 2.56 \times 10^{-27} kgm^{-3} \quad . \tag{4.12}$$

Given these values, Eq.(4.6) may be solved to get the expansion age of the universe, t_0 . For negligible pressure and a universal mass ($M_U = (4/3)\pi\rho\, a^3$), we have:

$$H = \left(\frac{\dot{a}}{a}\right) \approx \left[\frac{2GM_U}{a^3} + \frac{\Lambda c^2}{3}\right]^{1/2} \quad , \tag{4.13}$$

then upon integration,

$$t_0 = \left(\frac{1}{3H_0{\Omega_\Lambda}^{1/2}}\right) \ln\left[\frac{(1+{\Omega_\Lambda}^{1/2})}{(1-{\Omega_\Lambda}^{1/2})}\right] \approx 13.7 Gyr \ . \tag{4.14}$$

It is also possible to calculate the time when universal deceleration changed smoothly to acceleration. The general time /radius relationship is:

$$t = \left(\frac{2}{3H_0{\Omega_\Lambda}^{1/2}}\right) \ln\left[\left\{1 + \frac{\Lambda c^2/3}{2GM_U}(a^3)\right\}^{1/2} + \left\{\frac{\Lambda c^2/3}{2GM_U}(a^3)\right\}^{1/2}\right] , \tag{4.15}$$

and from Eq.(4.7), when $\ddot{a} = 0$ at radius $a = a_z$, and $p \approx 0$, we have:

$$\frac{4\pi G}{3}\rho_z \approx \frac{GM_U}{{a_z}^3} \approx \frac{\Lambda c^2}{3} \quad . \tag{4.16}$$

Therefore by substitution, the zero-field time is governed by the cosmological constant:

$$t_z \approx (0.439)\left(\frac{\Lambda c^2}{3}\right)^{-1/2} = 7.15 \text{ Gyr.} \tag{4.17}$$

In Eq.(4.16), the value of mass M_U depends on radius a_z , which has not been specified so far. By setting a_z equal to the proposed value, 6.06Gly in Section 6.2, we get the universal mass ($M_U = 1.073 \times 10^{+52}$ kg). Then given the present density from Eq.(4.12), the current universal outer radius must be ($a_u = 10.6$Gly).

It is possible to calculate the observed redshift of any supernovae which occurred at the time of zero-field, t_z . From Eq.(4.10) we have:

$$\frac{\Omega_\Lambda}{\Omega_m} = \frac{0.726}{0.274} = \frac{(\Lambda c^2/3)}{(2GM_U/{a_u}^3)} \quad , \tag{4.18}$$

which with the introduction of Eq.(4.16) yields a redshift independent of H_0 :

$$z_z = \frac{a_u}{a_z} - 1 = 0.74 \quad . \tag{4.19}$$

In conclusion, we may say that the standard big-bang model of the universe has always had non-Einsteinian characteristics, but now the expansion is also super-luminal at large radii, where Λ dominates. This super-luminal expansion of the space-time manifold occurs between galaxies but is not detectable within galaxies. The ethereal nature of space-time originating at the big-bang singularity, plus inexplicable inflation, is questionable. Apparently, there was not even void outside the primeval singularity into which the big-bang material could expand; yet infinite space and enough material were instantaneously created for the observed flat universe. Total energy, mass and size are not definitive, even though real mass density is postulated. Now continuous creation of dark energy throughout infinite space is also required, to add to inherent flatness- and horizon-problems. Together, these confounding characteristics represent a rejectable fantasy for any perspicacious physicist.

5 External coordinate observer cosmology: the ECO-model

The above standard cosmology model has not included the possibility that the local observer's time might be dilated by his own universal motion. We shall now consider the universal expansion from the point of view of an external coordinate observer, located at rest outside of the material universe, in field-free Minkowski spacetime. In order to satisfy Einstein’s most basic relativity principles, this model will be controlled by the velocity of light. And to eliminate the standard model's problems, the Cosmological Principle will have to be excluded.

The big-bang phenomenon is regarded here as an explosion of a primeval particle into a region of pre-existing empty space, at some arbitrary origin of coordinates. Before exploding, this particle of finite mass and complex structure was in equilibrium internally. The current material universe now occupies a spherical volume which is still expanding into free space, on the coordinate-frame time scale referred for simplicity to the big-bang event. This is different from the interior local universal-time scale used in the previous section. Our position within this material volume is unknown to us and not yet within sight of the material surface. Other regions of space beyond ours may be empty or occupied by separate material structures at various distances, ie. a multi-verse scheme. Such a realistic model is compatible with the world we experience and has not been disproved by observations.

5.1 The metric

The metric for the ECO-model is to be:

$$ds^2 = -\frac{a^2(t)}{(1+kr^2/4)^2}\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\} + \left(1 - \frac{v^2}{c^2}\right)c^2 dt^2 \ . \qquad (5.1)$$

As in Section 4, a(t) is initially a scale factor, but it will now take real units of radial length from *r* and represent the maximum radius of the material universe:

$$a(t) \to R(t) \ , \ \text{and} \ \dot{a}(t) \to \dot{R}(t) = v \ , \qquad (5.2)$$

for radius $R_\alpha < R(t) < \infty$, and $v < c$. The primeval particle dimension R_α will be defined in Section 6. Coordinate-frame time *t* is that measured by an external observer situated at rest outside of the expanding universal material. Local time for a co-moving observer is therefore dilated, due to the velocity of expansion, as [$d\tau = dt(1-v^2/c^2)^{1/2}$]. Upon introducing metric Eq.(5.1) into Einstein's field equations (see [8], Eq.(98.6)), we get after re-arranging:

$$\left(\frac{\dot{R}}{R}\right)^2 / \left(1 - v^2/c^2\right) = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \ , \qquad (5.3)$$

$$\left(\frac{\ddot{R}}{R}\right) / \left(1 - v^2/c^2\right)^2 = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \ , \qquad (5.4)$$

$$\dot{\rho} / \left(1 - v^2/c^2\right)^{1/2} = -3\left(\frac{\dot{a}}{a}\right)\left(\rho + \frac{p}{c^2}\right) / \left(1 - v^2/c^2\right)^{1/2} . \qquad (5.5)$$

And we shall specify a conserved universal mass for the expanding sphere of maximum radius R:

$$M_U = \frac{4}{3}\pi\left(\rho + \frac{3p}{c^2}\right)R^3 \ , \qquad (5.6)$$

where ρ is the average matter density in the matter dominated universe, and ($3p \ll \rho c^2$). The expansion velocity and deceleration are controlled by the velocity of light, see Figure 2, where the general coordinate time versus radius relationship has been calculated numerically:

$$t = \int \frac{dR}{v} = \int_{R_\alpha}^{R} \left[1 + \left\{ \frac{2GM_U}{c^2 R} + \frac{\Lambda R^2}{3} \right\}^{-1} \right]^{1/2} \frac{dR}{c} \quad . \tag{5.7}$$

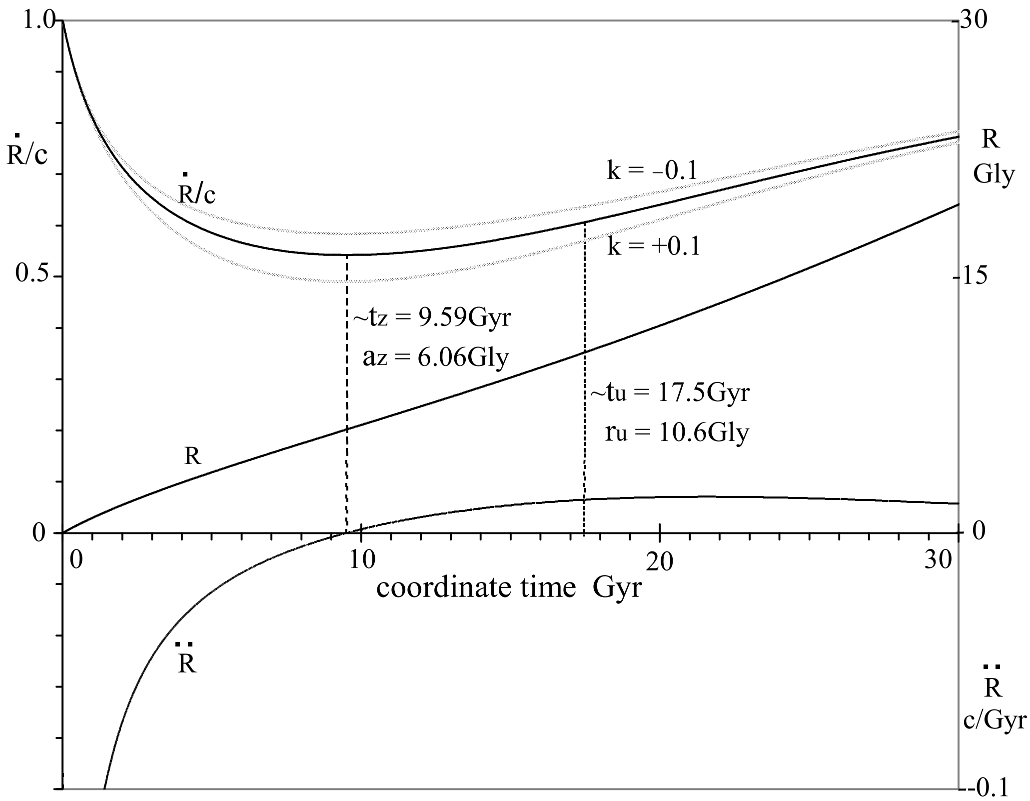

Figure 2. New ECO-model: the variation of expansion velocity relative to the velocity of light $(\dot{R}/c)$, radius (R, Gly), and acceleration $(\ddot{R})$ with *coordinate-frame time (t, Gyr).* Universal mass is $M_U = 1.073 \times 10^{52}$ kg, with the change from deceleration to accelerated expansion occurring at radius 6.06Gly, corresponding to 9.59Gyr from the big-bang. The present coordinate-frame age of the universe is 17.5Gyr, and its radius is 10.6Gly. For illustration purposes, the effect of a finite *k* value (±0.1) is also shown.

If $d\tau$ is substituted into Eqs.(5.3)-(5.5) in place of dt, then they look like Eqs.(4.6)-(4.8), and it follows that $\rho, k,$ and Λ must take the same local values as previously.

The Hubble parameter currently defined as H should now be defined as:

$$H_\tau = \frac{1}{R}\frac{dR}{d\tau} \quad , \tag{5.8}$$

so the left side of Eq.(5.3) may be written as H_τ^2 . Then Ω_Λ and Ω_m will take the same numerical values as previously, simply by changing H_0 to $H_{\tau 0}$ in Eqs.(4.10)-(4.12). The local time τ measured by a co-moving observer is analogous to Eq.(4.15) as:

$$\tau = \left(\frac{2}{3H_{\tau 0}\Omega_\Lambda^{1/2}}\right)\ln\left[\left\{1+\frac{\Omega_\Lambda}{\Omega_m}\left(\frac{R}{R_0}\right)^3\right\}^{1/2} + \left\{\frac{\Omega_\Lambda}{\Omega_m}\left(\frac{R}{R_0}\right)^3\right\}^{1/2}\right], \tag{5.9a}$$

and the corresponding local age of the universe is now analogous to Eq.(4.14):

$$\tau_0 = \left(\frac{1}{3H_{\tau 0}\Omega_\Lambda^{1/2}}\right)\ln\left[\frac{(1+\Omega_\Lambda^{1/2})}{(1-\Omega_\Lambda^{1/2})}\right] = 13.7\text{Gyr} \quad . \tag{5.9b}$$

It is time τ which has governed all atomic processes including star and galaxy evolution. Consequently, the graphs and superluminal velocities in Figure 1 are aberrations of the external observer's values in Figure 2, caused by time-dilation. That is, the 13.7Gyr worth of evolution which we have experienced has really taken 17.5Gyr to perform. Evolution *rate* began low at $t \sim 0$ and grew to a maximum rate at $t \sim$ 9.59Gyr, then declined thereafter. The co-moving local observer did not realise any variation.

Minimum expansion velocity, and zero acceleration in Eq.(5.4), occurred when:

$$\frac{\Lambda c^2}{3} = \frac{GM_U}{a_z^{\,3}} \quad , \tag{5.10}$$

where a_z = 6.06Gly, and $M_U = 1.073 \times 10^{52}$kg as in Sections (4) and (6.2). The effect on the velocity, of non-zero *k* values (-0.1, +0.1), is also demonstrated in Figure 2. Little difference occurs near the origin.

The primeval particle (see Section (6.1)) had all its material in viscous thermodynamic equilibrium while circulating coherently at velocity *c,* before exploding and converting to mass plus much radiation which would have been mostly lost from the slower expanding mass. No inflationary phase appears necessary because the expansion is moderated by the velocity of light, allowing time for equalisation of the radiation temperature. The term kc^2 in Eq.(5.3) is expected to be comparatively small theoretically, and if found to be negative it could be attributed to an extra impulse of KE from the reactive fireball, or if positive, to a viscous claw-back of KE by the gluonic constituents of the particle.

It is interesting that in Eqs.(5.3) and (4.6) the expansion velocity does not overtly depend on pressure *p*, even though acceleration does so in Eqs.(5.4) and (4.7). By substituting ρ from Eq.(5.6) into Eq.(5.3), we can see how pressure reduces the velocity of expansion (or collapse) because it shares some of the potential energy:

$$(\dot{R})^2/(1-v^2/c^2)+\frac{8\pi G}{c^2}pR^2=\frac{2GM_u}{R}-kc^2+\frac{\Lambda c^2}{3}R^2 \quad . \tag{5.11}$$

Likewise, *p* in Eq.(5.4) contributes to the gravitational force because pressure is stored energy. This use of Eq.(5.6) is only compatible with Eq.(5.5) if pressure has a realistic character (p = constant/R^2), where the constant could take today's values ($p_0R_0^2$). Then, according to Eq.(5.11), the pressure term behaves like kinetic energy, just as kc^2 does. Since the ratio of pressure relative to matter density is currently small, it must have been even smaller just after the fireball stage when the scale factor was 1000 times less. But the pressure term in these equations will only apply to gravitational energy, not thermonuclear processes.

If *k* really is zero, the total energy of the expanding matter is M_Uc^2, after any radiation has subsided. This energy is divided between the rest mass and kinetic energy because gravity is an inductive force field, see Paper 1. Consequently, kinetic energy was steadily converted into rest mass up until radius a_z ; thereafter the repulsive Λ-term has reversed the trend and induced conversion of mass to kinetic energy.

5.2 Cosmological redshift

The standard calculation of redshift done for the Robertson-Walker metric shows that measured light wavelengths are increased in proportion to the scale factor a(t), see Narlikar [12], p113. We need to calculate what an external observer would get for the redshift in the ECO-model.

According to the line element Eq.(5.1), plus Eq.(5.2), we can write for a null geodesic:

$$cdt\left(1 - v^2/c^2\right)^{1/2} = \frac{R(t)dr}{(1 + kr^2/4)} \quad . \tag{5.12}$$

This can then be developed in the usual way [12] to produce the redshift equation:

$$\frac{c\Delta t_o (1 - v_o{}^2/c^2)^{1/2}}{c\Delta t_1 (1 - v_1{}^2/c^2)^{1/2}} = \frac{R(t_o)}{R(t_1)} = 1 + z \quad , \tag{5.13}$$

where t_1 is the time of photon emission, and t_0 the time of detection. The first term is equal to $(c\Delta\tau_o / c\Delta\tau_1)$ and the second term is the ratio of scale factors, as seen by a coordinate-frame observer. Figure 2 confirms that the observed redshift of any supernova which occurred at the time of zero-field is given by the ratio of $R(t_o)$ = 10.6Gly, and $R(t_1)$ = 6.06Gly, as previously calculated for Eq.(4.19).

The redshift has been commonly attributed to the stretching of photon wavelengths by the expanding space-time continuum. An external observer might prefer to interpret the redshift as a Doppler velocity effect.

5.3 Luminosity distance

The luminosity distance d_L derived by Carroll, Press & Turner [13] for the standard model may be adapted for the present model, wherein $H_{\tau 0} \equiv H_0$. When $\Omega_k = 0$, and $\Omega_M + \Omega_\Lambda = 1$, and $Z = 1+z$, we get:

$$d_L = \frac{cZ}{H_{\tau 0}} \int_1^Z \frac{dZ}{\left(Z^3 \Omega_M + \Omega_\Lambda\right)^{1/2}} \quad . \tag{5.14}$$

Or for an empty open universe, when $\Omega_k = 1$, and $\Omega_M = \Omega_\Lambda = 0$, we have:

$$d_L = \frac{cZ}{H_{\tau 0}} \sinh \int_1^Z \frac{dZ}{Z^{1/2}} \quad . \tag{5.15}$$

Then for d_L in megaparsecs, the predicted distance modulus is:

$$\mu_p = 5 \log d_L + 25 \quad . \tag{5.16}$$

Given $H_{\tau 0} = 70.5 \text{km}^{-1}\text{Mpc}^{-1}$, these equations will produce exactly the same fit to the data gathered by Riess et al. ([14], Figure 7).

5.4 Flatness problem

This problem has effectively been removed for the new model. Let the density parameter be given as usual by $(\Omega_M = \rho / \rho_c)$, where critical density ρ_c exists for ($k = \Lambda = 0$). Then from Eq.(5.3), we can derive:

$$(\Omega_M + \Omega_\Lambda - 1) = k\left(c^2 / v^2 - 1\right) \quad . \tag{5.17}$$

According to Figure 2, (c/v) is always between 1.0 and 2.0 and therefore Ω_M approaches 1 in a very controlled manner, (while $\Omega_\Lambda \to 0$). There is no problem with this because mass is conserved in Eq.(5.6), and kc^2 is just a small constant amount of KE which becomes relatively unimportant as R decreases towards R_α . If this equation is differentiated, the elemental change in $(\Omega_M - 1)$ appears modest:

$$\frac{\delta(\Omega_M - 1)}{\delta(v/c)} = -2k\left(\frac{c}{v}\right)^3 \underset{(v\to c)}{\rightarrow} -2k \quad . \tag{5.18}$$

This control contrasts with the unfettered singularity of the standard model, wherein (c/v) can decrease without limit.

6. Properties of the primeval particle and gravitons

6.1 Primeval particle

The size of R_α in Section 5.1 can be specified if the primeval particle was of mass $M_{\alpha u} \approx 7.748 \times 10^{52}$ kg, such that a gravitational strength factor may be expressed as:

$$\frac{GM_{\alpha u} m_\ell}{\hbar c} = \frac{1}{137}\left(\frac{e^2}{Gm^2}\right) = \frac{1}{137}\left(\frac{E}{G}\right) \quad , \tag{6.1}$$

where $(m_\ell = m_p /9)$ is the proton-pearl mass [15], $\hbar$ is Planck's constant/2π, $(e^2/\hbar c \approx 1/137)$ is the fine structure constant or electromagnetic strength factor, $(e/m = E^{1/2})$ is the electronic charge/mass ratio, and $(E/G = 4.1656\times10^{42})$. The primeval mass relative to a pearl mass or electron mass is then like one of Eddington's large numbers:

$$\frac{M_{\alpha u}}{m_\ell} = \left(\frac{e^2}{Gmm_\ell}\right)^2 \quad \text{and} \quad \left(\frac{M_{\alpha u}}{m}\right)\left(\frac{m_\ell}{m}\right) = \left(\frac{E}{G}\right)^2 \quad . \tag{6.2}$$

A pearl classical *electromagnetic* radius is given by:

$$r_{o\ell} = e^2/m_\ell c^2 = 1.3812\times10^{-17}\,\text{m} \quad , \tag{6.3a}$$

whereas a *gravitational* radius for mass $M_{\alpha u}$ may be defined as:

$$R_{\alpha u} = GM_{\alpha u} / c^2 = 6.081\text{Gly} \quad . \tag{6.3b}$$

These characteristic parameters are connected by:

$$R_{\alpha u} = r_{o\ell}(E/G) \quad , \tag{6.4}$$

consequently, we will postulate that the original mass $M_{\alpha u}$ was like a *super-pearl* of radius ($R_{\alpha} = r_{o\ell}$), although its charge and structure were not identical to the pearl's. Such a particle *requires* the pre-existence of external coordinate space and time to contain it with its surrounding gravitational field; in addition to whatever else may have existed in the surrounding space, (other particles and anti-matter). This is different from Lemaitre's hypothesis of the 'primeval atom', which proposed that space and time only came into being following disintegration; see Godart & Heller [16].

According to our model [15], the super-pearl probably consisted of 24 helical loops of a toroid held together by a strong viscous gluon field. These were spinning and comprised of many smaller spinning elemental seeds, tied together by gluons. During disintegration, the seeds started decaying into radiation plus lesser particles but the pressure, generated between seeds by deflagration, caused segregation and prevented total conversion during the fireball expansion and cooling stage. Therefore, separate matter volumes, remaining from individual decaying seeds, survived the fireball to become early clusters of galaxies with large-scale velocity flow. Spaces between seeds may have survived also, to form low density voids. Early structure formation was thereby amplified above the standard model; see Perivolaropoulas [17].

Low-order multipole maps derived by Bielewicz et al. [18], Eriksen et al. [19], and Tegmark et al. [20], may be interpreted in terms of the hot-spots due directly to the surviving matter volumes. Some evidence of vorticity and toroidal field might eventually be detected in the cosmic microwave background anisotropy maps from WMAP; see Jaffe et al. [21] and de Oliveira-Costa et al. [22]. Fine granularity in the form of minor seeds and gluons would help account for early production of galaxies and stars, in addition to producing some correlation between the CMB anisotropy and galaxy clusters; see Cole et al. [23].

The viscous gluon material between and within each expanding volume helped equalise the overall density, and a trace may have survived the fireball to initiate great strings and super-clusters of galaxies. Consequently, these did not form entirely from accreted *homogeneous* matter. [Such great structures did not appear in the Millenium Simulation produced by Springel et al. [24]] The above segregated seed structure could account for dark matter, which survived the fireball without immediate conversion into baryons or being involved in nucleosynthesis.The observed small degree of anisotropy is all that remains of granular structure, so thermalisation of the cosmic microwave background radiation involved multiple scattering of the radiation by photons, matter, and dark matter. In the next section we will show how 86% of the primeval particle mass must have been completely lost from the fireball, as radiation into surrounding free space.

6.2 Evaluation of r_m , r_a and Λ

For simple interpretation of Eqs.(2.5) to (2.12), the cosmological constant Λ should depend on the central mass M through r_o as in Eq.(2.8); but logically, r_m and r_a are constant properties of the fundamental particles which constitute M. An estimate of graviton maximum radius r_m , which will satisfy astronomical observations, may be derived by using a separate theory of the proton [15]. First of all, *for an electron*, the electric field strength relative to the gravitational field is given by:

$$(e^2/Gm^2) = (E/G) \approx 4.1656x10^{42} \quad , \qquad (6.6)$$

A practical application of this ratio is possible if the electromagnetic field from an electron also ends at radius r_m , rather then extending to infinity. Then the electric field energy saved beyond r_m is given by:

$$\int_{r_m}^{\infty} \frac{1}{8\pi}\left(\frac{e^2}{r^4}\right)4\pi r^2 dr = \frac{1}{2}\frac{e^2}{r_m} \quad . \qquad (6.7a)$$

This saving could conveniently provide the total *gravitational* field energy for the electron, which is emitted from an effective internal source radius r_s ; namely from Eq.(2.10a) we integrate field energy density T_4^4 and propose:

$$\frac{1}{2}\frac{Gm^2}{r_s} \approx \int_{r_s}^{r_m} \frac{1}{8\pi}\left(\frac{Gm^2}{r^4}\right)4\pi r^2 dr \quad . \qquad (6.7b)$$

Consequently, by equating Eqs.(6.7a) and (6.7b), the ratio in Eq.(6.6) may be expressed as (r_m/r_s = E/G), for the electron.

On the assumption that most of the universal mass now comprises proton-pearls in matter or cold dark matter we will relate r_s and r_m to *pearl* rather than electron dimensions. So according to our proton model [15], it consists of 3 'quarks', each containing 3 'pearls'; where a pearl radius is given by:

$$r_\ell = \left(\frac{e^2}{m_p c^2}\right)\left(\frac{\pi}{48}\right) = 1.00446\text{x}10^{-19}\,\text{m} \quad , \tag{6.8}$$

for proton mass m_p. A graviton is then proposed to have a wavelength ($\lambda_G = 137 \times 2\pi r_\ell$), which will be taken as the characteristic source dimension r_s . (This is analogous to the electron Compton wavelength, $\lambda_C = 137 \text{ x } 2\pi(e^2/mc^2)$). The graviton maximum extent is therefore equal to (E/G) wavelengths:

$$r_m = r_s(E/G) = (137 \times 2\pi r_\ell)(E/G) = 38.08\text{Gly} \quad . \tag{6.9a}$$

Further, we shall propose that the radius a_z at zero acceleration be given by:

$$a_z = r_a = r_m/2\pi = 6.06\text{Gly} \quad ; \tag{6.9b}$$

which is close to the theoretical gravitational radius of the primeval particle Eq.(6.3b), as if Λ was involved in specifying Eq.(6.1). This a_z has been employed in previous sections, such that the current universal mass ($M_U = 1.073\text{x}10^{52}$kg), used in Eq.(5.10), follows from the measured value for Λ in Eq.(4.11). If the *original* primeval mass was ($M_{\alpha u} \sim 7.748\text{x}10^{52}$kg) as calculated in Eq.(6.1), then 86% of the mass must have been lost from the fireball and material universe as radiation. The radiation energy lost was therefore around 2π times greater than the surviving matter.

It is interesting to calculate a real value of the repulsive field. The universal *cosmic* repulsion field term at radius a_z is from Eq.(5.4):

$$F_\Lambda = \left(\Lambda c^2/3\right)a_z = GM_u/a_z{}^2 \approx 2.173 \times 10^{-10}\,\text{ms}^{-2}. \tag{6.10}$$

For comparison, the gravitational field inside a spherical galaxy, of included mass $10^{11}M_\Theta$ within a 10kpc radius, is of the same order:

$$F = \frac{G \times 10^{11} M_\Theta}{r^2} \approx 1.4\text{x}10^{-10}\,\text{ms}^{-2}. \tag{6.11}$$

Overall, according to these different solutions, the cosmological *constant* is proportional to the source mass which generates it, as given by:

$$\Lambda = 3GM / c^2 r_a^{\,3} = 3r_o / r_a^{\,3} \quad , \tag{6.12}$$

where r_a depends upon the type of source particle (eg. proton-pearl, electron). The change in graviton behaviour from attraction to repulsion may be understood as a reversal of helicity within its structure due to longitudenal stress, see Figure 3.

Figure 3. Pictorial representation of a graviton's reversal of helicity/attraction due to internal stress; cf. tendril of passiflora.

7. Cosmic flow and variation of the fine structure constant

7.1 Cosmic flow

Cosmic flow, alias dark flow, has recently been observed and looks real, see [25] [26] [27]. Its magnitude implies that it must be due to a distant mass attracting large clusters of galaxies in a particular direction. This mass appears to be beyond our own universal material and is therefore not visible to us. It has previously been attributed to inhomogeneities in the pre-inflation space-time.

Our model starts with a primeval super-pearl having complex circulating structure, which would account for early seeding and large-scale structure in the universe, with possible cosmic flow. Alternatively, there could have been a neighbouring pearl or anti-pearl to produce an extra (anti-)universe, too far away to be visible to us. The gravitational field of our super-pearl existed before it disintegrated into the Big Bang, so the material in these separating universes will have attracted gravitationally from the beginning to affect the general kinetic energy of expansion. Today this process might reveal itself as flow of galaxy clusters against the smooth general cosmic expansion. In view of the complex primeval particle structure [15], the cosmic flow in detail may be forked into several component directions. There could even be rough expanding giant rings or arcs, visible in the cosmic microwave background [28].

7.2 Variation of alpha

The apparent variation of the fine structure constant reported by Webb et al [29] might be attributed to the various absorption lines originating from inhomogeneous clouds with different turbulence and radial velocities. In addition, the spectrograph is sensitive to variable illumination of the slit by the QSO scintillating image position, plus variable vignetting, see Suzuki et al [30]. This problem could be eliminated by using a light-pipe image diffuser to mix the QSO light, and reference source, into a uniform source on the slit, see Wayte et al [31] [32]. Intuitively, the tiny apparent variation in α over 5Gyr is indicative of zero actual variation. One specific derivation of natural α makes it a constant [33], which can only be increased in high pressure environments like e^+e^- collisions, [34].

8. Conclusion

Repulsive gravity at large distances has been accommodated within the universal solution of Einstein's equations, by introducing a cosmological constant Λ. This represents inherent graviton-graviton repulsion at large radii, rather than dark energy. A cosmological model for the external coordinate observer was then developed, to replace standard Friedmann cosmology because of its non-Einsteinian features. It was logically necessary to limit the graviton field extent from matter to a definite maximum radius. This radius was related to proton structure and led to an estimated onset of universal repulsion at 7.15Gyr (local time) after the big-bang, or 9.59Gyr in the coordinate-frame, when the universe radius was 6.06Gly. The present age of the universe is 13.7Gyr local time, corresponding to 17.5Gyr coordinate time. Evolution of stars and galaxies is governed by the local time rate so cosmological redshift and other observed features (luminosity, etc.) take the same values as found for the standard model. The horizon problem has been ameliorated by prescribing a granular primeval particle in equilibrium, which disintegrates to produce a viscous fireball. The singularity problem is redundant, and the flatness problem is no more.

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References

1. Riess AG, Filippenko, AU, Challis P, Clocchiatti A, Diercks A, Garnavich PM, and 14 co-authors: *Astron J* 1998, **116**:1009.
2. Perlmutter S, Aldering G, Goldhaber G, Knop RA, Nugent P, Castro PG, and 27 co-authors: *Astrophys J* 1999, **517**:565.
3. Riess AG, Nugent PE, Gilliland RL, Schmidt BP, Tonry J, Dickinson M: *Astrophys J* 2001, **560**:49.
4. Tonry JL, Schmidt BP, Barris B, Candia P, and 21 co-authors: *Astrophys J* 2003, **594**:1.
5. Kirshner RP: *Science* 2003, **300**:1914.
6. Kirshner RP, Aguilera C, Barris B, Becker A, Challis P, Chornock R, and 22 co-authors: *Amer Astro Soc* 2003, **202**:2308.
7. Wayte R: *Astrophys Space Sci* 1983, **91**:345.
8. Tolman RC: *Relativity, Thermodynamics and Cosmology*: Clarendon Press: Oxford; 1934.
9. Davis TM, Lineweaver CH: *Pub Astr Soc Australia* 2004, **21**:97.
10. Coles P: *Nature* 2005, **433**:248.
11. Komatsu E, Dunkley J, Nolta MR, Bennett CL, and 15 co-authors: *Astrophys J Suppl Ser* 2009, **180**:330.
12. Narlikar JV: *An introduction to cosmology. 3rd ed*. CUP: Cambridge; 2002.
13. Carroll SM, Press WH, Turner EL: *Ann Rev Astron Astrophys* 1992, **30**:499.
14. Riess AG, Strolger LG, Tonry J, Casertano S, Ferguson HC, Mobasher B, and 13 co-authors: *Astrophys J* 2004, **607**:665
15. Wayte R: 2010, www.vixra.org/abs/1008.0049
16. Godart O, Heller M: *Cosmology of Lemaitre*. Pachart Pub House: Tucson; 1985.
17. Perivolaropoulos L: 2008, arXiv:0811.4684v2.
18. Bielewicz P, Gorski KM, Banday AJ: *Mon Not R Astr Soc* 2004, **355**:1283.
19. Eriksen HK, Banday AJ, Gorski KM, Lilje PB: *Astrophys J* 2004, **612**:633.
20. Tegmark M, de Oliveira-Costa A, Hamilton AJS: *Phys Rev D* 2003, **68**:123523.
21. Jaffe TR, Banday AJ, Eriksen HK, Gorski KM, Hansen FK: *Astrophys J* 2005, **629**:L1.

22. De Oliveira-Costa A, Tegmark M, Zaldarriaga, M., Hamilton A: *Phys Rev D* 2004, **69**:063516.

23. Cole S, Percival WJ, Peacock JA, Norberg P, Baugh CM, Frenk CS, and 25 co-authors: *Mon Not R Astr Soc* 2005, **362**:505.

24. Springel V, White SDM, Jenkins A, Frenk CS, Yoshida N, Gao L, and 11 co-authors: *Nature* 2005, **435**:629.

25. Kashlinski A, Atrio-Barandela F, Ebeling H, Edge A, Kocevski D: *Astrophys J Lett* 2010, **712**:L81.

26. Watkins R, Feldman HA, Hudson MJ: *Mon Not R Astr Soc* 2009, **392**:743.

27. Lavaux G, Tully RB, Mohayaee R, Colombi S: *Astrophys J* 2010, **709**:483.

28. Kovetz ED, Ben-David A, Itzhaki N: 2010, arXiv: 1005.3923v2.

29. Webb J K, King JA, Murphy MT, Flambaum VV, Carswell RF, Bainbridge MB: 2010, arXiv: 1008.3907v1, and see the many references therein.

30. Suzuki N, Tytler D, Kirkman D, O'Meara J M, Lubin D: *PASP* 2003, **115**:1050.

31. Wayte RC, Ring J: *Mon Not R Astr Soc* 1977, **181**:131.

32. Wayte RC, Blades JC, Wynne-Jones I: *Mon Not R Astr Soc* 1978, **182**:5P.

33. Wayte R: 2010, www.vixra.org/abs/1007.0055.

34. Wayte R: 2010, www.vixra.org/abs/1010.0046.