A complete graph model of the Schwarzschild black hole

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Abstract

1 Introduction

Where $\hbar = c = G = 1$, the following components will be used to model a Schwarzschild black hole of rest mass-energy E in $\mathbf{R}3$:

1. A 2-sphere (event horizon) S_0 at $R_0 = 2E$, upon which lies $N_0 = E$ uniformly distributed vertices V_0.
2. A complete graph of edges E_0 generated by V_0.
3. An exterior region at $R > R_0$, upon which lies a countable number of vertices V_{ext}.
4. A countable number of (non-complete) graph edges E_{ext} generated by the Delaunay tetrahedralization of V_0 and V_{ext}.

The following presumptions are made *a priori*:

1. The event horizon and interior region of a Schwarzschild black hole do not contain any physical singularities.
2. The complete graph edges E_0 define a universal edge coordinate length of
$$L = \frac{1}{\sqrt{1 - R_0/R_{\text{mid}}}}, \tag{1}$$
where R_{mid} refers to the coordinate distance between an edge's midpoint and the black hole centre. Accordingly, edge proper length is $l = L^2$.
3. The complete graph edges E_0 define a universal minimum edge coordinate length of $L = 1$ (e.g., the Planck length).

2 Method

The following steps are used to construct the model's components:

1. With regard to the 2-sphere S_0, numerically solve for the coordinate radial distance R_1 of a second 2-sphere S_1, using the formula for the height of regular tetrahedron as a guide:
$$H = L\sqrt{2/3}, \tag{2}$$
$$R_1 = R_0 + H_0, \tag{3}$$
$$\frac{H_0}{\sqrt{2/3}} \approx \frac{1}{\sqrt{1 - \frac{R_0}{H_0/2 + (R_0 + H_0)}}}. \tag{4}$$

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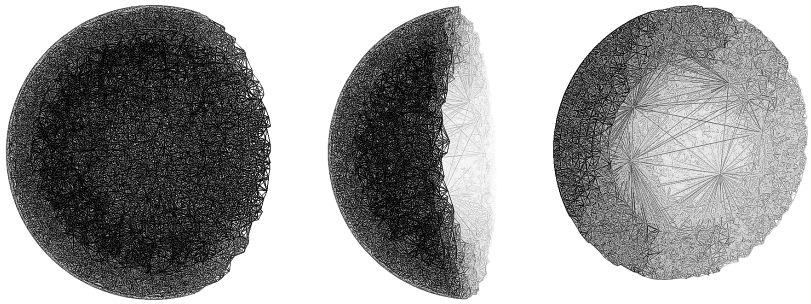

Figure 1: One half of the edges for 2-spheres S_0 through S_{10}, where $N_0 = 10$. The manifold is geodesically complete, since it does not contain any infinitely small or large edges, or "dead end" paths.

2. Calculate the number of vertices V_1 that lie upon S_1, using the formula for the area of a regular triangle as a guide:

$$A = (1/4)\sqrt{3}L^2, \tag{5}$$

$$V_1 + F_1 - E_1 = 2, \tag{6}$$

$$F_1 = 2V_0 - 4 = \frac{16\pi R_1^2}{\sqrt{3}(\frac{1}{1-R_0/R_1})}, \tag{7}$$

$$V_1 = \frac{F_1 + 4}{2}. \tag{8}$$

3. Calculate H_1:

$$\frac{H_1}{\sqrt{2/3}} \approx \frac{1}{\sqrt{1 - \frac{R_0}{H_1/2+(R_1+H_1)}}}. \tag{9}$$

4. Repeat steps 2 and 3 for each subsequent 2-sphere $S_{\geq 2}$.
5. Generate the vertices V_0 that lie upon S_0. Use Coulomb repulsion on S_0 to make the vertex distribution roughly uniform.
6. Obtain the complete graph edges E_0 generated by V_0.
7. Generate the vertices $V_{\geq 1}$ that lie along each 2-sphere $S_{\geq 1}$. Use Coulomb repulsion on each 2-sphere to make its vertex distribution roughly uniform, if desired.
8. Obtain the (non-complete) graph edges generated by the Delaunay tetrahedralization of all vertices $V_{\geq 0}$.

3 Results

Depending on how well the vertices $V_{\geq 1}$ are uniformly distributed along their respective shells, one will have to multiply $H_{\geq 0}$ and $V_{\geq 1}$ by some small constant values (e.g., ~ 1) in order to meet the edge coordinate length requirement given in Eq. 1 with accuracy. See [1] for a public domain C++ code that generates this model's vertices and edges. Edge analysis code is included. The default configuration produces an edge coordinate length accuracy of ~ 0.99. As with other discretization models, edge length accuracy is based on an average. Unlike other discretization models however, one is not free to arbitrarily choose the scale of the simplices (e.g., tetrahedra in this case), and so the manifold is geodesically complete by definition, not by choice.

References

[1] Google Code. `http://code.google.com/p/cgmetric/downloads/list`