GENERALIZATIONS OF DEGARGUES THEOREM*

Florentin Smarandache
University of New Mexico
200 College Road
Gallup, NM 87301, USA
E-mail: smarand@unm.edu

Let’s consider the points $A_1, \ldots, A_n$ situated on the same plane, and $B_1, \ldots, B_n$ situated on another plane, such that the lines $A_i B_i$ are concurrent. Let’s prove that if the lines $A_i A_j$ and $B_i B_j$ are concurrent, then their intersecting points are collinear.

Solution. Let $\alpha$ be the plane that contains the points $A_1, \ldots, A_n$ (in the case in which the points are non-collinear $\alpha$ is unique), and analogously, let $\beta = P(B_1, \ldots, B_n)$, and consider $\alpha \cap \beta = d$.

Because the lines $A_i A_j$ and $B_i B_j$ are concurrent, $A_i A_j \subset \alpha$, and $B_i B_j \subset \beta$, therefore their intersection belongs to line $d$.

Remark 1.

For $n = 3$ and $A_1, A_2, A_3$ non-collinear, $B_1, B_2, B_3$ non-collinear, and $A_i \neq B_j$ we obtain Desargues theorem.

Remark 2.

An extension of this generalization is: If we consider $A_1, \ldots, A_n$ situated in a plane, and $B_1, \ldots, B_m$ situated on another plane, prove that if $A_i A_j$ and $B_k B_r$ are concurrent, then their intersection points are concurrent.

Remark 3.

For $n = m$, and $A_i B_i$ concurrent lines, we obtain the first generalization.

Remark 4.

If in addition we also have $n = m = 3$ along with the previous conditions, we obtain the Desargues theorem.