Neutral particle gravity with nonassociative braids

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Bilson-Thompson has characterised the fundamental leptons and quarks using simple three strand ribbon diagrams. A minimal extension to this set is specified by a doubling of the neutrino sector. The resulting braid set is reinterpreted in a categorical framework for localization, which assumes a non local view of the neutral particle oscillations that are responsible for gravity. A few observational consequences, such as CPT violation in the neutrino sector, are discussed.

I. INTRODUCTION

One strong motivation for studying physics with diagrams from category theory is the potential of such methods to provide real world computations without invoking unnecessary and difficult continua. The banishment of the absolute background is seen as an essential element of an emergent classical spacetime in quantum gravity.

A braid or ribbon diagram consists of strands with endpoints in the upper and lower sides of a diagram. These diagrams make sense in a large interesting class of categories, such as representation categories for Hopf algebras. Using ribbon strands, we find many connections with deep mathematics, such as Grothendieck's children's drawings.

As discussed by Shum [1], a ribbon segment may be described by two bounding strands, placed near to one another. In general, it is useful to group strands according to the bracketing of the tensor product of objects that is being represented by the full strand set. That is, once we insist on distinguishing ribbon edge pairs from separated strands, we might as well look at diagrams with mixtures of ribbon and strand elements.

In [2] the fundamental fermions of the Standard Model are listed in terms of three braided ribbon segments. A ribbon segment is represented by two nearby strands, for a total of six single strands in each diagram. However, since the ribbons of neutral fermions remain untwisted, in this case the two ribbon edges may be replaced by a single strand, as in the right hand side of figure 4. Thus all possible neutral particle braids are given by a three strand diagram, including all possible reflections both vertically and horizontally in the page.

A reflection across the vertical axis performs both a left right mirror and a matter to antimatter conjugation, while the horizontal axis reflection is a left right mirror only. The four possible braids of the type shown in figure 4 then correspond to the four chiral neutrinos and antineutrinos, ν_L, ν_R, $\overline{\nu}_R$ and $\overline{\nu}_L$. The chiral charged leptons and quarks are described by putting twisted ribbon charges in place of these neutral strands.

Ignoring the natural color ordering on the fractionally charged quark diagrams, there are eight such leptons and eight quarks. With the introduction of mass quantum numbers there would be a total of 24 chiral particles for the electroweak sector. Although these particles may be mapped to boson diagrams via a generalized quantum Fourier transform, there is no Higgs boson. In [2] it was shown that basic interaction rules are obeyed by braid compositions.

The question is: why do these particular diagrams capture the low energy fermion spectrum? In this paper we will reinterpret the particle braids using trivalent vertices which are mixtures of single strands and ribbon segments. As for morphisms in the symmetric monoidal categories of quantum mechanical computation [3][4] these vertices have distinct reflections horizontally and vertically, although they remain unlabeled.

With an aim to understanding the quantum gravitational context of such diagrams, observe that Standard Model localization is viewed as an emergent feature in modern on-shell twistor space techniques for particle scattering [5][6][7][8][9]. Here scattering amplitudes for n particles may be constructed using a recursive procedure that begins with the three particle case. However, physical amplitudes begin at $n = 4$, which appears with the decomposition of a 4-valent graph into two trivalent pieces, as in figure 1. Categorically speaking, the four particle case is associated to an associahedron point, whereas a trivalent node denotes an empty associahedron. In other words, space begins with four things, but it arises from nothingness.

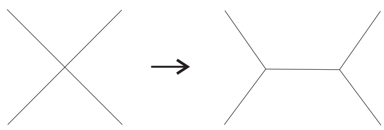

FIG. 1: Localization as factorization

The next section outlines the particle diagram elements that now appear natural from a categorical perspective, and the final section discusses a few physical consequences of this framework.

II. MIXED STRAND AND RIBBON DIAGRAMS

Braid or ribbon diagrams are neatly oriented in the plane of the page, so that they may be composed via concatenation. In mixed diagrams, ribbons are joined to ribbons and strands to strands. We consider diagrams from three points to three points, where the fixed three point set is associated to the generation mass quantum number as a ternary analogue of dualities associated to two ribbon edges.

An algebraic (or coalgebraic) object in a symmetric monoidal category, where the braid crossings are ignored, is usually represented by a trivalent node. However, the real conundrum of figure 1 is that the tree diagram for the empty three particle associahedron should be represented by a two leg tree, as shown in figure 2, where the signs stand for particle helicity [6]. For each particle number n there should be a reduction to $n-1$ tree legs, through the selection of a root ribbon leg. Thus a four particle diagram becomes the simplest possible rooted tree with two branches, known as the associahedron point. Such a replacement of nodes by braid crossings is also familiar in the modern renormalisation theory of Kreimer, Broadhurst, Connes and others.

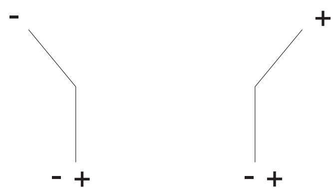

FIG. 2: Three particle rooted trees

The allowed trivalent mixed diagrams are then given in figure 3. Observe that all six are generated, under reflections, by two cases: one uncrossed and one crossing diagram. Arbitrary networks may be created from these six vertex types. By shrinking ribbons to strands, and ignoring crossings, we recover the unique unlabeled trivalent node of a symmetric monoidal category.

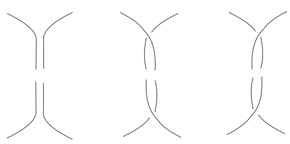

FIG. 3: Six trivalent vertex types

Let us consider now the particle braid on the right hand side of figure 4. This is constructed out of four trivalent diagrams as shown, where ribbon segments join ribbons and strands join strands. The lower pair is the dual of the upper pair, where duality is given by both vertical and horizontal reflections. As a categorical morphism $f : \mathbf{3} \rightarrow \mathbf{3}$, this braid takes the form $f = (F \otimes I) \circ (I \otimes F^{\dagger})$, with F a four leg diagram with one crossing. The two ribbon segments on the left hand side are introduced as a categorical localization procedure a la figure 1, without which the morphism f would just be given by two 4-valent symmetric crossings.

Note that the symmetric quantum mechanical reduction to a unique vertex [4] assumes a time reversibility that is most definitely not a feature of the mass generation mechanism under consideration. On the contrary, the observer's thermodynamic arrow of cosmic time is clearly time asymmetric. This will be discussed in the following section.

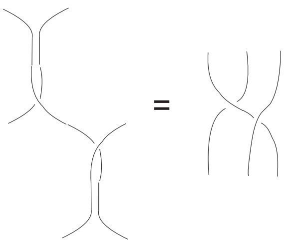

FIG. 4: Composition of 4-valent segment with its dual

In the expanded braid diagram the upper and lower strand ends are ungrouped, whereas the ribbon segments may denote a bracketing of object pairs. Let the three objects be named A, B and C. Reading top to bottom, the diagram then stands for a sequence of six steps

$$ABC \to (AB)C \to (BA)C \to BAC \to B(AC) \to B(\overline{CA}) \to BCA$$

with strand switching only allowed inside brackets. The bar denotes the opposite crossing sign on the second braiding. Such diagrams were first analysed by Bar-Natan in [10] and they lead to a generalisation of the associahedra polytopes. The morphism F is then the composition

$$F: \quad AB \to (AB) \to (BA)$$

of structure morphisms in the category. The full morphism picks up all three even permutations on ABC, defining one half of the 12-gon polytope that replaces the usual hexagon of braided monoidal categories [10]. In order to pick up a cyclic permutation, one would consider the braid rotatable in three dimensions. In other words, the endpoints of a braid would live not on an infinite horizontal line, but on a small loop in the plane emerging from the page. This allows the missing bracketings of A and C.

Such three strand diagrams, if given same sign crossings, are associated to the hexagon laws of a braided monoidal category. Recently it has been shown by Furusho [11] that solutions to certain (Hopf) algebraic hexagon equations follow from the associator solution to the pentagon rule on four strands, which has no crossings in it. Given such associators, the opposite sign crossings of the neutral particle braid give the simplest representation of *permutoassociahedra* structure [10], where the morphism F is now constructed from a single associator.

But we are not only interested in monoidal structure. A time asymmetry is directly associated with a breaking of monoidal structure in tricategories, where the pentagon law lies on five sides of a cube [9]. That is, bracket orderings now matter and on four objects we distinguish two varieties of the object $(AB)(CD)$. In this context, pentagons do indeed lead to hexagons labeled by the permutation group S_3.

A true three dimensional arrow may fill this axiomatic cube for tricategories. The most interesting feature in this dimension is the existence of a *dimension raising* tensor product. Space is being created out of two dimensional pieces, and from then on one can construct spaces of any dimension.

III. A FEW CONSEQUENCES

We would like to understand how mass operators are associated to particle braids. Three strand diagrams $f : \mathbf{3} \rightarrow \mathbf{3}$ will be associated to 3×3 matrices, which are not necessarily viewed as elements of a linear algebra. These matrices have rows and columns indexed by the set A, B and C, so the nine entries cover all possible length two noncommutative paths in these letters. Similarly, higher dimensional arrays arise from longer paths.

Assuming that positive rest mass is extracted from a zero mass energy operator Z, with three zero eigenvalues split by the complex decomposition $Z = \sqrt{M} + i\sqrt{M}$, we begin with the ansatz that $\sqrt{M}$ selects half of the S_3 permutations (coming from length three paths) as a basis. A Koide mass operator $\sqrt{M}$ is such a 3×3 Hermitian 1-circulant matrix, the eigenvalues of which give a triplet of rest masses after squaring. This Fourier transform of a diagonal matrix has been used by Brannen to fit all the lepton and many hadron masses [12], reducing the number of parameters required to specify these inputs to the Standard Model. A Koide matrix has only one off diagonal phase. In particular, the neutrino and antineutrino mass predictions follow from the phase choices

$$\nu : \quad \theta = \frac{2}{9} + \frac{\pi}{12} \qquad\qquad \overline{\nu} : \quad \theta = \frac{2}{9} - \frac{\pi}{12}$$

where Brannen's [13] Berry phase of $\pi/12$ appears naturally in a noncommutative path integral representation of generation number. Here it represents the 24 information dimensions spanned by the allowed leptons. The phase $2/9$ appears in all Koide triplet fits, and must therefore be a universal mass splitting parameter. It's origin remains unclear, but it may represent the fraction $6/27$ of noncommutative paths that involve all three spin directions: X, Y and Z. After all, these are the paths used to derive the other component of the phase [13]. The reason for the missing π could then be the mismatch between a local orbital Keplerian time and Riofrio's radial cosmic time, which represents an evolving universal mass from a zero mass cosmological horizon. Note that these Koide eigenvalues may also be expressed directly in terms of the nine entries of the tribimaximal neutrino mixing matrix, supporting the view that neutrino mixing is associated to mass generation.

The resulting rest mass predictions include an (electron) antineutrino mass of 0.00117 eV [14]. Graham Dungworth [15] has pointed out that this corresponds precisely, under Wien's law for black body radiation, to a temperature of 2.73 K, which we recognise as the observed temperature of the cosmic microwave background radiation. This sets the human observer's thermodynamic cosmic clock. Dungworth also discusses other astrophysical consequences of both the light neutrino masses and the mirror dark matter that results from the right handed braids. That is, since dark matter appears only to interact gravitationally, we might as well consider a model where dark matter is given by the mirror braids. In the weak interaction, there is both a left handed and a right handed neutrino involved. The right handed particle lives in the mirror world, and carries the gravitational interaction non locally into our own matter sector.

The CPT violation recently observed by MINOS [16] agrees quantitatively with the Δm^2 values given by the Koide triplets [14]. Note that the precisely known value of the CMB temperature may then be used to better constrain the Koide fits to MINOS results, and tighten other predictions. The difference in neutrino and antineutrino masses relies on the existence of one negative eigenvalue, for the electron neutrino, which has the lightest of all masses at 0.00038 eV. This does not occur for the charged leptons, whose mass equivalence appears to be exact. In an ongoing study of particle mixing, we suppose a tight relation between particle oscillations and potential CPT violation. A matter antimatter mass difference is also observed for neutrons, although the statistical significance of this result is currently poor.

We now consider the positively charged leptons to arise from a Koide phase of $-2/9$, in line with the idea that antimatter uses negative phase components. With only a single phase component, the lepton masses are unaltered under such a sign change, as observed. The ν and $\overline{\nu}$ triplets are thus distinguished only by a sign match or mismatch, rather than an absolute choice of overall sign. This is in line with Dungworth's suggestion [15] that the ν and $\overline{\nu}$ triplets be split into annihilating equal mass pairs, giving the doubling of neutral braid states.

This conveniently associates the four neutrino phase choices precisely with the four neutrino braids, in such a way that the $+\pi/12$ is for right handed objects, while the $-\pi/12$ is for left handed objects. Similarly, the phase $+2/9$ is for the $(\overline{\nu}_L, \nu_R)$ pair, while $-2/9$ selects the other neutral pair. It is therefore natural to insist on a same sign double phase for ordinary left handed matter, as is done for the hadron Koide matrix fits [12].

Further consequences of neutrino gravity will be considered in other work.

Acknowledgements

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