Comment on arXiv:1009.2266v2, "On the fine structure constant in the Machian universe"

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Abstract

We consider a recent attempt by Gogberashvili and Kanatchikov to connect the value of the fine structure constant α with cosmological parameters. After pointing out an error in the authors' account, we show that by modifying their treatment of dark energy a suitable value of α can still be obtained using current values of the relevant cosmological parameters. We also address the question of α's determination in earlier cosmological eras.

In [1], Gogberashvili and Kanatchikov (hereafter "GK") attempt to derive the value of the fine structure constant α from cosmological parameters using a Machian theory in which all particles in the universe are "gravitationally entangled," so that they interact nonlocally with each other. GK refer to the "Machian energy" of particles, by which they mean the energy arising from collective, nonlocal interactions between gravitationally entangled particles. In connection with their derivation of α's value, GK identify the total Machian energy of all particles in the universe with dark energy, which leads them to conclude that the ratio M_{Mach}/M equals the relative dark energy density Ω_Λ, where M_{Mach} is the total Machian mass of all particles in the universe (i.e., the mass equivalent of the total Machian energy). It is important to realize that M, for GK, is not simply the total mass of all particles in the universe; it is the total active gravitational mass of the cosmic fluid, and hence it includes all dark components [2]. Since dark energy thus contributes to M, and since M_{Mach} represents energy over and above the "normal" energy of matter, it is natural to associate dark energy with M_{Mach}, and to equate the ratio M_{Mach}/M with Ω_Λ, as GK do. (For other aspects of the Machian model, see [3].)

GK's derivation, which uses the current values of cosmological parameters, purports to yield a value of α that is in good agreement with the measured value of 7.297×10^{-3}. In what follows, we adopt the basic framework of GK's cosmological model but argue that one of the equations used in deriving α's value contains an erroneous term, and that the corrected equation does not in fact yield a suitable value of α. We therefore consider a modified version of GK's account of dark energy, and we show that it does produce the desired value of α given the current values of the relevant cosmological parameters. We conclude by addressing the question of why α has remained constant or nearly constant over time, even though the values of cosmological parameters have changed significantly over the same time period.

All references to equations (e.g., "eq. 3") are to the equations of [1], and the numbering is the same as in [1]. The equations cited here are listed below in the Appendix. As in [1], we use [4] as the source for the current values of cosmological parameters.

GK's eq. 3, which is an expression for the total Machian energy produced by all particles in the universe, contains the term "N(N-1)", which is approximately equal to N^2, since N is extremely large; and their eq. 13, which is an expression for the Machian energy $E_{b\,Mach}$ of baryons, contains the term "$2N_bN-N_b^2$", which as they note is approximately equal to $2N_bN$. One may wonder why the expression for $E_{b\,Mach}$ does not contain instead the term "N_bN", since intuitively this seems to be the "baryonic counterpart" of "N^2" in the expression for the total Machian energy. (In fact, GK themselves used "N_bN" in the original version of their paper (arXiv:1009.2266v1, eq. 12).) The answer, evidently, is that GK take the *non*-baryonic counterpart of "N^2" to be "$(N\text{-}N_b)^2$", so that the baryonic counterpart is "$N^2\text{-}(N\text{-}N_b)^2$", which is just $2N_bN\text{-}N_b^2$. Now, it is important to realize that the baryonic component of the total Machian energy results from the interaction of each baryonic particle with all other particles in the universe, an interaction expressed by "N_bN". The way to obtain the non-baryonic counterpart of "N^2", therefore, is to subtract this baryonic component from the total Machian energy. Taking "$(N\text{-}N_b)^2$" as the non-baryonic counterpart of "N^2", however, fails to express this subtraction of the baryonic energy-component. Instead, "$(N\text{-}N_b)^2$" describes the Machian energy of a universe that lacks baryons altogether, i.e. a universe whose total Machian energy has no baryonic component. This is clear from the fact that (i) using "$(N\text{-}N_b)^2$" involves subtracting the number N_b of baryons from the total number N of particles *prior* to taking the square, and (ii) the operation of taking the square here reflects or expresses the Machian interactions between particles that give rise to Machian energy. In other words, the prior subtraction of baryons entails that baryons do not contribute at all to Machian energy, which is manifestly false in a universe such as ours that contains baryons (assuming, of course, that we accept GK's idea of Machian energy in the first place). Hence, "$(N\text{-}N_b)^2$" is not the correct non-baryonic counterpart of "N^2", because it (wrongly) treats the baryonic component of Machian energy as *non-existent* rather than simply subtracting this component from the total Machian energy. The correct expression for the non-baryonic counterpart of "N^2" is "$N^2\text{-}N_bN$", since in this expression the baryonic component N_bN of Machian energy is subtracted from the term "N^2" that represents the total Machian energy. Therefore, the baryonic counterpart of "N^2" is "$N^2\text{-}(N^2\text{-}N_bN)$" = N_bN, as one would intuitively expect. The resulting use of "N_bN" rather than "$2N_bN$" in eq. 13 means that the value of $E_{b\ Mach}$ is lessened by half; this entails that the value of α that GK obtain in eq. 20 must likewise be reduced by half, giving us a value of α that is *not* in good agreement with α's measured value of 7.297×10^{-3}. Specifically, the factor "8π" in eq. 20 becomes "4π"; and so, instead of eq. 21's "good" value of $\alpha \approx 7.1 \pm 0.6 \times 10^{-3}$, we have $\alpha \approx 3.55 \pm 0.6 \times 10^{-3}$.

As we will show, however, this problem can be remedied by hypothesizing the existence of other nonlocal interactions – besides those involving particles of matter – that contribute to dark energy, so that nonlocal interactions between particles give rise to only a *part* of the total dark energy. In particular, we consider the idea proposed in [5] that there exist nonlocal interactions involving elements or "particles" of spacetime itself, interactions that give rise to a "quantum potential of spacetime" Q that acts as dark energy. This Q can be thought of as the Machian energy of spacetime particles, and hence Q contributes to the total Machian energy. Thus, GK's identification of dark energy with Machian energy is upheld. What we have, then, is that Machian dark energy consists of two components, a Q-component and a non-Q-component (the latter being the Machian energy that GK describe). We propose that these two components of Machian dark energy are roughly equal at present, so that each is approximately 0.37 of the total energy density of the universe (taking $\Omega_\Lambda = 0.74 \pm 0.03$). This proposal is motivated by two considerations. First, the derivation of Q's value in [5] yields a value of Q that equals approximately 0.37 of the universe's total energy density (where the magnitude of this total density is $\sim V^{-½} \sim H^2$, with V being the four-volume of the universe in Planck units); this is explained in more detail below. And second, taking the two components to be equal, and using the current values of cosmological parameters, leads to a value for α that is in good agreement with the measured value, as will be seen shortly.

The key consequence of this modification of GK's model is that it invalidates GK's eq. 14, which states that the ratio of the Machian energy of baryons to the total energy of baryons equals the ratio of total Machian energy to the total energy of the universe, where the latter ratio is just

Ω_Λ. Instead, the former ratio has the value 0.586 = 0.792Ω_Λ, using the current values of the relevant cosmological parameters. Hence, eq. 14 needs to be modified to read: $E_{b\ Mach}/E_{b\ tot}$ = 0.792Ω_Λ. Likewise, all occurrences of "Ω_Λ" in eqs. 15, 17 and 18 need to be replaced by "0.792Ω_Λ," since these equations all follow (in part) from eq. 14; as a result, the value of "1- Ω_Λ" in these equations becomes 0.414 instead of 0.26. It is important to notice, however, that the "Ω_Λ" in eq. 19 is just the usual Ω_Λ (which has a current value of 0.74 ± 0.03), since eq. 19 is obtained from eqs. 1, 7 and 8 alone and hence is completely independent of eq. 14. Therefore, when eq. 19 is plugged into eq. 18 to get eq. 20, the "Ω_Λ" term in eq. 19 does not exactly cancel the "Ω_Λ" term in eq. 18. Instead, we have Ω_Λ/0.792Ω_Λ, which equals 1.26; and therefore, the rhs of eq. 20 must be multiplied by 1.26 in order to obtain the value of α. Furthermore, as already noted, the factor "8π" in eq. 20 must be replaced by "4π". Taking all of this into account, we get the following modified version of GK's eq. 20: $\alpha \approx 4\pi(0.414 \times 1.26)\Omega_r/\Omega_b$. For the current value $\Omega_r/\Omega_b = 1.09 \pm 0.03 \times 10^{-3}$, and taking Ω_Λ's current value to have a margin of error of ± 0.03, we obtain $\alpha \approx 7.14 \pm 0.6 \times 10^{-3}$, in good agreement with α's measured value of 7.297×10^{-3}.

Let us say something more about the dark energy Q described in [5]. The idea is that nonlocal interactions between Planck-sized elements of spacetime that fluctuate in volume produce a widespread cancellation of these fluctuations. At any given time t, and using Planck units for V, there are $\sim \sqrt{V}$-many elements whose volume-fluctuations are uncanceled; hence, there is a net volume-fluctuation $\Delta V \sim \pm\sqrt{V}$ at t for the entire volume V. The volume-fluctuations here amount to fluctuations in the *density* of the spacetime elements; and we take the nature of these elements to be described by causal set theory [6]. At a given time t, the widespread cancellation of these density-fluctuations, combined with the presence of sparsely distributed *un*canceled fluctuations, gives rise to a quantum potential Q of spacetime with energy density ρ_Q. If the net volume-fluctuation is a *contraction*, i.e. if $\Delta V \sim -\sqrt{V}$, we have $\rho_Q \approx 0.35V^{-1/2}$; and if the net volume-fluctuation is an *expansion*, so that $\Delta V \sim +\sqrt{V}$, we have $\rho_Q \approx 0.39V^{-1/2}$. Hence, the average value of ρ_Q is approximately $0.37V^{-1/2}$, which is just what is needed in the present context, as indicated above. We conclude, therefore, that when GK's account of α's value is suitably corrected, and is modified due to introducing the quantum potential Q of spacetime as a component of dark energy, this account does indeed yield a "good" value of α.

The value of α derived above is obtained using the *current* values of cosmological parameters such as Ω_Λ and Ω_r. Given the strong constraints on any temporal variation of α, however, and given the relatively large temporal variability of the cosmological parameters in question, it is extremely implausible to suppose that the values of these parameters in earlier cosmological eras can yield values of α that satisfy these constraints on α's variation. It appears, therefore, that the only way to deal with this issues within the framework of GK's Machian model is to regard the current values of the relevant cosmological parameters as fixing α's value at *all* past times, and hence acausally (see [7] for a different example of acausality in a cosmological context). Such a move, while certainly radical and controversial, is nonetheless in the spirit of a Machian approach [8], and hence it deserves to be taken seriously from a Machian standpoint.

Suppose, then, that we adopt GK's view of the universe as fundamentally Machian. From this standpoint, cosmological evolution can be viewed as an evolution of the universe's Machian character, with the final stage of this evolution consisting of the emergence of Machian dark energy as the dominant component of the total energy density ρ_{tot} of the universe. This suggests – and here we draw on [7] – that the true infrared scale of GK's Machian model is given by the time needed for dark energy to become dominant. As a result, α's value at early times is fixed acausally by the deep future behavior of the Machian model, i.e. by the cosmological conditions that prevail when dark energy becomes dominant – in other words, by conditions in the current cosmological epoch. Hence, α's value at early times is just the value obtained above using the present values of cosmological parameters, namely $\alpha \approx 7.14 \pm 0.6 \times 10^{-3}$. Obviously, this acausal treatment of α implies that α is constant at all past times (future variation of α is not ruled out, however). This constancy of α has been questioned [9], but it does enjoy significant observational and theoretical support [10]. In any case, GK's model, as it currently stands, clearly allows far too much variation of α to satisfy the strong observational constraints on α's temporal and spatial variation. As a result, *something* needs to be done if the basic ideas of this model are to be retained; and the adoption of the sort of acausal account of α's value proposed here represents one possible course of action. This acausal approach, as mentioned above, fits in with a Machian standpoint, and it has been used to obtain a value of zero for the "true" cosmological constant Λ [8], thereby resolving one part of the cosmological constant problem. (Resolving the other part, of course, requires a positive account of dark energy, such as the account in [5], an account that may be combined with the ideas in [1], as described above.)

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APPENDIX

Equations from [1]:

(1) $c^2 \approx 2MG/R$
(R here is the radius of the universe.)

(3) $E = [N(N-1)/2](2Gm^2/R) = N(N-1)Gm^2/R$
(E here is Machian energy, and N is the number of particles in the universe; in the simplified model of [1], these particles are all identical, and each particle has mass m.)

(4) $M_{Mach} = [N(N-1)/2]m \approx N^2m/2$

(7) $-mc^2\Delta t \approx -2\pi\hbar$
(The rationale for eq. 7 is explained in [1].)

(8) $\Delta t \sim R/Nc \sim 1/NH$

(12) $\Omega_\Lambda = M_{Mach}/M \approx N^2m/2M$

(13) $E_{b\,Mach} = (2N_bN - N_b^2)\,Gm^2/R \approx 2N_bN\,Gm^2/R$
(In eq. 12 of [1]'s *first* version (arXiv:1009.2266v1), the Machian energy of baryons is written as "E_b", and we have $E_b \approx N_bNGm^2/R$ rather than $2N_bN\,Gm^2/R$.)

(14) $E_{b\,Mach}/E_{b\,tot} = \Omega_\Lambda$

(15) $\Omega_b \approx (E_{b\,tot} - E_{b\,Mach})/Mc^2 \approx [E_{b\,Mach}(1-\Omega_\Lambda)]/Mc^2\Omega_\Lambda$

(16) $E_r \approx (N_b/2)(k_ce^2/R) = (N_b/2)(\alpha\hbar c/R)$, where k_c is the Coulomb constant.

(17) $\Omega_r/\Omega_b \approx (E_r/E_{b\,Mach})(\Omega_\Lambda/1-\Omega_\Lambda) \approx (\alpha\hbar c/4NG\,m^2)\,(\Omega_\Lambda/1-\Omega_\Lambda)$
(This is obtained from eqs. 13-16.)

(18) $\alpha = [(1-\Omega_\Lambda)/\Omega_\Lambda](\Omega_r/\Omega_b)(4NG/c)(m^2/\hbar)$
(This is just a re-arrangement of eq. 17.)

(19) $m^2/\hbar = (2\pi c/NG)\,\Omega_\Lambda$
(This is obtained from eqs. 1, 7 and 8.)

(20) $\alpha \approx 8\pi(1-\Omega_\Lambda)\Omega_r/\Omega_b$
(This is obtained by plugging eq. 19 into eq. 18.)

(21) $\alpha \approx 7.1 \pm 0.6 \times 10^{-3}$
(This is obtained from eq. 20 using the contemporary values $\Omega_\Lambda = 0.74 \pm 0.03$ and $\Omega_r/\Omega_b = 1.09 \pm 0.03 \times 10^{-3}$.)