A Multiple Particle System Equation Underlying the Klein-Gordon-Dirac-Schrödinger Equations

D.T. Froedge

Formerly Auburn University
Phys-dtfroedge@glasgow-ky.com

V120811

Abstract

The purpose of this paper is to illustrate a fundamental, multiple particle, system equation for which the Klein-Gordon-Dirac-Schrödinger equations are single particle special cases. The basic concept is that there is a broader picture, based on a more general equation that includes the entire system of particles. The first part will be to postulate an equation, and then, by modifying the methods of Path Integrals, develop a solution which describes the internal dynamics as well as particle interactions of quantum particles. The complete function has both real and imaginary, as well as timelike and spacelike parts, each of which are separable into independent expressions that define particle properties. In the same manner that eigenvalues of the Schrödinger equation represents energy levels of an atomic system, particle are eigenvalues in an interacting universe of particles. The Dirac massive and massless equation and solution will be shown as factorable independent components. A clear distinction between the classical and quantum properties of particles is made, increasing the scope of QM.

INTRODUCTION

The success and the accuracy of Quantum Mechanics has been one of the most outstanding achievements in the history of science, but its application to anything other than microscopic systems quite limited. In essence QM deals with a single system acted on by an external potential that allows determination of the properties, motion, and energy associated with that system. The standard QM, coupling is by the insertion of a representation of the potential through the correspondence relations, which is an action on the wavefunction, but not a part of the wavefunction. It is asserted that isolating the system, and acting on it by an external potential, however relativistically correct, limits the scope, and limits the understanding of the system as a whole. The defining function introduced will include the entire system of particle and the interaction between those particles. It will be developed by defining a vector action for a free particle as an integral of a vector Lagrangian over the classical path from the initial big bang event to the current time, and then formulating the function by a method similar to formulating a solution to Schrodinger equation from the method of path integrals. Particle masses are shown to be separation constants and eigenvalues of the general expression. This paper is an attempt to define a function representing an integrated universe of discrete particles, which obey the well known rules of energy and motion.

The defining differential equation is termed the "System Equation", and the solution a "Systemfunction", $\tilde{\Theta}$, to distinguish it from standard QM wave equations and wavefunctions.

Reviewing the standard QM relations

Field free KG:

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial ct^2}\right)\psi=-\left(1/\lambda\!\!\!^{-}{}_0^2\right)\psi \tag{1}$$

Field free Dirac:

$$\gamma^\mu\partial_\mu\psi=i\left(1/\lambda\!\!\!^{-}{}_0\right)\psi \tag{2}$$

Field free Schrödinger:

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}-2i\frac{1}{\lambda\!\!\!^{-}{}_0}\frac{\partial}{\partial ct}\right)\psi=\ 0 \tag{3}$$

Where $\lambda\!\!\!^{-}{}_0$ is the particle Compton radius $\lambda\!\!\!^{-}{}_0=m_0c/\hbar$. Of the rest mass of a particle (For general conventions, and notation, see **Appendix I**)

I. THE GENERAL SYSTEM EQUATION

The Systemfunction $\tilde{\Theta}$ for a universe of discrete particles is proposed to be a solution to the general System Equation:

$$\left(-\frac{\partial^2}{\partial\left(X^2\right)^2}-\frac{\partial^2}{\partial\left(Y^2\right)^2}-\frac{\partial^2}{\partial\left(Z^2\right)^2}+\frac{\partial^2}{\partial\left(\Re^2\right)^2}\right)\tilde{\Theta}=-\left(\frac{1}{2\Re\lambda\!\!\!^{-}}\right)^2\tilde{\Theta} \tag{4}$$

That is, the derivative with respect to a virtual displacement of the square of the expansion of the universe. $\Re^2=\left(cT\right)^2$, and X, Y, & Z are the coordinates of the expanding sphere of the universe, along the light cone. $\tilde{\Theta}$ is the general Systemfunction.

Using standard Clifford operations this can be:

$$\left(\gamma^{\nu}\frac{\partial}{\partial\left(X^{2}\right)_{\nu}}\right)\left(\gamma^{\mu}\frac{\partial}{\partial\left(X^{2}\right)_{\mu}}\right)\tilde{\Theta}=-\left(\frac{1}{2\Re\lambda\!\!\!^{-}}\right)^{2}I\tilde{\Theta}, \tag{5}$$

and the linear function is expected to be:

$$\gamma^{\mu}\frac{\partial}{\partial\left(X^{2}\right)_{\mu}}\tilde{\Theta}=\frac{i}{2\Re\lambda\!\!\!^{-}}I\tilde{\Theta}\quad , \tag{6}$$

where equivalence only implies similarity.

The plan is to develop a function which is a solution to the System Equation based on a summation, of the vector path integral over a vector Lagrangian field, of individual particles in the system. The proposed methodology expands the general method to include the detailed particle interactions, that have eluded the conventional approach, and overcomes a shortcoming noted by Feynman of the standard path integral formulation [2].The Systemfunction will be shown to be a product of several independent functions, one of which being the solution to the Dirac equation.

A. Action re-defined

In the Path Integral formulation of QM, the normal action for the m particle is in general the path integral, over a scalar action, from one event to another, over all possible paths. From the standard path integral formulation of QM [4] The path integral depends on its final coordinate and time in such a way that it obeys the Schrödinger equation, thus heuristically for a Schrödinger wavefunction:

$$\psi\left(x,t\right)=\int\mathcal{D}q\left(t\right)e^{i\int\frac{L}{\hbar}dt}=\int\mathcal{D}q\left(t\right)e^{i\frac{S}{\hbar}} \tag{7}$$

And for the function of a collection of independent particles this would be:

$$\psi(x,t) = \int \mathcal{D}q(t)_1 \mathcal{D}q(t)_2 \mathcal{D}q(t)_3\, e^{i\left(\int \frac{L_1}{\hbar}dt + \int \frac{L_2}{\hbar}dt \;\cdots\right)} \rightarrow \int \mathcal{D}q(t)\, e^{\frac{i}{\hbar}\sum_n S_n} \quad (8)$$

This has the presumption that the individual particles, though summing to the total function, do not interact, but are a linear sum of the non interacting parts.

The departure from standard methods starts here. Proposed is a systemfunction solution to the System Equation Eq., which is:

$$\tilde{\Theta} = Ae^{\left(\sum_N \vec{S}_m\right)\left(\sum_N \vec{S}_m\right)}, \quad (9)$$

where $\vec{S}_n$ is a vector action of a vector Lagrangian for a particle over the path interval from the start of the universe to the current event. The sum is the total free particle actions of all the particles in the system is a four-space event function.

$$f\left(\vec{S}\right) = \sum_N \vec{S}_m \quad , \quad (10)$$

that is evaluated at a point at a contemporary retarded time for all the particles. Note that these are vector actions of presumably a conservative Lagrangian, and thus the value of the integral along any vector path depends only on the end points. In fact a particle can take no longer or shorter time to arrive at its time retarded current position on the light cone.

The proposal for the Systemfunction requires defining the action as a vector, being a single classical path integral over a vector Lagrangian, rather than a scalar:

$$\frac{i}{\hbar} S_m \rightarrow \vec{S}_m \quad (11)$$

and:

$$\vec{S}_m = \; ic\int_{t_1}^{t_2} \vec{L}_m \, d(t) \quad , \quad (12)$$

where the classical Lagrangian for a free particle would go to :

$$L = \frac{m_n c}{\hbar}\sqrt{1 - \frac{\vec{v}_n^{\,2}}{c^2}} \quad \rightarrow \quad \vec{L} = \frac{m_n c}{\hbar}\not{v}_n = \frac{1}{\lambda\!\!\!^{-}}\not{v}_n \quad , \tag{13}$$

and the Lagrangian for the m charged particle residing in the universe is proposed to be:

$$\vec{L} = \underset{m}{\pm}\frac{\alpha}{|r_m|}\not{v}_m \tag{14}$$

Where the distance to the particle is:

$$|\vec{r}_m| = \left|\not{\gamma}\vec{r}_m - \not{\gamma}\alpha\lambda\!\!\!^{-}_m\right|_S = r_m - \alpha\lambda\!\!\!^{-}_m \ , \tag{15}$$

which is the distance from an observation point to the classical charge radius not to the center of mass. $\not{v}_m$ is the unitless four-velocity of the m particle, $\not{v}_m = \left(\gamma^k (v/c)_k + \gamma^0\right)$, and $\lambda\!\!\!^{-}_m$ is the Compton radius of the m particle. $(\lambda\!\!\!^{-} = \hbar / mc)$, $\underset{m}{\pm}$ is the charge sign of the m particle.

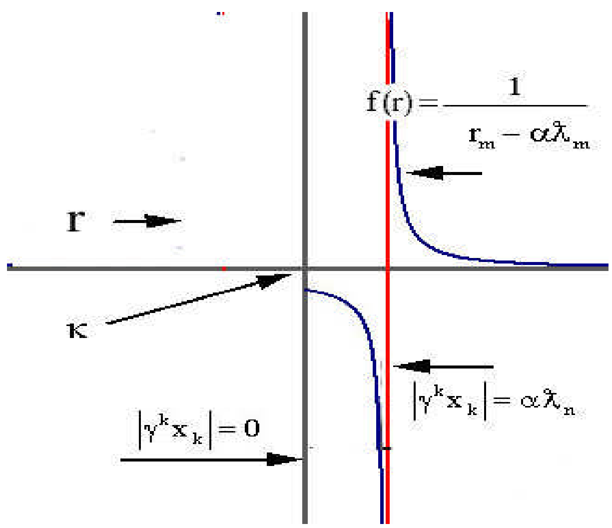

Figure 1. Eq.(19), Radial distance $|\vec{r}_n|$ to classical radius.

Using Eq.(14), as the Lagrangian for a particle, the action can be integrated from the initiation of the timelike universe to its current position. The addition of these vectors requires contemporaneous world lines, and thus connection through a null vector interval.

$$\not\eta\,\vec{S}_m = \not\eta\, i c \int_{t_1}^{t_2} \vec{L}_m \, dt \;\to\; i c \int_{t=0}^{T-\frac{r_m}{c}} \frac{\alpha}{|r_m|} \not\psi_m \not\eta\, dt \tag{16}$$

Where $\not\eta$ is the unit directional null vector $\not\eta = \left(\gamma^0 + \vec{\eta}\bullet\vec{\gamma}\right)$ from the particle to the observation point, making $\not\eta\, c\,dt$ an invariant differential distance. $T - \frac{r_m}{c}$, is the retarded time of the endpoint for a particle at a distance r_m from the observation point. Integrating this over the life of the universe this is then:

$$\not\eta\,\vec{S}_m = i\alpha \not\psi_m \left(\frac{\not\eta\left(\Re - |r_m|\right)}{|r_m|} \right)\Bigg|_{\Re >> r_m} \approx i\alpha \left(\frac{\not\eta\,\Re}{|r_m|} \right) \not\psi_m \tag{17}$$

It could be presumed that $|r_m|$, and $\not\psi_m$ are functions of time, and should be considered in the integral, however since the endpoint is the current position, the exact history is not necessary. It is presumed that the function of the final state satisfies the System Equation.

$\Re = cT$ is the radius of the universe α is the fine structure constant, and $\not\eta\,\Re$ is the null vector to the expanding sphere of universe
.

$$\not\eta\,\Re = +\gamma^k X_k + \gamma^0 \Re \tag{18}$$

From Eq.(10), the total “system” action is, thus.

$$\vec{S}_{total} = \not\eta \sum_{m=1}^{N} \vec{S}_m = \sum_{m=1}^{N} \pm_m i\alpha \frac{\not\eta\,\Re}{|\vec{r}_m|} \not\psi_m \,, \tag{19}$$

This sum represents a collective action of the system at any event in the system, where the $1/r_m$ dependence of each of the elements results in the value of the system action being dependent on the relative position of all the parts.

When the function is evaluated at the origin of a particle (i.e. the space distance being zero, $r_n = 0$), Eq.(19), becomes the total action for the n particle. At that point, the Systemfunction becomes an eigenfunction, and that particle's rest mass is an eigenvalue.

$$\left|\vec{r}_n\right| \; = \; r_m - \alpha \lambda\!\!\!^{-}_m \quad \to -\alpha \lambda\!\!\!^{-}_n \tag{20}$$

Thus $\vec{S}_n$ for $r_n = \left|\gamma^k x_k\right| = 0$, the n free particle action of Eq.(12), becomes:

$$\vec{S}_n = \underset{n}{\mp} i \frac{\cancel{\eta}\Re}{\lambda\!\!\!^{-}_n} \cancel{\mathcal{V}}_n \tag{21}$$

$\vec{S}_n$, which is the free particle vector action for the n particle (Eq.(13). The action contribution of the m particle to the n particle action is:

$$\vec{S}_m = \underset{m}{\pm} i\alpha \frac{\cancel{\eta}\Re}{r_m} \cancel{\mathcal{V}}_m \qquad\qquad r_m >> \alpha \lambda\!\!\!^{-}_n\,, \tag{22}$$

where $\underset{m}{\pm}$ is the sign for the m particle, and r_m is the three space distance from the evaluation point to the m particle.

with Eq., and Eq., the product for the function evaluated at the κ point at the n particle is:

$$\left(\sum_N \vec{S}_m\right)\left(\sum_N \vec{S}_m\right) = \left(\vec{S}_n{}^2 + \vec{S}_n \sum_m \vec{S}_m + \sum_m \vec{S}_m \vec{S}_n \bullet\bullet\bullet\right), \tag{23}$$

where $\vec{S}_n \sum_m \vec{S}_m + \sum_m \vec{S}_m \vec{S}$ represents the interactions of the n particle with the other particles in the system. All terms in this expression are scalar.

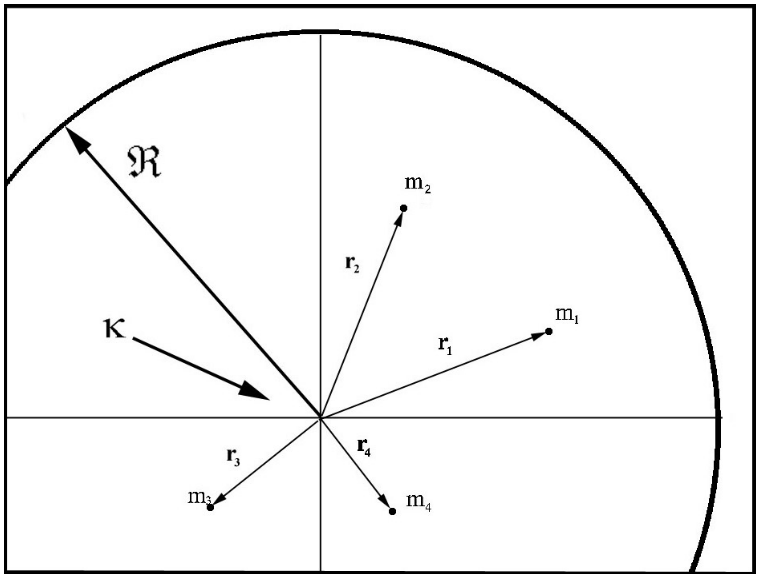

Figure 2. *Total action evaluation point* κ *and radius of universe* $\Re$

II. THE GENERAL ACTION IN A TIMELIKE UNIVERSE

The Systemfunction $\tilde{\Theta}$ with the product of the action defined in Eq.(9), and evaluated at a selected point, κ for a single particle, has a scalar exponent, all the terms of the exponent being real and scalar. The κ point is a single event, but that event can observed simultaneously on an invariant interval anywhere along its future light cone $\cancel{\eta}\left(\Re + r_o\right)$. The future light cone subsequently functions as an observation field for the evaluation event at the n particle center.

It is expected that the initiation of the universe at T=0 there is a minimum value for the sum of the action. Since the least action for any particle is $\hbar/2$ it is proposed and later determined that minimum for a system is $\hbar/2$. This would seem quite arbitrary, but, will be shown necessary to produce proper physical results. Starting with the action function in Eq.(19), The generalized total action for the system is proposed to be:

$$\sum_{m=1}^{N} \vec{S}_m \rightarrow \left(\cancel{\eta} \sum_{m=1}^{N} \vec{S}_m + 1/2 \right) \qquad (24)$$

And the complete Systemfunction of Eq.(9), is then:

$$\tilde{\Theta} = Ae^{\left(\not{\eta}\sum i\vec{S}+1/2\right)\left(\sum i\vec{S}\not{\eta}+1/2\right)} \tag{25}$$

Since the product of the sums is scalar, (See **Appendix IV)** the Systemfunction becomes:

$$\tilde{\Theta} = Ae^{\left(\not{\eta}\not{\eta}\sum i\vec{S}\sum i\vec{S} + \not{\eta}\bullet\sum i\vec{S}\right)}, \tag{26}$$

The square of the null vectors is zero, so for quantum issues the first term can temporarily be ignored. The scalar imaginary portion of the function is then:

$$\tilde{\Theta} = Ae^{\not{\eta}\cdot\sum i\vec{S}}. \tag{27}$$

This is the imaginary Systemfunction, representing a field of observation, connecting the evaluation point κ worldline of the observer, through an invariant interval.

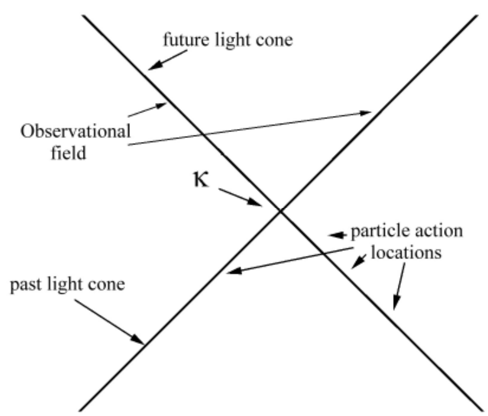

Figure 3. *The observational field of the* κ *event.*

Eq.(6), now becomes:

$$\gamma^{\mu}\frac{\partial}{\partial\left(X^{2}\right)_{\mu}} e^{\not{\eta}\bullet\sum i\vec{S}} = \frac{i}{2\Re\lambda} Ie^{\not{\eta}\bullet\sum i\vec{S}} \tag{28}$$

Or by a change of coordinates to the local coordinates (shown in Appendix II), we have:

$$\gamma^{\mu}\partial_{\mu}\ \tilde{\Theta}\ = \pm\frac{i}{\lambda}I\tilde{\Theta} \tag{29}$$

This has the appearance of the Dirac expression, but is a matrix similarity relation. The eigenvectors of this will be shown equivalent to the Dirac eigenvectors.

III. THE QUANTUM PARTICLE

Putting in the specific values Eq.(27), it becomes. (See **Appendix VIII** for the details)

$$\tilde{\Theta} = Ae^{\pm_{n} i\left(\frac{\Re}{\bar{\lambda}_{n}}\cancel{\mathcal{V}}_{n} - \sum \pm_{n}\pm_{m}\alpha\frac{\Re}{r_{m}}\cancel{\mathcal{V}}_{m}\right)\bullet\cancel{\eta}} \tag{30}$$

Now since $\Re = \Re_0 + ct + r_o$, where $i\Re_0$ represent an ignorable arbitrary phase, and r_o is the space distance from κ, to an arbitrary observation position, (See Figure 3) the function is then:

$$\tilde{\Theta} = Ae^{\pm_{n} i\left(ct + r_o\right)\left(\frac{\cancel{\mathcal{V}}_{n}}{\bar{\lambda}_{n}} - \sum \pm_{n}\pm_{m}\alpha\frac{\cancel{\mathcal{V}}_{m}}{r_{m}}\right)\bullet\cancel{\eta}} \tag{31}$$

or in more elementary terminology:

$$\tilde{\Theta} = Ae^{\pm_{n} i\left(ct - r_o\right)\left[\frac{E_n}{c\hbar}\left(1 - \frac{\vec{v}_n}{c}\bullet\frac{\vec{r}_o}{\left|\vec{r}_o\right|}\right) - \sum \pm_{n}\pm_{m}\frac{\alpha}{r_m}\left(1 - \frac{\vec{v}_m}{c}\bullet\frac{\vec{r}_o}{\left|\vec{r}_o\right|}\right)\right]} \tag{32}$$

Where $\vec{r}_o$ is the spatial three vector of $\cancel{\eta}$, making :

$$r_o\frac{\vec{r}_o}{\left|\vec{r}_o\right|} \to \vec{r}_o, \tag{33}$$

which designates a specific point on an observational sphere.

This is the imaginary Systemfunction has frequencies and wavelengths associated with the Compton and deBroglie waves, and a phase velocity of c. The Compton waves are spherical and the deBroglie waves are plane, in the direction of motion.

Notably, the field defined by this imaginary function, Eq.(32), $\tilde{\Theta}$ has a value representing the linear components of the action of the n particle evaluated at κ, as observed on the invariant null interval throughout the space. This is in contrast with the Dirac positional probability amplitude, and *is the conceptual difference between the Systemfunction and the wavefunction*. The reconciliation of this with the probability amplitudes, will be left for later.

A. Separating Dirac Massive, and Massless Functions

The matrix function Eq.(32), can be separated into a product of functions, with eigenvectors equal to those of the Dirac wavefunction for a massive function, and a massless particle.

The Dirac massive type parts of this function have derivatives with no dependence on the observational field, and thus create a clear demarcation, between the Dirac massive part of the function and the Dirac massless part of the function.

Starting from Eq.(32), and separating into a & b products of the imaginary Systemfunction:

$$\tilde{\Theta} = \Theta_{Ia}\Theta_{Ib} = Ae^{\frac{\pm}{n}i\left[\left(\frac{E_n}{c\hbar}\mp\frac{\alpha}{r_m}\right)ct+\left(\frac{\vec{p}_n}{\hbar}\pm\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\vec{r}_o\right]} \times Ae^{\pm i\left[-\left(\frac{E_n}{c\hbar}\mp\frac{\alpha}{r_m}\right)r_o-\left(\frac{\vec{p}_n}{\hbar}\mp\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{\left|\vec{r}_o\right|}(ct)\right]}. \quad (34)$$

or:

$$\Theta_{Ia} = Ae^{\frac{\pm}{n}i\left[\left(\frac{E_n}{c\hbar}\mp\frac{\alpha}{r_m}\right)ct+\left(\frac{\vec{p}_n}{\hbar}\pm\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\vec{r}_o\right]} \quad (35)$$

and:

$$\Theta_{Ib} = Ae^{\pm i\left[-\left(\frac{E_n}{c\hbar} \mp \frac{\alpha}{r_m}\right)r_o - \left(\frac{\vec{p}_n}{\hbar} \mp \frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]} \tag{36}$$

Applying the chain rule as before to Eq.(29):

$$\gamma^{\mu}\partial_{\mu}\left(\Theta_{Ia}\Theta_{Ib}\right) = \left(\Theta_{Ib}\ \gamma^{\mu}\partial_{\mu}\ \Theta_{Ia} + \Theta_{Ia}\ \gamma^{\mu}\partial_{\mu}\ \Theta_{Ib}\right) = \pm\left(\frac{i}{\lambda_n}\right)I\left(\Theta_{Ia}\Theta_{Ib}\right)\ , \tag{37}$$

As noted earlier Eq.(37), implies a similarity relation, and diagonalizng both sides produces back the trivial standard relativistic velocity mass relation, but finding the eigenvectors of this relation $\mathbf{a}$, can be done, and the process reduces the expression into a very familiar relation.

$$\frac{1}{\Theta_{Ia}}\gamma^{\mu}\partial_{\mu}\Theta_{Ia}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} + \frac{1}{\Theta_{Ib}}\gamma^{\mu}\partial_{\mu}\ \Theta_{Ib}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} = \pm\frac{i}{\lambda_n}I\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} \tag{38}$$

Examination will show that the derivatives of the “Ia” function are independent of the observation vector $\vec{r}_0$, whereas the derivatives of “Ib” are not.

If the first term in Eq.(37), is set equal to the mass constant, making it homogeneous, and identical to the Dirac massive particle equation. This requires the second term to be set to zero, which makes the second equation inhomogeneous or massless. This particular separation illustrates that the imaginary Systemfunction is a product of the Dirac expressions for a massive, and massless particle.

The first massive:

$$\gamma^{\mu}\partial_{\mu}\Theta_{Ia}\mathbf{a} = \gamma^{\mu}\partial_{\mu}\ e^{\pm_n i\left[\left(\frac{m_n c}{\hbar} \mp \frac{\alpha}{r_m}\right)ct + \left(\frac{\vec{p}_n}{\hbar} \pm \frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\vec{r}_o\right]}\ \mathbf{a} = \pm\frac{i}{\lambda_n}I\mathbf{a}\Theta_{Ia}\ , \tag{39}$$

and the second, massless:

$$\gamma^{\mu}\partial_{\mu}\Theta_{Ib} = \gamma^{\mu}\partial_{\mu}\ Ae^{\pm i\left[-\left(\frac{m_n c}{\hbar} \mp \frac{\alpha}{r_m}\right)r_o - \left(\frac{\vec{p}_n}{\hbar} \mp \frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]}I\ \mathbf{a} = 0 \tag{40}$$

B. The Dirac Massive Particle

Rearranging the homogeneous Eq.(39), gives:

$$\gamma^{\mu}\partial_{\mu}\; e^{\pm\frac{i}{n}\left[\frac{m_n c}{\hbar}\left(ct+\frac{\vec{v}_n}{c}\bullet\vec{r}_o\right)\mp\frac{\alpha}{r_m}\left(ct+\frac{\vec{v}_m}{c}\bullet\vec{r}_o\right)\right]}\;\mathbf{a}=\pm\frac{i}{\lambda\!\!\!^{-}_n}I\,\Theta_{Ia}\mathbf{a}\,. \tag{41}$$

An interesting point is the second term in the exponent of Eq.(41), is the potential, and it can be separated and included in the operator.

$$\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}}\left[\mp\, i\frac{\alpha}{r_m}\left(ct+\frac{\vec{v}_m}{c}\bullet\vec{r}_o\right)\right]= i\frac{\alpha}{r_m}\left(\gamma^0+\frac{\vec{v}_m}{c}\right)= i\frac{Q}{c\hbar}\vec{A}_m\,, \tag{42}$$

Where $\vec{A}$ is just the vector potential. Eq. (39), can then be rewritten as:

$$\left(\gamma^{\mu}\partial_{\mu}+i\frac{Q}{c\hbar}\vec{A}_m\right)\mathbf{a}\,\Theta_{Ia}=\pm\frac{i}{\lambda\!\!\!^{-}_n}\mathbf{a}\,I\,\Theta_{Ia}\,. \tag{43}$$

where in this case Θ_{Ia} does not contain the potential terms.

This is identical to the Dirac expression with the vector potential [3], noting that the vector potential was part of the particle action, and not included by way of the correspondence relation. This $\vec{A}_m$ term becomes the source term for the spin potential. (See **Appendix VII**.)

This, equation and function, is now equivalent in form to the Dirac particle in a potential field, and if the potential is removed it is just the Dirac free particle.

Appendix VI, demonstrates the modes for Eq.(39), and Eq.(40), . For the Dirac particle, they are well known but shown for comparison. They are:

$$a_1\sqrt{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}=a_3\sqrt{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}\qquad a_2\sqrt{\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}=a_4\sqrt{\left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}, \tag{44}$$

C. The Dirac Massless Particle Function

Taking the derivatives of the second expression Eq.(40), the “Ib” function, results in:

$$-\pm i\left[\left(\frac{m_n c}{\hbar}\mp\frac{\alpha}{r_m}\right)\frac{\vec{r}_o}{|r_o|}+\left(\frac{\vec{p}_n}{\hbar}\mp\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}\right]\mathbf{a}\,\Theta_{Ib}=0\,, \qquad (45)$$

Examination of this function shown in **Appendix VI,** shows that it can only be valid it the observation vector is collinear with the momentum, and thus Eq.(40), ignoring the potential, and observed along the direction of motion is:

$$\Theta_{Ib}\rightarrow Ae^{\mp i\frac{m_n c}{\hbar}\left[r_o-v_n t\right]}I\mathbf{a}\;\;, \qquad (46)$$

which is the Dirac wavefunction for the massless particle.

Appendix VI, demonstrates the modes for Eq.(45), for the free massless particle which are:

$$\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right)a_3=0\,,\;\left(-\frac{m_n c}{\hbar}-\frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right)a_4=0\;\;,\;\left(-\frac{m_n c}{\hbar}-\frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right)a_1=0\;\;,\;\left(\frac{m_n c}{\hbar}-\frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right)a_2=0$$
$$(47)$$

To review:

The imaginary Systemfunction is identical to a product of the Dirac wavefunctions for a massive and massless particle.

The functions defined in Eq.(35), and Eq.(36), represent the values of the Systemfunction evaluated at the κ event, with that event’s value being observed on an arbitrary invariant interval, at a distant location, The value of that point as seen at a distant point has the characteristics of a wavefunction defined in the observation space . Though having the same values, as a probability amplitude.

IV THE CLASSICAL PARTICLE

The classical particle as discussed here is not a different particle, but the development of the properties of the particle, that are generally considered to be classical. Primarily this will be to show a real mass obeying a real differential equation.

The systemfunction defined in Eq.(26), can be revisited.

$$\tilde{\Theta} = \mathrm{A}e^{\left(\not{\eta}\not{\eta}\sum i\vec{S}\sum i\vec{S} + \not{\eta}\bullet\sum i\vec{S}\right)} \qquad (48)$$

The first term in the exponent is zero, but it is also real and is separable into spacelike and timelike parts, making the function a product of separable spacelike and timelike function.

$$\tilde{\Theta} = \mathrm{A}e^{\left(-X^2-Y^2-Z^2\right)\left(\sum i\vec{S}\sum i\vec{S}\right)} \times e^{+\Re^2\left(\sum i\vec{S}\sum i\vec{S}\right)} \times e^{\left(\not{\eta}\bullet\sum i\vec{S}\right)} \qquad (49)$$

Designation these functions as Θ_{R1}, Θ_{R2} , & Θ_I , and applying the chain rule to this, from Eq.(5):

$$\left(\gamma^\mu \frac{\partial}{\partial\left(X^2\right)_\mu}\right)\left(\Theta_{R2}\Theta_I\gamma^k \frac{\partial}{\partial\left(X^2\right)_k}\Theta_{R1} + \Theta_{R1}\Theta_I\gamma^0 \frac{\partial}{\partial\left(\Re^2\right)}\Theta_{R2} + \Theta_{R1}\Theta_{R2}\gamma^\mu \frac{\partial}{\partial\left(X^2\right)_\mu}\Theta_I\right)\tilde{\Theta}$$

$$= \left(K_{R1} + K_{R2} + K_I\right)\Theta_{R1}\Theta_{R2}\Theta_I$$

(50)

The last term in Eq.(50), is the imaginary term which has been developed earlier, and from Eq.(29), $K_I = i/\lambda$

With the assumption that each of these terms are equal to or at least similar to independent constants, and if:

$$K_{R1} = -K_{R2} \qquad (51)$$

the relation is still valid.

The K's then represent a separation constant between the spacelike and timelike universe of particles, and it is asserted that this separation constitutes the mass of the particles.

$$
\begin{aligned}
K_{R1} &= 1/\bar{\lambda}_0^2 \\
K_{R2} &= -1/\bar{\lambda}_0^2
\end{aligned}
\tag{52}
$$

This is then for the timelike function:

$$
\gamma^0 \frac{\partial}{\partial\left(\Re^2\right)} \Theta_{R2} = -1/\bar{\lambda}_0^2 \; I\Theta_{R2} \quad , \tag{53}
$$

and the spacelike function:

$$
\gamma^k \frac{\partial}{\partial\left(X^2\right)_k} \; \Theta_{R1} = 1/\bar{\lambda}_0^2 \, I \; \Theta_{R1}. \tag{54}
$$

The separation of the Systemfunction into positive and negative masses at the initial event would seem to be the initial separation of the universes at the Big bang.

(55)

It will now be shown that the timelike expression acting on our general systemfunction yields the proper equations of motion for a real particle in a timelike universe.

Inserting timelike function from Eq.(49), into Eq.(53), leads to the classical relativistic Lagrangian equation of motion for a real particle.

$$
\gamma^0 \frac{\partial}{\partial\left(\Re^2\right)} \Theta_{R1} \;=\; \gamma^0 \frac{\partial}{\partial\left(\Re^2\right)} \exp\left(\vec{S}_n^{\,2} + \vec{S}_n \sum_m \vec{S}_m + \sum_m \vec{S}_m \, \vec{S}_n \bullet\bullet\bullet \right) \;=\; \left(\frac{1}{\bar{\lambda}_0^2} \right) I\Theta_{R1} \tag{56}
$$

(See **Appendix IV,** *Classical particle*, for details.)

The exponent of this function is scalar and real, thus, Θ_{R1} is the “Real” part of the Systemfunction for the n particle. Explicitly this is:

$$
\Theta_{R1} = Ae^{\left(\frac{\Re}{\bar{\lambda}_n} \not{\mathcal{V}}_n \right)^2 - \sum_m \pm_n \pm_m 2\alpha \frac{\Re^2}{\bar{\lambda}_n |r_m|} \not{\mathcal{V}}_m \cdot \not{\mathcal{V}}_n} \tag{57}
$$

Putting this into Eq.(53), and expressing in more elementary notation is:

$$\left(\frac{m_{n0}c}{\hbar}\right)^2 = \left(\frac{m_n c}{\hbar}\right)^2\left[\left(1-\frac{\vec{v}_n^{\,2}}{c^2}\right)-\sum_{n}\pm\sum_{m}\pm\frac{2}{m_n c^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right], \tag{58}$$

Eq.(57), is the coordinate free, square of the classical scalar Lagrangian for a moving charged particle in the presence of other moving charged particle. Taking the square root gives the familiar classical linear Lagrangian expression for a particle the presence of other electrically charged particles. Note that there could not be the proper electromagnetic interaction without the vector actions.

$$m_{n0}c^2 \approx +\left[m_n c^2\left(1-\frac{1}{2}\frac{\vec{v}_n^{\,2}}{c^2}\right)\mp\sum_m\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right] \tag{59}$$

The real Systemfunction defines the “classical properties” of the same quantum particle defined earlier.

Arriving at this relativistic interaction Lagrangian is not unusual in itself, but arriving at it using the afore described procedure is unusual, and gives a degree of credence to the general System Equation.

V POINTS OF INTEREST

A. Quantum and classical connection

It has been shown that the real part of the general Systemfunction Eq.(26):

$$\tilde{\Theta}_R = Ae^{\left(\not{h}\not{h}\sum i\vec{S}\sum i\vec{S}\right)}, \tag{60}$$

which is a summation of single particle vector path integrals over the classical path squared, and leads to the classical relativistic equations of motion Eq.(59). This would suggest that the real part of the Systemfunction (Eq.(60),), which only includes the symmetric inner products of the individual vector paths, has reduced the multiplicity of paths to the path of least action.

On the other hand the imaginary (non-squared) part of the systemfunction has components identical to the Dirac solution, which can be arrived at as the result of some multiple scalar path integral formulation [7], [8].

$$\tilde{\Theta}_I(x,t) = Ae^{\left(\not{\eta}\cdot\sum i\vec{S}\right)} \rightarrow \int \mathcal{D}q(t)e^{i\sum_n S_n} \tag{61}$$

The classical properties are the result of the square of this action function, which would lead more directly to the conclusion that the path integral formulation of QM is the underlying mechanism of the principle of least action for the classical, as well as the quantum particle properties.

B. Factorable Systemfunction

Throughout the paper we have factored out parts of the Systemfunction creating independent equations and functions. First the parts of the imaginary function (Eq.(37),) then the photon functions and then the timelike and spacelike real function (Eq.(50),)

It can be subsequently be pointed out that the total Systemfunction is a product of all the functions that define the properties of the particles.

So we have:

Dirac Massive, Massless. $\quad \Theta_I = \Theta_{Ia}\Theta_{Ib}$

Spacelike, Timelike. $\quad \tilde{\Theta} = \Theta_{R1}\Theta_{R2}\Theta_{Ia}\Theta_{Ib}$ (62)

Thus, the entire systemfunction is a product of the separable functions that represent distinguishable properties of the particle defined by the action function.

$$\tilde{\Theta} = \Theta_{R1}\Theta_{R2}\Theta_{Ia}\Theta_{Ib} = e^{\left(\not{\eta}\sum i\vec{S}+1\right)\left(\sum i\vec{S}\not{\eta}+1\right)} \tag{63}$$

It could be noted that as the time T goes to zero, the value of the product of the S sums, which are negative approach zero. As this happens the value of the exponent goes to negative infinity in the spacelike, and timelike universes, but not in the product since $\not{\eta}\not{\eta} = 0$. This should be the expectation in the separation of positive and negative masses between timelike and spacelike universes (Eq.(50),) in a “big bang”.

CONCLUSION

A multiple particle system equation for the universe and its connection to Classical and Quantum Mechanics has been demonstrated. If viable, it represents a new approach to understanding particle dynamics, and particle interactions. It has been the intention of this paper to show the generation of standard well known physical relations as the result of the application of the System Equation to the Systemfunction. The fact that standard physical relations, as well as the Dirac relations are derived, and not contravened, lends a degree of confidence in the methodology. The fact that the particle electromagnetic interaction comes automatically out of the theory without special hypothesis is an extraordinary feature. Extension to other arenas of physics not well understood would be the logical next step.

Référence :

1. John David Jackson, Classical Electrodynamics Second Edition pp 404
2. Feynman, Hibbs, Quantum Mechanics and Path Integrals McGraw-Hill (1965) pp 24, "Today, any general law that we have been able to deduce from the principle of superposition of amplitudes, such as the characteristics of angular momentum, seem to work. But the detailed interactions still elude us. This suggests that amplitudes will exist in a future theory, but their method of calculation may be strange to us."
3. P. A. M. Dirac, The Principles of Quantum Mechanics, 4th ed., Oxford University Press,London (1958).
4. Notes on Quantum Mechanics, K. Schulten, Beckman Institute University April 18, 2000)pp324 http://www.ks.uiuc.edu/Services/Class/PHYS480/qm_PDF/QM_Book.pdf
5. G.A. Blass, Theoretical Physics,Meredith Publishing,1962, pp439
6. Chris Doran, Geometric Algebra for Physicists, Cambridge University Press, 2003
7. Janos Polonyi, Path Integral for the Dirac Equation, arXiv:hep-th/9809115v1
8. G. J. Papadopoulos, J. T. Devreese, Path-integral solutions of the Dirac equationPhys. Rev. D 13, 2227–2234 (1976)

Appendix I

Definitions, Notation, and Conventions

The radius of the universe $\Re = cT = \Re_0 + ct$

Four velocity of particle $\gamma^{\mu}\left(\frac{v}{c}\right)_{\mu} = \not{v}$ unitless

Three velocity of m particle $\gamma^{k}\left(v_{m}\right)_{k} = \vec{v}_{m} = \gamma^{k}\bullet\vec{v}_{m}$

Null unit vector $\not{\eta} = {}^{\mu}\eta_{\mu} = \left(\gamma^{0} + \vec{\eta}\bullet\vec{\gamma}\right)$ $\vec{\eta}\bullet\vec{\eta} = -1$

Mass m

Rest mass m_0

Compton radius $\lambdabar = \frac{\hbar}{mc}$

Vector 4 potential $\vec{A}$

Compton radius rest mass $\lambdabar_0 = \frac{\hbar}{m_0 c}$

Sign of m particle charge $\underset{m}{\pm}$

Shortened derivatives $\gamma^{\mu}\frac{\partial}{\partial\left(x\right)_{\mu}} = \gamma^{\mu}\partial_{\mu}$

$$\gamma^{\mu}\frac{\partial}{\partial\left(X^{2}\right)_{\mu}} = \gamma^{\mu}\partial X^{2}{}_{\mu}$$

The Dirac matrix convention:

$$\gamma^{1} = \begin{bmatrix} & & & +1 \\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \quad \gamma^{2} = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix} \quad \gamma^{3} = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix} \quad \gamma^{0} = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix}$$

(1.1)

and

$$\gamma^{1}\gamma^{1} = -1, \quad \gamma^{2}\gamma^{2} = -1, \quad \gamma^{3}\gamma^{3} = -1, \quad \gamma^{0}\gamma^{0} = +1.$$

Special vectors:
Trivector:

$$\bar{\sigma} = \gamma^1\gamma^2\gamma^3 \qquad -\bar{\sigma} = \gamma^3\gamma^2\gamma^1 \qquad \bar{\sigma} = \begin{bmatrix} & & -i & \\ & & & -i \\ i & & & \\ & i & & \end{bmatrix} \tag{1.2}$$

Pseudo-scalar:

$$\gamma^5 = \bar{\sigma}\gamma^0 \tag{1.3}$$

Square are:

$$\bar{\sigma}^2 = 1 \qquad \gamma^5\gamma^5 = -1$$

Spin vector:

$$\sigma_1 = \gamma_2\gamma_3 \qquad \sigma_2 = \gamma_3\gamma_1 \qquad \sigma_3 = \gamma_2\gamma_1 \tag{1.4}$$

The vector four velocities:

$$\not{v} = \left(\gamma^1 v_x + \gamma^2 v_y + \gamma^3 v_z + \gamma^0 c\right)/c \tag{1.5}$$

The product of two four velocities:

$$\not{v}_n \not{v}_m = \left(\gamma^1 v_{xn} + \gamma^2 v_{yn} + \gamma^3 v_{zn} + \gamma^0 c\right)\left(\gamma^1 v_{xm} + \gamma^2 v_{ym} + \gamma^3 v_{zm} + \gamma^0 c\right)/c^2 \tag{1.6}$$

or

$$\not{v}_n \not{v}_m = \left[\vec{v}_n \bullet \vec{v}_m + \bar{\sigma}\left(\vec{v}_n \times \vec{v}_m\right) + \gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right]/c^2 \tag{1.7}$$

The inner product:

$$\not{v}_n \not{v}_m + \not{v}_m \not{v}_n = 2\,\not{v}_n \bullet \not{v}_m \tag{1.8}$$

The outer product:

$$\not{v}_n \not{v}_m - \not{v}_m \not{v}_n = \left[2\bar{\sigma}\left(\vec{v}_n \times \vec{v}_m\right) + 2\gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right]/c^2 \tag{1.9}$$

Appendix II

The Change to Local Co-ordinates

From Eq.(5):

$$\sum_{\mu}\frac{\partial^2}{\partial\left(X^2\right)^2}\tilde{\Theta}=-\left(\frac{1}{2\Re\lambda\!\!\!^{-}}\right)^2\tilde{\Theta} \tag{2.1}$$

Where the X coordinates represent the location coordinates of the sphere of the expanding universe, or using Clifford algebra by the Dirac procedure:

$$\left(\gamma^{\nu}\frac{\partial}{\partial\left(X^2\right)_{\nu}}\right)\left(\gamma^{\mu}\frac{\partial}{\partial\left(X^2\right)_{\mu}}\right)\tilde{\Theta}=-\left(\frac{1}{2\Re\lambda\!\!\!^{-}}\right)^2 I\tilde{\Theta} \tag{2.2}$$

Note that this is the change in the function with respect to a displacement of the expanding light cone at the edge of the universe.

Since in a spherical universe, X,Y,Z are the location coordinates of the expanding light-cone of the universe, and each have a magnitude equal to the radius $\Re$.

Along the light cone, $\partial X = c\partial T$ and locally, $\partial x = c\partial t$. Since $\partial t = \partial T$ then $\partial x = \partial X$, thus:

$$\begin{aligned}
\partial\left(X^2\right)&=2X\,\partial x=2\Re\,\partial x\\
\partial\left(Y^2\right)&=2Y\,\partial y=2\Re\,\partial y\\
\partial\left(Z^2\right)&=2Z\,\partial z=2\Re\,\partial z\\
\partial\left(\Re^2\right)&=2\Re\,\partial\left(ct\right)=2\Re\,\partial\left(ct\right)
\end{aligned}\quad , \tag{2.3}$$

Then Eq.(6), for the timelike systemfunction would be:

$$\gamma^0\frac{\partial}{\partial\left(x\right)}\tilde{\Theta}=\frac{i}{\lambda\!\!\!^{-}}I\tilde{\Theta} \tag{2.4}$$

.

and general Eq.(2.2),:

$$\left(\gamma^{\nu}\frac{\partial}{2\Re\partial(x)_{\nu}}\right)\left(\gamma^{\mu}\frac{\partial}{2\Re\partial(x)_{\mu}}\right)\tilde{\Theta}=-\left(\frac{1}{2\Re\bar{\lambda}}\right)^{2}I\tilde{\Theta} \tag{2.5}$$

To the extent that $\Re$ is large slow changing, and can be treated as constant, and the expression becomes similar to the Dirac equation, thus:

$$\left(\gamma^{\nu}\frac{\partial}{\partial(x)_{\nu}}\right)\left(\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}}\right)\tilde{\Theta}=-\left(\frac{1}{\bar{\lambda}}\right)^{2}I\tilde{\Theta} \tag{2.6}$$

and:

$$\gamma^{\mu}\partial_{\mu}\tilde{\Theta}=\frac{i}{\bar{\lambda}}I\tilde{\Theta} \tag{2.7}$$

If $\Re$ is not treated as a constant, there are other terms.

Note that this a matrix equation, and the equality implies similarity. The eigenvector equation for this is identical to the Dirac equation.

Appendix IV
Details, Classical Particle

This is the details of the products of the actions of the individual particles for. Starting with Eq.(53):

$$\Theta_{R2}=e^{\left(\sum_{m-1}^{N}\vec{S}_m\right)\left(\sum_{m-1}^{N}\vec{S}_m\right)} \tag{4.1}$$

Putting in the particle actions:

$$\frac{\partial}{\partial\Re^{2}}\Theta_{R2}=\frac{\partial}{\partial\Re^{2}}\exp\left(\vec{S}_n{}^{2}+\vec{S}_n\sum_{m}\vec{S}_m+\sum_{m}\vec{S}_m\,\vec{S}_n\bullet\bullet\bullet\right)=-\frac{1}{\tilde{r}_0^{2}}\Theta_{R2}$$

(4.2)

The actions for the central and distant particles Eq.(21), and Eq.(22):

$$\vec{S}_n = \underset{n}{\pm} i \frac{\Re}{\lambda_n} \not{v}_n \qquad \vec{S}_m = \underset{m}{\pm} i \frac{\alpha \Re \not{v}_m}{|r_m|} \tag{4.3}$$

$$\begin{aligned} \vec{S}_n &= \left[\pm i \frac{\Re}{\lambda_n} \not{v}_n \right] \qquad r_n \to 0 \\ \vec{S}_n \vec{S}_n &= \left[\pm i \frac{\Re}{\lambda_n} \not{v}_n \right] \left[\pm i \frac{\Re}{\lambda_n} \not{v}_n \right] \\ S_n^2 = \vec{S}_n \vec{S}_n &= -\left(\frac{\Re}{\lambda_n} \not{v}_n \right)^2 \end{aligned} \tag{4.4}$$

$$\vec{S}_n \vec{S}_m = \left(\pm i \frac{\Re}{\lambda_n} \not{v}_n \right) \left(\underset{m}{\pm} i \frac{\alpha \Re \not{v}_m}{|r_m|} \right) = -\underset{n}{\pm}\underset{m}{\pm} \frac{\alpha \Re^2}{\lambda_n |r_m|} \not{v}_n \not{v}_m$$

and (4.5)

$$\vec{S}_m \vec{S}_n = -\underset{n}{\pm}\underset{m}{\pm} \frac{\alpha \Re^2}{\lambda_n |r_m|} \not{v}_m \not{v}_n$$

$$\vec{S}_n \vec{S}_m + \vec{S}_m \vec{S}_n = -\underset{n}{\pm}\underset{m}{\pm} 2\alpha \frac{\Re^2}{\lambda_n |r_m|} \not{v}_m \bullet \not{v}_n \tag{4.6}$$

So that

$$\vec{S}_n{}^2 + \vec{S}_n \sum_m \vec{S}_m + \sum_m \vec{S}_m \vec{S}_n = +\left(\frac{\Re}{\lambda_n} \not{v}_n \right)^2 - \underset{n}{\pm}\underset{m}{\pm} 2\alpha \frac{\Re^2}{\lambda_n |r_m|} \not{v}_m \bullet \not{v}_n \tag{4.7}$$

Note that the m,m terms are ignored since their mutual interaction is only an indirect correction to the n term action.

Thus:

$$\Theta_{R2} = A e^{-\left(\frac{\Re}{\lambda_n} \vec{V}_n \right)^2 - \sum_m \underset{n}{\pm}\underset{m}{\pm} 2\alpha \frac{\Re^2}{\lambda_n |r_m|} \not{v}_m \bullet \not{v}_n} . \tag{4.8}$$

Taking the differential with respect to $\Re^2$, and noting that the function is only a function of the time coordinate:

$$\left(\frac{m_{n0}c}{\hbar}\right)^2 = +\frac{m_n{}^2c^2}{\hbar^2}\left(1-\frac{\vec{v}^2{}_n}{c^2}\right)-\sum_m \underset{n}{\pm}\underset{m}{\pm}2\frac{m_n}{\hbar^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right) \quad , \tag{4.9}$$

which is the square of the relativistic Lagrangian for a classical mass particle. Taking the square root gives the familiar linear expression.

$$\frac{m_{n0}c}{\hbar} = +\sqrt{\left(\frac{m_n c}{\hbar}\right)^2\left(\not{v}_n{}^2-\sum_m \underset{n}{\overset{C}{\pm}}\underset{m}{\overset{C}{\pm}}\frac{2}{m_n c^2}\frac{Q^2}{r_m}\not{v}_m\bullet\not{v}_n\right)}, \tag{4.10}$$

or

$$m_{n0}c^2 \approx m_n c^2\left(1-\frac{1}{2}\frac{\vec{v}_n^2}{c^2}\right)\pm\sum_m\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right) \quad . \tag{4.11}$$

Appendix V
Details, Quantum Particle

Starting with the Systemfunction Eq.(27),

$$\tilde{\Theta} = Ae^{\not{\eta}\cdot\sum i\vec{S}} \tag{5.1}$$

The action of the n particle at κ is:

$$\vec{S}_n = \underset{n}{\pm} i\frac{\Re}{\bar{\lambda}_n}\not{v}_n \qquad\qquad r_n \to 0 \tag{5.2}$$

and the m particles are:

$$\sum_m \vec{S}_m = \sum_m \pm\alpha i\frac{\Re}{r_m}\not{v}_m \qquad\qquad r_m >> \alpha\bar{\lambda} \tag{5.3}$$

Then the sums from Eq.(27), are,

$$\cancel{\eta}\bullet\sum i\vec{S}=\left[\underset{n}{\pm} i\frac{\Re}{\bar{\lambda}_n}\cancel{v}_n+\sum_m \underset{m}{\pm}\ \alpha i\frac{\Re}{r_m}\cancel{v}_m\right]\bullet\vec{\eta}\ , \tag{5.4}$$

and the systemfunction is:

$$\tilde{\Theta}=Ae^{\underset{n}{\pm} i\left(\frac{\Re}{\bar{\lambda}_n}\cancel{v}_n-\sum \underset{n}{\pm}\underset{m}{\pm}\alpha\frac{\Re}{r_m}\cancel{v}_m\right)\bullet\cancel{\eta}} \tag{5.5}$$

Now since $\Re=\Re_0+ct+r_o$, where $i\Re_0$ represents an ignorable arbitrary phase, and r_o is the distance from κ, thus:

$$\tilde{\Theta}=Ae^{\underset{n}{\pm} i(ct-r_o)\left(\frac{\cancel{v}_n}{\bar{\lambda}_n}-\sum \underset{n}{\pm}\underset{m}{\pm}\alpha\frac{\cancel{v}_m}{r_m}\right)\bullet\cancel{\eta}} \tag{5.6}$$

The imaginary Systemfunction is then a spatially defined wavefunction being composed of spherical and plane waves having frequencies of the Compton and deBroglie waves, and a phase velocity of c.

This function is the corresponding relation for the Systemfunction, as the Dirac particle in a potential.

Appendix VI
Connection to Dirac Expression

The purpose of this appendix is to illustrate the properties of the two parts of the imaginary systemfunction and the connection to the Dirac function,

Dirac Function

This development is well known, but shown here for comparison. By letting $r_m\to\infty$ in Eq.(40), the Systemfunction becomes that for the free particle condition, or:

$$\gamma^\mu\partial_\mu\ e^{\underset{n}{\pm} i\left(\frac{m_n c}{\hbar}ct+\frac{\vec{p}_n}{\hbar}\bullet\vec{r}_o\right)}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}=\pm\frac{1}{\bar{\lambda}_n}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}\Theta_{Ia} \tag{6.1}$$

or:

$$\Theta_I = \mathbf{a}e^{\pm_n i\left(Et+\vec{p}_n \bullet \vec{r}_o\right)/\hbar} \tag{6.2}$$

This is the same as the Dirac function for a free particle which is:

$$\psi = u_{\vec{p}}e^{i\left(Et-\vec{p}\bullet\vec{r}\right)/\hbar} \tag{6.3}$$

From Eq.(6.2), for the free particle of our "Ia" imaginary Systemfunction, is the same as the Dirac free particle wavefunction)

$$\gamma^{\mu}\partial_{\mu}e^{\pm_n i\left(\frac{m_n c}{\hbar}ct+\frac{\vec{p}_n}{\hbar}\bullet\vec{r}_o\right)}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix} = i\left(\gamma_0\frac{m_n c}{\hbar}-\gamma_1\frac{\vec{p}_x}{\hbar}-\gamma_2\frac{\vec{p}_y}{\hbar}-\gamma_3\frac{\vec{p}_z}{\hbar}\right)\Theta_1\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}, \tag{6.4}$$

or:

$$\left(\gamma_0\frac{m_n c}{\hbar}-\gamma_1\frac{\vec{p}_x}{\hbar}-\gamma_2\frac{\vec{p}_y}{\hbar}-\gamma_3\frac{\vec{p}_z}{\hbar}\right)\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}\Theta_{Ia} = \pm\frac{1}{\lambda_n}\begin{bmatrix}a_1\\a_2\\a_3\\a_4\end{bmatrix}\Theta_{Ia} \tag{6.5}$$

Using a common set of Dirac matrix which is:

$$\gamma_1 = \begin{bmatrix} & & & +1\\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix},\ \gamma_2 = \begin{bmatrix} & & & -i\\ & & i & \\ & i & & \\ -i & & & \end{bmatrix},\ \gamma_3 = \begin{bmatrix} & & 1 & \\ & & & -1\\ -1 & & & \\ & 1 & & \end{bmatrix},\ \gamma_0 = \begin{bmatrix} & & 1 & \\ & & & 1\\ 1 & & & \\ & 1 & & \end{bmatrix} \tag{6.6}$$

the "Ia" expression is just the normal Dirac equation.

$$\begin{bmatrix} & & \frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar} & -\frac{\vec{p}_x}{\hbar}+i\frac{\vec{p}_y}{\hbar} \\ & & -\frac{\vec{p}_x}{\hbar}-i\frac{\vec{p}_y}{\hbar} & \frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar} \\ \frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar} & \frac{\vec{p}_x}{\hbar}-i\frac{\vec{p}_y}{\hbar} & & \\ \frac{\vec{p}_x}{\hbar}+i\frac{\vec{p}_y}{\hbar} & \frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar} & & \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\Theta_{Ia} = \frac{m_0 c}{\hbar}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\Theta_{Ia} \tag{6.7}$$

Now for simplicity the velocity is set to be along the z axis, then the normal four simultaneous equations is.

$$\begin{bmatrix} -\frac{m_0 c}{\hbar}a_1 & & \left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)a_3 & \\ & -\frac{m_0 c}{\hbar}a_2 & & \left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)a_4 \\ \left(\frac{m_n c}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)a_1 & & -\frac{m_0 c}{\hbar}a_3 & \\ & \left(\frac{m_n c}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)a_2 & & -\frac{m_0 c}{\hbar}a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6.8}$$

Solving for the a's

$$\frac{m_0 c}{\hbar}a_1 = \left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)a_3 \quad , \text{ or } \qquad \frac{m_0 c}{\hbar}a_3 = \left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)a_1 \tag{6.9}$$

We can then have for the modes:

$$\frac{a_1}{a_3}=\frac{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}{\frac{m_0c}{\hbar}}\left[\frac{\sqrt{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}}{\sqrt{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}}\right]\quad=\frac{\sqrt{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}}$$

or:

$$\frac{a_1}{a_3}=\frac{\frac{m_0c}{\hbar}}{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}\left[\frac{\sqrt{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}}\right]\quad=\frac{\sqrt{\left(\frac{\vec{p}_z}{\hbar}-\frac{mc}{\hbar}\right)}}{\sqrt{\left(\frac{\vec{p}_z}{\hbar}+\frac{mc}{\hbar}\right)}}\tag{6.10}$$

Similarly for a_2 and a_4:

$$\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)a_2-\frac{1}{2}\frac{m_0c}{\hbar}a_4\qquad\frac{a_2}{a_4}=\frac{\frac{m_0c}{\hbar}}{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}\left[\sqrt{\frac{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}}\right]\quad=\frac{\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}}$$

$$\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)a_4-\frac{1}{2}\frac{m_0c}{\hbar}a_2\qquad\frac{a_2}{a_4}=\frac{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}{\frac{m_0c}{\hbar}}\left[\sqrt{\frac{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}}\right]\quad=\frac{\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}}{\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}}\tag{6.11}$$

The modes and options are:

$$\Theta_1=\left\{\begin{array}{l}\alpha_1\begin{bmatrix}\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}\\0\\\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}\\0\end{bmatrix}+\beta_1\begin{bmatrix}0\\\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}\\0\\2\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}\end{bmatrix}\\[2em]+\ \alpha_2\begin{bmatrix}\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}\\0\\\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}\\0\end{bmatrix}+\beta_2\begin{bmatrix}0\\\sqrt{\left(\frac{m_nc}{\hbar}-\frac{\vec{p}_z}{\hbar}\right)}\\0\\\sqrt{\left(\frac{m_nc}{\hbar}+\frac{\vec{p}_z}{\hbar}\right)}\end{bmatrix}\end{array}\right\}e^{\pm\left(\frac{E}{\hbar}t-\frac{\vec{p}_n}{\hbar}\bullet r_0\right)}\tag{6.12}$$

Where the α's and β's are arbitrary constants.

Dirac Massless Function

Now looking at the second equation, “Ib”, from Eq.(36):

$$\Theta_{Ib} = Ae^{\pm i\left[-\left(\frac{m_n c}{\hbar}\mp\frac{\alpha}{r_m}\right)r_o-\left(\frac{m_n\vec{v}_n}{\hbar}\mp\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]}, \qquad (6.13)$$

and Eq.(40):

$$\gamma^{\mu}\partial_{\mu} Ae^{\pm i\left[-\left(\frac{m_n c}{\hbar}\mp\frac{\alpha}{r_m}\right)r_o-\left(\frac{m_n\vec{v}_n}{\hbar}\mp\frac{\alpha}{r_m}\frac{\vec{v}_m}{c}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]} I\,\mathbf{a} = 0 \qquad (6.14)$$

Where $\vec{r}_o / r$ is the three space part of the null vector η_o
Observing only the free particle by letting the potential be small $r_m \to \infty$

$$\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}} Ae^{\pm i\left[-\frac{m_n c}{\hbar}r_o-\frac{m_n\vec{v}_n}{\hbar}\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]} = 0 \qquad (6.15)$$

With multiplication by the column eigenvector **a**, becomes:

$$i\left[\frac{m_n c}{\hbar}\left(\frac{\vec{r}}{r}\right) - \gamma_0\frac{\vec{p}_n}{\hbar}\bullet\left(\frac{\vec{r}}{r}\right)\right]\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\Theta_{Ib} = 0 \qquad (6.16)$$

Noting the corresponding term from the Dirac massive particle :

$$i\left(\frac{\vec{p}_n}{\hbar} - \gamma^0 \frac{m_n c}{\hbar}\right)\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} \tag{6.17}$$

Both terms in Eq. (6.16), have r the observation vector in the terms whereas the Dirac massive particle terms do not.

Explicitly Eq. (6.16), is:

$$\begin{bmatrix} & & \frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|} & \frac{m_n c}{\hbar}\left(\frac{x}{r} - i\frac{y}{r}\right) \\ & & \frac{m_n c}{\hbar}\left(\frac{x}{r} + i\frac{y}{r}\right) & -\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|} \\ -\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|} & \frac{m_n c}{\hbar}\left(\frac{x}{r} + i\frac{y}{r}\right) & & \\ \frac{m_n c}{\hbar}\left(\frac{x}{r} - i\frac{y}{r}\right) & \frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|} & & \end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

(6.18)

Where $A = a'_1, a'_2, a'_3, a'_4$,

Now the system is taken to be such that the observation vector is aligned with the momentum along the z axis $r = z$, and multiply the components of the a's, thus:

$$\begin{bmatrix} & & \left(\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right) & \\ & & & \left(-\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right) \\ \left(-\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right) & & & \\ & \left(\frac{m_n c}{\hbar}\frac{z}{r} - \frac{\vec{p}_n}{\hbar}\bullet\frac{\vec{r}_o}{|r_o|}\right) & & \end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \tag{6.19}$$

so:

$$\begin{bmatrix} \left(\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_3 \\ \left(-\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_4 \\ \left(-\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_1 \\ \left(\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \qquad (6.20)$$

thus:

$$\left(\frac{m_n c}{\hbar} + \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_1 = 0, \quad \left(\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_2 = 0 , \quad \left(\frac{m_n c}{\hbar} - \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_3 = 0 , \quad \left(\frac{m_n c}{\hbar} + \frac{\vec{p}_n}{\hbar} \bullet \frac{\vec{r}_o}{|r_o|}\right) a_4 = 0$$

(6.21)

Which are satisfied if the two modes are $E = \pm c \ \vec{P} \bullet \frac{\vec{r}_o}{|r_o|}$. Thus, the modes, and are only observable along the direction of motion, one in the positive direction, and one in the negative direction, and under that circumstance, the function is identical to the well known Dirac massless particle function.

$$\gamma^{\mu} \partial_{\mu} \Theta_{Ib} = \gamma^{\mu} \partial_{\mu} \ A e^{\mp i \frac{m_n c}{\hbar} \left[r_o - v_n t \right]} I \mathbf{a} = 0 \qquad (6.22)$$

Thus, our second function of the imaginary Systemfunction alludes to a second particle having zero rest mass, and two modes with a velocity of $\pm$ v., but the phase velocity from Eq.(6.15), is v, so that if the particle is not moving, the phase velocity is zero.
The Systemfunction of the free particle then represents a composition particle, consisting of a Dirac type massive, and massless particles. An interesting factoring of this is included in **Appendix X.**

Appendix VII
Spin

Taking Eq.(2.6),

$$\left(\gamma^{\nu}\frac{\partial}{\partial(x)_{\nu}}\right)\left(\gamma^{\mu}\frac{\partial}{\partial(x)_{\mu}}\right)\tilde{\Theta} = -\left(\frac{1}{\lambda\!\!\!^{-}}\right)^{2} I\tilde{\Theta}, \tag{7.1}$$

and putting the operator defined in Eq.(43):

$$\left(\gamma^{\mu}\partial_{\mu} \;+\; \frac{Q}{c\hbar}\vec{A}_{m}\right) \tag{7.2}$$

We will have a term such that:

$$\left(\gamma^{\mu}\partial_{\mu} + \frac{Q}{c\hbar}\vec{A}\right)\left(\gamma^{\nu}\partial_{\nu} + \frac{Q}{c\hbar}\vec{A}\right)\Theta_{Ia} \tag{7.3}$$

Noting that:

$$\gamma^{\mu}\partial_{\mu}\frac{Q}{c\hbar}\vec{A} = \frac{Q}{c\hbar}\sigma\bullet B + i\gamma^{5}\frac{Q}{c\hbar}\sigma\bullet E \tag{7.4}$$

We have:

$$\left(\gamma^{\mu}\partial_{\mu} + \frac{Q}{c\hbar}\vec{A}\right)\left(\gamma^{\nu}\partial_{\nu} + \frac{Q}{c\hbar}\vec{A}\right)\Theta_{Ia} = \left[\left(\gamma^{\mu}\partial_{\mu}\right)^{2} \;+\left(\;\frac{Q}{c\hbar}\vec{A}_{m}\right)^{2} + \frac{Q}{c\hbar}\left(\sigma\bullet B_{m} + i\gamma^{5}\sigma\bullet E_{m}\right)\right]\Theta_{Ia}, \tag{7.5}$$

which illustrates that the Systemfunction exhibits the same spin energy associated with a 2g electron in the presence of a charged particle, (i.e. the Bohr magnetron) as does the Dirac particle[5].

Appendix VIII

Developing the General System Equation **and Systemfunction**

Eq.(19), which is the sum of all the free particle actions, and is thus the space of the total "system" action.

$$\vec{S}_{total} = \sum_{m=1}^{N}\vec{S}_{m} \;=\; i\alpha\sum_{m=1}^{N}\pm_{m}\frac{\Re}{\left|\vec{r}_{m}\right|}\not{\mathcal{P}}_{m}, \tag{8.1}$$

This sum represents the function of collective action of the system, where the contribution of each of the particles to the function at the κ point has a $1/r_m$, dependence.
If the $\vec{S}_{total}$ vector field is evaluated at an event, the system is then an instant point function of time, which is acceptable for the point, but if this point is observed from another location, the interval would not be proper.

With $\Re$ being the time from the initiation of the system the observability of all the particle actions are along the past lightcone. And the invariant interval connecting the particles is a null vector, thus $\Re$ in the action which has been the Lagrangian integrated from the initial event should be replaced with the null vector along the past lightcone.

$$\Re \to \not{\Re} = \gamma^{\mu} X_{\mu} = \not{\eta}\Re \tag{8.2}$$

An assumption now is, that the start of the expansion of the universe, T_0 the minimum for the action sum is not zero, but ½ the quantum of *action,* $\hbar$. The units of $\vec{S}$ are in $\hbar$, so Eq.(8.1), should become:

$$\vec{S} = \left(\not{\eta}\sum i\vec{S} + 1/2\right)\left(\sum i\vec{S}\not{\eta} + 1/2\right) \qquad (8.3)^*$$

Putting this into Eq.**Error! Reference source not found.**, gives.

$$\tilde{\Theta} = Ae^{\left(\sum i\vec{S}\right)\left(\sum i\vec{S}\right)} \to Ae^{\left(\not{\eta}\sum i\vec{S}+1/2\right)\left(\sum i\vec{S}\not{\eta}+1/2\right)}, \tag{8.4}$$

Rearranging the exponent of this is:

$$\left(\not{\eta}\sum i\vec{S}+1/2\right)\left(\sum i\vec{S}\not{\eta}+1/2\right) = \not{\eta}\sum i\vec{S}\sum i\vec{S}\not{\eta} + 1/2\not{\eta}\sum i\vec{S} + /2\sum i\vec{S}\not{\eta} + 1/4 \tag{8.5}$$

As noted earlier $\sum i\vec{S}\sum i\vec{S}$ is a scalar, then:

$$\not{\eta}\not{\eta}\sum i\vec{S}\sum i\vec{S} + \not{\eta}\bullet\sum i\vec{S} \quad +1/4 \tag{8.6}$$

Now since $\vec{S}$ has a factor $\Re$, (which will be shown for illustration), as the product of the first term:

$$\cancel{\eta}\,\cancel{\eta}\Re^2 = \left(\Re^2 - X^2 - Y^2 - Z^2\right) = \left(\Re^2 - \gamma^k X^2{}_k\right) = 0 \tag{8.7}$$

Separating the space and time terms in Eq.(8.5), there is the space, time symmetric exponents, so that Eq.(8.5), becomes:

$$= \left(\Re^2 \sum i\vec{S} \sum i\vec{S}\right) - \left(\left(\gamma^k X_k\right) \sum i\vec{S} \sum i\vec{S}\right) + \cancel{\eta} \bullet \sum i\vec{S} + 1/4 \tag{8.8}$$

If this represents the factors of the Systemfunction, which by the procedure outline Eq.(49), - Eq.(54), leading to:

The imaginary function:

$$\gamma^\mu \frac{\partial}{\partial\left(X^2\right)_\mu} \Theta_I = \frac{i}{2\Re\lambda} I\Theta_I \tag{8.9}$$

The real timelike function:

$$\gamma^0 \frac{\partial}{\partial\left(\Re^2\right)} \Theta_{R2} = -1/\lambda_0^2 I\Theta_{R2} \tag{8.10}$$

and the spacelike function:

$$\gamma^k \frac{\partial}{\partial\left(X^2\right)_k} \Theta_{R1} = 1/\lambda_0^2 I\Theta_{R1} \tag{8.11}$$

These functions represent the Systemfunction for two universes, Θ_{R2}, which is a timelike universe inside the light cone, in which we reside, and Θ_{R1}, a spacelike universe, exterior to the light cone.

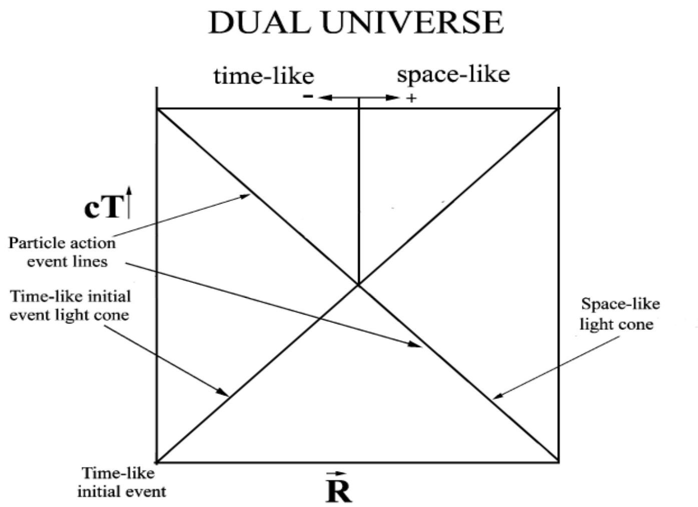

Figure 8.1

.

Appendix IX

Particle Interaction Lagrangian

From our Lagrangian we have for the interaction of the m particle with the n particle:

$$\frac{m_{n0}c}{\hbar} = +\frac{m_n c}{\hbar}\sqrt{\left[1-\frac{\vec{v}_n^{\,2}}{c^2}\right] \overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}} 2\left(\frac{1}{M_n c^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right)} \tag{9.1}$$

or

$$\frac{m_{n0}c^2}{\hbar} = +\frac{m_n c^2}{\hbar}\left[1-\frac{1}{2}\frac{\vec{v}_n^{\,2}}{c^2} \overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}} \left(\frac{1}{M_n c^2}\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right)\right] \tag{9.2}$$

$$m_{n0}c^2 = +\left[m_n c^2 - \frac{1}{2}m_n \vec{v}^{\,2}_n \overset{C}{\underset{n}{\pm}}\overset{C}{\underset{m}{\pm}} \left(\frac{Q^2}{r_m}\left(1-\frac{\vec{v}_m}{c}\bullet\frac{\vec{v}_n}{c}\right)\right)\right] \tag{9.3}$$

So the interaction Lagrangian is:

$$L_{Int} = \mathop{\pm}_{n}^{C} \mathop{\pm}_{m}^{C} \left(\frac{Q^2}{r_m} \left(1 - \frac{\vec{v}_m}{c} \bullet \frac{\vec{v}_n}{c} \right) \right) \tag{9.4}$$

From Jackson p407 [1], the interaction Lagrangian for two particles is:

$$L_{int} = \frac{1}{\gamma} \frac{Q}{mc} (p \bullet A) = Q\left(\not{v} \bullet \not{A} \right) \tag{9.5}$$

Where:

$$m_0 = m\sqrt{1 - \left(\frac{v}{c}\right)^2} \qquad m = m_0 \gamma \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{9.6}$$

And from Doran Geometric Algebra p242 [6], the Liénard- Wiechert Potentials for a particle is:

$$A = \frac{Q\not{v}}{r\left(\vec{X} \bullet v\right)} \qquad \left(\vec{X} \bullet v\right) \to 1 \qquad A = \frac{Q}{r}\not{v} \tag{9.7}$$

We are not looking at r as the distance to the actual position, but the distance as observed at n

Thus

$$L_{int} = Q\frac{\vec{v}_n}{c} \bullet A_m = Q\left[\gamma^0 + \gamma^k \left(\frac{v_n}{c}\right)_k\right] \bullet \left[\frac{Q}{r_m}\left(\gamma^0 + \gamma^k\left(\frac{v_m}{c}\right)_k\right)\right] = \frac{Q^2}{r_m}\left(1 - \left(\frac{v_m}{c}\right) \bullet \left(\frac{v_n}{c}\right)\right), \tag{9.8}$$

which is the same as our interaction Lagrangian in the classical particle Eq.(9.4).

Appendix X

Factoring the Dirac massless function

An interesting feature of the Dirac massless particle function Θ_{Ib} is the ability to factor it into a product of photon type function.

$$\Theta_{Ib} = Ae^{\pm i\left[-\left(\frac{m_n c}{\hbar}\right)r_o - \left(\frac{\vec{p}_n}{\hbar}\right)\bullet\frac{\vec{r}_o}{|\vec{r}_o|}(ct)\right]} \qquad (10.1)$$

without the potential and observed along the direction of motion has a phase velocity equal to the particle velocity v_n, is:

$$\gamma^{\mu}\partial_{\mu}\Theta_{Ib} = \gamma^{\mu}\partial_{\mu}\ Ae^{\mp i\frac{m_n c}{\hbar}\left[r_o - v_n t\right]}\,\mathbf{I}\,\mathbf{a} = 0 \qquad (10.2)$$

Which, with the substitutions of:

$$m_n c^2 = \left[(m_1 + m_2)c\ \right]c \qquad\qquad \vec{v} = \frac{m_1\vec{c} - m_2\vec{c}}{(m_1 + m_2)} \qquad (10.3)$$

The first is the total energy is set equal the sum of the energy of two photons, and the second is the velocity Is set equal to the velocity of the center of mass of the two photons going in opposite directions.

Eq.(10.2) , can then be written as:

$$\Theta_{Ib} = \Theta_{P1}\Theta_{P2} = Ae^{\mp\frac{i}{\hbar}m_1 c(r_o + ct)} \times\ e^{\mp\frac{i}{\hbar}m_2 c(r_o - ct)}, \qquad (10.4)$$

or:

$$\Theta_{Ib} = \Theta_{P1}\Theta_{P2} = Ae^{\mp i2\pi\frac{(r_o + ct)}{\lambda_1}} \times\ e^{\mp i2\pi\frac{(r_o - ct)}{\lambda_2}} \qquad (10.5)$$

which is just the product of wavefunctions for two opposite going light waves.

Appendix XI

Distance and size of a simple charged particle

From Eq., the free particle action for a simple charged particle has been asserted to be:

$$\vec{S}_m = \underset{m}{\pm} i\alpha \frac{\Re}{|\vec{r}_m|} \cancel{\mathcal{D}}_m \tag{11.1}$$

Where the distance to the particle is:

$$|\vec{r}_m| = \left| \cancel{\eta}\vec{r}_m - \cancel{\eta}\alpha\hat{\lambda}_m \right|_S = r_m - \alpha\hat{\lambda}_m \tag{11.2}$$

Note that $|\vec{r}_m|$ is the distance from an observation point to the classical radius of the particle

This action would apply to an electron, positron, and perhaps the leptons, but not likely compost particles that have a more complex structure. The detail structure does not come into play for atomic systems, or the issues of large scale phenomena, although it seems plausible that actions for other particles could be established.

An issue of course is the boundary at $\alpha\hat{\lambda}_m$, which is the classical electrostatic energy radius as well as the Thompson photon scattering radius. This radius is in contrast with electron-electron scattering experiments which are consistent with the hypothesis that the electron is a 'point particle.'

The apparent answer to this dichotomy is that as the energy of the scattering particles is increased, the radius being inversely proportional decreases. At an energy of 29Gev (Bender 1984) the radius would be 10E-5 smaller or < 10E-16 cm, which was the observed upper limit. On the other hand, the arrival of a photon, which has no potential, at the boundary photon, would not alter the radius other than by impact, and thus the $\alpha\hat{\lambda}_m$ radius would be consistent.