A Multiple Particle System Equation Underlying the Klein-Gordon-Dirac-Schrödinger Equations

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Abstract

The purpose of this paper is to illustrate a fundamental, multiple particle, system equation for which the Klein-Gordon-Dirac-Schrödinger equations are single particle special cases. In the same manner that eigenvalues of the Schrödinger equation represents energy levels of an interacting atomic system, eigenvalues represent particle energies in an interacting system of particles. An equation is proposed that has vector solutions defined in Dirac, or Clifford algebra, that treats all of the particles in the universe as a single system. The proposed solution is a descriptor of a symmetric, light speed expanding group of interacting particles having real, as well as the familiar QM constituents.

INTRODUCTION

This paper presents a system equation, and solutions termed a "Systemfunction", $\vec{\Theta}$ which in effect, can be considered a space, of propagators for an expanding system of point particles, and the interactions between those particles. The connection with standard QM is straightforward and plausible.

The interrelations between the standard QM equations are well known, but it is only under special conditions, that being field free, that there is a degree of connectivity [1]. Their most notable omission is the inability to define more than just a single particle, and one of the most notable issues, is the lack of a common physical properties, after the inclusion of the vector potential.

Field free KG

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial ct^2} \right)\psi = -\frac{1}{\mathbf{r}^2}\psi \tag{1}$$

Field free Dirac

$$\left(+\gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} + \gamma_4 \frac{\partial}{\partial ct} \right)\psi = \pm\frac{1}{\mathbf{r}}\psi \tag{2}$$

Field free Schrödinger

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - \frac{2i}{\mathbf{r}}\frac{\partial}{\partial ct} \right)\psi = 0 \tag{3}$$

Where $\mathbf{r}$ is the particle Compton radius $\mathbf{r} = \hbar/mc$. (For general conventions see appendix [II])

The general system equation, we are proposing, is a descriptor of a time expanding system of charged, half spin particles. The inclusion of particles other than half-spin should be straightforward, but the particle action (Eq. (14)), for such particles have not been included. Normal quantum mechanical expressions, (Klein-Gordon-Dirac-Schrödinger), represent single particles systems, separated from the rest of the universe, interacting through a coupling potential. The standard QM, coupling is by the insertion of a representation of the potential through the correspondence relations, rather than an interaction of quantum mechanical wavefunctions. (See Appendix I, for a heuristic view of the issue.)

One can view this Systemfunction as a space containing the sum of the path integrals of particles from the initiation of the universe, to the current time. This being somewhat the same as viewing a summation of paths, of a particle traveling through a slit arriving at a position on a screen. In the current universe we will consider these positions as particle event eigenstates.

I. THE EQUATION

The proposed general expression for the Systemfunction will be. (See Appendix VI for more detail)

$$\frac{\partial}{\partial\left(\Re^2\right)}\vec{\Theta}_1 = K^2\,\vec{\Theta}_1 \tag{4}$$

That is the derivative with respect to a virtual displacement of the square of the expansion of the universe, has constant eigenvalues. Where $\Re^2$ the square of radius of the universe, and is expanding at ct ($\Re = cT$). (This should not be confused with the second derivative). The "Systemfunction" $\vec{\Theta}$ being proposed, as a solution to the system equation, is a matrix representation defined by Dirac or Clifford matrix, not the normal scalar, or scalar vector component wavefunction. i.e.:

$$\vec{\Theta}_n = e^{\,f\left[\gamma_1 f(x)\,+\,\gamma_2 f(y)\,+\,\gamma_3 f(z)\,+\,\gamma_4 f(ct)\right]} \tag{5}$$

This function will have real as well as imaginary components that are separable, leading to well known energy and frequency relations.

Preliminaries

Since the Systemfunction is expected to have a composition of both real and imaginary parts, we will require separablility, such that:

$$\vec{\Theta} = \vec{\Theta}_R\,\vec{\Theta}_i \tag{6}$$

and thus:

$$\vec{\Theta}_R\,\frac{\partial}{\partial\Re^2}\vec{\Theta}_i + \vec{\Theta}_i\,\frac{\partial}{\partial\Re^2}\vec{\Theta}_R = \left(\frac{1}{r_0^{\,2}} + \frac{i}{2\Re r_0}\right)\vec{\Theta}_R\,\vec{\Theta}_i\,, \tag{7}$$

and:

$$\frac{\partial}{\partial\Re^2}\vec{\Theta}_R = \frac{1}{r_0^{\,2}}\vec{\Theta}_R \quad \text{Real}, \tag{8}$$

$$\frac{\partial}{\partial\Re^2}\vec{\Theta}_i = \frac{i}{2\Re r_0}\vec{\Theta}_i \quad \text{Imaginary}, \tag{9}$$

The real equation is somewhat new, but the imaginary part may be a little more familiar if we mote that $\partial\mathfrak{R}^2 = 2\mathfrak{R}\partial\mathfrak{R} = 2\mathfrak{R}\partial ct$ thus:

$$\frac{\partial}{c\partial t}\vec{\Theta}_i = \frac{i}{\mathbf{r_0}}\vec{\Theta}_i \tag{10}$$

Where we have changed from a virtual displacement of the radius of the universe to local time differential. This is possible since the value of the change in the expansion of the universe ∂cT is the same as ∂ct a local time differential. The imaginary function $\vec{\Theta}_i$, now takes on the properties of a particle propagator. which we will assert later, is the case.

$$\psi_T = \psi_0\vec{\Theta}_i \quad , \tag{11}$$

and a solution to the Dirac equation.

$$\left(+\gamma_1\frac{\partial}{\partial x} + \gamma_2\frac{\partial}{\partial y} + \gamma_3\frac{\partial}{\partial z} + \gamma_4\frac{\partial}{\partial ct}\right)\psi_0\vec{\Theta}_I = \pm\frac{1}{\mathbf{r}}\psi_0\vec{\Theta}_I \tag{12}$$

A. **Single Particle Action**

It is asserted that the current expanding universe of particles, can be viewed as a space of extremum points, which represent the action end point of a particles, integrated over the path from the initial event, to the present.

The "Systemfunction" Θ, will be defined in the space inside the radius $\mathfrak{R} = cT$ having a value in the space proportional to the inverse of the distances to the individual particles. In this space the extremums represent the vector action of the particles from the initial event to the current time. We will presume a vector velocity thus a vector Lagrangian. That is:

$$\vec{s}_m = \int_0^T \frac{\vec{L}_m}{\hbar}dt , \tag{13}$$

where $\vec{s}_m$ is the action of the m particle.

For the purpose of this development it is important to define the particle action properly, otherwise the results diverge from reality. Based on trial and error we will define action of these particles to be:

$$\vec{s}_m = \pm \frac{i\Re M_m \vec{V}_m \pm \frac{1}{2}\hbar(\bar{\sigma}+i\gamma_4)}{(Mcr_m/\alpha)-\hbar} \tag{14}$$

Where M is the mass, $\Re$ is the expanding radius of the universe, $\vec{V}_m$ is the four velocity, and $\bar{\sigma}$ is the classic spin matrix $\gamma_1\gamma_2\gamma_3$. Conceptually, if this is the location of the particle integrated from the initial event to the current position, the summation of all the paths, is the spinnor particle propagator.

The standard propagator for this would normally be:

$$K = \lim_{n\to\infty} \sum_{j=1}^{n} e^{\vec{S}_m} \tag{15}$$

Where the sum is over the multiple path for the particle from the initial event to the current time.

If we would presumed K is the Hartree-Fock product wavefunction representing all the wavefunctions of the particles in the system, we could have:

$$K = K_1 K_2 K_3 K_4 ... \left(\lim_{N\to\infty}\sum_{j=1}^{N} e^{\vec{S}_n}\right)\left(\lim_{N\to\infty}\sum_{j=1}^{N} e^{\vec{S}_m}\right)... \tag{16}$$

Now since all the particle start from the same initial event, and are, for now, presumed independent:

$$K = \lim_{N\to\infty} \sum_{j=1}^{N}\sum_{k=1}^{N} e^{\vec{S}_m+\vec{S}_n..} \tag{17}$$

So that heuristically at least, the system action would be just the sum of the particle actions.

$$\vec{S} = \vec{S}_n + \vec{S}_m + ... = \int_0^T \frac{(L_n+L_m)}{\hbar}dt \tag{18}$$

Unfortunately since the paths are independent this would represent a system of non-interacting particles, and insufficient to describe a system of interacting particles.

We will propose instead, that "total" system action of a particle $\vec{s}'$ be defined as the **square** of the sum of the actions defined in Eq. (14). Thus:

$$\vec{s}'_n = \left(\vec{s}_n + \vec{s}_{m\,+}.\right)^2 \tag{19}$$

Thus $\vec{s}^2 \to \vec{s}'$ becomes a "Total" particle actions and $\vec{s}_n \vec{s}_m + \vec{s}_m \vec{s}_n$ are the interacting action of the m and n particles.

Our Systemfunction is then:

$$\vec{\Theta}_n = \exp\left(\sum_I^N \vec{S}_I \sum_J^N \vec{S}_J\right) \tag{20}$$

Note that from our definition of $\vec{S}$ from Eq.(14) the $\vec{\Theta}$ function has a value that is dependent on the inverse distance to individual particles, but is generally zero accept at the locus of one of those particles. We have a presumption that at that point there is an eigenvalue for that particle, and that is an eigenstate of the system. The Systemfunction has values throughout the space of particles up to the extent of cT, at the radius of the universe. Points in the Systemfunction are generally zero accept at extrema, which are particle locations $r_n = 0$ that have discrete values.

For the function evaluated at the locus of the nth particle this would be:

$$\vec{\Theta}_n = \exp\left(\vec{S}_n^2 + \left(\sum_m^N \vec{S}_m\right)\vec{S}_n + \vec{S}_n\left(\sum_m^N \vec{S}_m\right) + + +\right) \tag{21}$$

Notable here is that the first term $\vec{S}_n^2$ is the square of the action for the single particle, and the cross terms would be the Clifford dot products of the interacting actions of the other particles in the system.

Illustrating: If we let the observation point be at the location of the nth particle then the actions (Eq. (14)) for the n ($r_n = 0$), and m particles ($r_m >> \mathbf{r}_m$), where $\left(\mathbf{r}_m = \hbar/Mc\right)$ become:

$$\vec{S}_n = \pm\left(i\frac{\Re\,\vec{V}_n}{\mathbf{r}_n}\frac{}{c} \pm \frac{1}{2}\left(i\overline{\sigma} + \gamma_4\right)\right), \qquad \vec{S}_m = \pm\,\alpha\left(i\frac{\Re\,\vec{V}_m}{r_m}\frac{}{c} \pm \frac{1}{2}\frac{\mathbf{r}_m}{r_m}\left(i\overline{\sigma} + \gamma_4\right)\right) \tag{22}$$

B. **Lagrangian Equation of Motion**

Putting in the moment for the nth particle evaluated at $r = 0$ for $\vec{S}_n^2$ gives the Systemfunction representation of the **free particle** or:

$$\vec{\Theta}_n = \exp\left[-\left(\frac{\Re \, \vec{V}_n}{\mathbf{r}_n \, c} \right)^2 \pm i \frac{\Re \left(i\sigma \bullet V_n + \gamma_4 c \right)}{\mathbf{r}_n} + \frac{1}{4} \right], \tag{23}$$

or, for illustration, explicitly in terms of mass and velocities:

$$\vec{\Theta}_n = \exp\left[-\left(\frac{\Re M_n c}{\hbar} \right)^2 \left(1 - \frac{v^2}{c} \right) \pm i \frac{\Re M_n}{\hbar} \left(i\sigma \bullet V_n + \gamma_4 c \right) + \frac{1}{4} \right] \tag{24}$$

Note that this free particle Propagator is defined at a given point, and has an extremely small real value for an electron $\left(\exp(-75) \right)$, also note the velocity V_n is the three velocity. Since $e^{i\Re} \to e^{ict+2n\pi} \to e^{ict}$, the imaginary part is just:

$$\vec{\Theta}_{In} = e^{\pm i \frac{M_n c t}{\hbar} \left(i\sigma \bullet V_n + \gamma_4 c \right)} \tag{25}$$

Taking the magnitude of the exponent With a position phase shift along the x axis would be:

$$\left\| \Theta_{In} \right\| = e^{+i \frac{p}{\hbar} \bullet r \, \pm i \sqrt{\left(M_n^2 c^4 - M_n^2 c^2 V^2 \right)} \frac{c \, t}{\hbar}} \tag{26}$$

Contrasting, the Dirac free particle is:

$$\psi(x) = u_{\vec{p}} e^{i \left(\vec{p} \bullet \vec{x} - \sqrt{p^2 c^2 + M_0^2 c^4} \, t \right) / \hbar} \tag{27}$$

Which are equivalent, if the Dirac solution is presumed to be in the particle frame, dilated time, and Eq, (26) is in stationary reference frame. This can be seen from the fact that the energy defined in Eq. (26) is the invariant mass energy and the energy defined in Eq, (27), is the relativistic mass energy. Thus if Eq. represents the particle propagator, the Dirac free particle equation is satisfied.

Including the interaction terms from Eq. (21) our particle propagator becomes:

$$\vec{\Theta}_{in} = e^{\pm i\left[\frac{M_n c t}{\hbar}(\sigma\bullet V_n +\gamma_4 c) \pm\frac{Q^2}{r_m}\left(\frac{M_n\vec{V}_n(\sigma\bullet V_n +\gamma_4 c)\pm M_m\vec{V}_m(\sigma\bullet V_m +\gamma_4 c)}{M_m c}\right)\sigma\right]\frac{t}{\hbar}}$$

(28)

Which now includes the vector potentials resulting from the other particles in the system

Putting this into Eq. (12) and taking the differential with respect to the potential leaves the free particle propagator in a vector field:

$$\left(+\gamma_k\frac{\partial}{\partial x_k} \pm\frac{Q^2}{r_m}\left(\frac{M_n\left(\sigma\bullet V_n +\gamma_4 c\right)_n \pm M_m\left(\sigma\bullet V_m +\gamma_4 c\right)_m}{M_m c}\right)\right)\psi_0 \ \vec{\Theta}_{FI} = \pm\frac{1}{r}\psi_0\vec{\Theta}_{FI},$$

(29)

which is the Dirac equation for a particle in a vector field of other moving particles.

The imaginary portion of Eq. (24), and yields the wavelength:

$$\frac{1}{\lambda} = \frac{1}{2\pi}\left|\frac{M_n}{\hbar}\left(\sigma\bullet V_n +\gamma_4 c\right)\right| = \frac{p}{h}$$

(30)

Which is the correct free particle deBroglie wavelengths for the velocity and the total energy. The frequencies are.

$$\omega_d = \frac{M_n c v}{\hbar} \ \& , \ \omega_C \ \frac{M_n c^2}{\hbar}$$

(31)

Which are the deBroglie kinetic frequency, and the Compton frequency. Note that these are the values in the rest frame and change in the moving frame.

Real Terms

Putting the values from Eq.(22) into Eq.(21) , separating the real terms, gives for the free particle action.

$$\vec{S}_n' = \vec{S}_n\vec{S}_n = -\mathfrak{R}^2\left(\frac{M_n c}{\hbar}\right)^2\left(1-\frac{v_n^2}{c^2}\right) = -\mathfrak{R}^2\left(\frac{M_{0n} c}{\hbar}\right)^2 \tag{32}$$

and the electromagnetic interacting, or cross terms of Eq. (21), are:

$$\vec{S}_n' = \vec{S}_n\left(\sum_m^N \vec{S}_m\right)+\left(\sum_m^N \vec{S}_m\right)\vec{S}_n = \pm\frac{\mathfrak{R}\mathfrak{R}}{\mathbf{r}_n}\left[\frac{\vec{V}_n}{c}\left(\sum\frac{\alpha}{r_m}\frac{\vec{V}_m}{c}\right)+\left(\sum\frac{\alpha}{r_m}\frac{\vec{V}_m}{c}\right)\frac{\vec{V}_n}{c}\right] \tag{33}$$

Note that this is the inner product relation between the velocity vectors, so that the complete expression is:

$$\vec{S}_n' = \vec{S}_n^2 +\left(\sum_m^N \vec{S}_m\right)\vec{S}_n +\vec{S}_n\left(\sum_m^N \vec{S}_m\right) = -\mathfrak{R}^2\left(\frac{M_n c}{\hbar}\right)^2\left(1-\frac{v_n^2}{c^2}\right)\pm 2\mathfrak{R}^2\frac{M_n}{\hbar^2}\sum\frac{Q^2}{r_m}\left(\frac{\vec{V}_n}{c}\cdot\frac{\vec{V}_m}{c}\right) \tag{34}$$

Inserting into the real portion of Eq. (8). And taking the derivative with respect to the virtual displacement of $\mathfrak{R}^2$ gives:

$$-\frac{\left(M_{0n}c^2\right)^2}{\hbar^2 c^2} = -\frac{\left(M_n c^2\right)^2}{\hbar^2 c^2}\left[\left(1-\frac{v_n^2}{c^2}\right)\pm\frac{2}{M_n c^2}\sum\frac{Q^2}{r_m}\left(\frac{\vec{V}_n}{c}\cdot\frac{\vec{V}_m}{c}\right)\right], \tag{35}$$

Which is the square of the classic Lagrangian for a moving particle in an electromagnetic field defined by the interaction of the n and m particles. This is a remarkable result, in that the vector potential is arising from the product of the actions of the central particle, with the actions of the other particles in the system, The effects of the electromagnetic interaction are not the result of the inclusion of a vector field, but as the properties of the actions of the other particles in the system.

Simplifying:

$$M_{0n}c^2 = M_n c^2\sqrt{\left(1-\frac{v_n^2}{c^2}\right)\pm\frac{2}{M_n c^2}\sum\frac{Q^2}{r_m}\left(\frac{V_n}{c}\cdot\frac{V_m}{c}\right)}, \tag{36}$$

and:

$$M_{0n}c^2 = M_nc^2\left(1-\frac{1}{2}\frac{v_n^2}{c^2}\right)-\sum\frac{Q^2}{r_m}\left(1-\frac{V_n}{c}\bullet\frac{V_m}{c}\right) \qquad (37)$$

Which is just the Lagrangian for a particle in the presence of a collection of charged particles.

In addition to the previous noted terms, there is a small spin-spin term which has no derivatives with respect to $\mathfrak{R}^2$ and thus doesn't contribute to the total energy. A detail representation of this is presumed to illustrate its participate in the mechanism of the exclusion principle.

$$\pm\frac{1}{2M_mc^2}\left(\frac{Q^2}{2r_m}\right) \qquad (38)$$

CONCLUSION

A multiple particle system equation for the universe, and its connection to quantum mechanics has been demonstrated. Although some detail have been left out for simplicity, and a lot of the obvious aspects have not been explored, it is clear that it represents a new approach to particle dynamics, and perhaps opens a window into the relation between the particles, and the expanding universe. A point to note is, that by considering only the relativistic dynamics, and property of particles, interactions can be defined without the necessity of defining fields. Though one could add gravitational contributions into the system equation, for our purposes here it does not seem make a useful contribution.

References :

[1] E. Comay, "Difficulties with the Klein-Gordon Equation", Journal-ref: Apeiron 11, No. 3, 1 (2004) arXiv:quant-ph/0312206,v3,8, Sep,2004.

[2] John David Jackson, Classical Electrodynamics Second Edition pp 411

[3] P. A. M. Dirac, The Principles of Quantum Mechanics, 4th ed., Oxford University Press,London (1958).

Appedix I

The current base equations for QM are the Relativistic Schrödinger equation or the Klein-Gordon and the Dirac equation with the potential incorporated by use of the "correspondence relation" . This method asserts that the total momentum of a charged particle in an external field is modified as such that.

$$p \rightarrow p - \frac{q}{c} A \qquad (1.1)$$

$$\frac{\partial}{\partial x_\mu} \rightarrow \left(\frac{\partial}{\partial x_\mu} - \frac{q}{c} A_\mu \right) \qquad (1.2)$$

The Schrödinger equation with fields included is:

$$\left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} - i\frac{2}{r} \left[\frac{\partial}{\partial ct} - \frac{\alpha}{r} \right] \right) \psi = 0 \qquad (1.3)$$

And the field included Dirac expression becomes:

$$\left(+\gamma_1 \left(\frac{\partial}{\partial x} - \frac{q}{c} A_x \right) + \gamma_2 \left(\frac{\partial}{\partial y} - \frac{q}{c} A_y \right) + \gamma_3 \left(\frac{\partial}{\partial z} - \frac{q}{c} A_z \right) + \gamma_4 \left(\frac{\partial}{\partial ct} - \frac{q}{c} \varphi \right) \right) \psi = \pm \frac{1}{r} \psi \qquad (1.4)$$

Before this modification, that is for the free field solution, the Klein-Gordon, and the Dirac equations are identical, in that the Dirac equation is a factorization of the KG equation using the Dirac matrix. Once the modification has been included via the correspondence substitution, the equations, are not equivalent, not even in interpretation [1]. The KG describes electromagnetic spin one particles in a potential, and the Dirac is a probability distribution of half spin particles.

It is asserted that including the potential, via the correspondence relation is the fundamental error plaguing QM, in explaining physical phenomena. The insertion of an infinite potential has to be considered an approximation and only accurate to the extent that the approximation of the inverse square force is an accurate representation.

For illustration, consider the field free Dirac expression:

$$\gamma^\mu \left(\partial_\mu \psi \right) + \frac{1}{r} \psi = 0 \qquad (1.5)$$

Where we replace the isolated particle wavefunction with:

$$\psi \rightarrow \psi U \qquad (1.6)$$

Where U is the Hartree-Fock product wavefunction representing the effect of the entirety of a system not including ψ.

$$U = \psi_1 \psi_2 \psi_3 \psi_4 \dots \qquad (1.7)$$

Substituting this into Eq. (1.5) then gives:

$$\gamma^\mu \left(U \partial_\mu \psi + \psi \partial_\mu U \right) + \frac{1}{r} \psi U = 0, \qquad (1.8)$$

or:

$$\gamma^\mu \left(\partial_\mu \psi + \psi \frac{\partial_\mu U}{U} \right) + m\psi = 0 \qquad (1.9)$$

Comparing this with the Dirac expression Eq. (1.4)   with the correspondence related potential:

$$\gamma^\mu \left(\partial_\mu \psi - \frac{q}{c} A\psi \right) + \frac{1}{r} \psi = 0 \qquad (1.10)$$

We find:

$$\frac{\partial_\mu U}{U} = -\frac{q}{c} A \qquad (1.11)$$

Thus, A, which is normally inserted through the correspondence relation actually is an approximation of the electromagnetic effects of the rest of the system. In the case of the electron inverse radial potential, this only works to the extent that the potential is an accurate representation of the entirety of the rest of the system. The most notable defect of course is that the integration of the electrons energy becomes infinite, requiring renormalization.

One should point out that the Hartree-Fock product wavefunction Eq. (1.7), cannot represent multi-fermionic anti-symmetric system, and so this is only an heuristic argument

Appendix II

Definitions and Conventions

The radius of the universe:

$$\mathfrak{R} = cT = \mathfrak{R}_0 + ct$$

The Dirac matrix convention used in this development is

$$\gamma_1 = \begin{bmatrix} & & & -1 \\ & & -1 & \\ & +1 & & \\ +1 & & & \end{bmatrix} \quad \gamma_2 = \begin{bmatrix} & & & i \\ & & -i & \\ & -i & & \\ i & & & \end{bmatrix} \quad \gamma_3 = \begin{bmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{bmatrix} \quad \gamma_4 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix},$$

and

$$\gamma_1^2 = -1, \ \gamma_2^2 = -1, \ \gamma_3^2 = -1, \ \gamma_4^2 = +1.$$

The product of the space coordinates is termed the spin matrix:

$$\overline{\sigma} = \gamma_1 \gamma_2 \gamma_3 \qquad -\overline{\sigma} = \gamma_3 \gamma_2 \gamma_1 \qquad \tau = \overline{\sigma} \gamma_4 \qquad S = \begin{bmatrix} & & i & \\ & & & i \\ -i & & & \\ & -i & & \end{bmatrix}$$

The square is:

$$\overline{\sigma}^2 = 1 \qquad\qquad \tau^2 = -1$$

The product of $\overline{\sigma}^2$ with the coordinate vectors:

$$\overline{\sigma}\gamma_{1,2,3} = \overline{\sigma}\gamma_{1,2,3}$$
$$\overline{\sigma}\gamma_4 = -\gamma_4\overline{\sigma}$$

$$\overline{\sigma} \ \gamma_n = \ \gamma_2\gamma_3, \ \ \gamma_1\gamma_3, \ \ \gamma_1\gamma_2$$

Which are the elements of the spin vector:

$$\sigma_1 = \gamma_2\gamma_3 \qquad \sigma_2 = \gamma_3\gamma_1 \qquad \sigma_3 = \gamma_2\gamma_1$$

The vector four velocity:

$$\vec{V} = \gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c$$

Commutation relation with V and S

$$\overline{\sigma}\vec{V} = \overline{\sigma}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)$$

$$\vec{V}\overline{\sigma} = \left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\overline{\sigma}$$

$$\overline{\sigma}\vec{V} + \vec{V}\overline{\sigma} = 2\,\vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z\right)$$

$$= 2\left(\gamma_2\gamma_3 v_x + \gamma_1\gamma_3 v_y + \gamma_1\gamma_2 v_z\right)$$

$$= 2\,\sigma\bullet V$$

$$\left(\sigma\bullet V_n + \gamma_4 c\right)\left(\sigma\bullet V_n + \gamma_4 c\right) = \overline{\sigma}\left(V_n + \overline{\sigma}\gamma_4 c\right)\left(V_n + \gamma_4\overline{\sigma}c\right)\overline{\sigma}$$

$$\left(\sigma\bullet V_n + \gamma_4 c\right)\left(\sigma\bullet V_m + \gamma_4 c\right) = \overline{\sigma}\left(V_n + \overline{\sigma}\gamma_4 c\right)\left(V_m + \gamma_4\overline{\sigma}c\right)\overline{\sigma}$$

$$= V_m V_n + c^2$$

$$\left(\sigma\bullet V_n + \gamma_4 c\right)\left(\sigma\bullet V_n + \gamma_4 c\right) + \left(\sigma\bullet V_n + \gamma_4 c\right)\left(\sigma\bullet V_n + \gamma_4 c\right)$$

$$= 2 V_m V_n + 2c^2 = 2\vec{V}_n\bullet\vec{V}_m$$

:

The product of two four velocities:

$$\vec{V}_m \vec{V}_n = \left(\gamma_1 v_{xn} + \gamma_2 v_{yn} + \gamma_3 v_{zn} + \gamma_4 c\right)\left(\gamma_1 v_{xm} + \gamma_2 v_{ym} + \gamma_3 v_{zm} + \gamma_4 c\right)$$

or

$$\vec{V}_m \vec{V}_n = -\vec{V}_m\bullet\vec{V}_n + \sigma\bullet V_m \times V_n + \gamma_4 c\,\Delta V$$

The inner product :

$$\vec{V}_m \vec{V}_n + \vec{V}_m \vec{V}_n = 2\vec{V}_m\bullet\vec{V}_n$$

The outer product:

$$\vec{V}_m \vec{V}_n - \vec{V}_m \vec{V}_n = 2\overline{\sigma}\vec{V}_m \times \vec{V}_n$$

Appendix III

Discussion of the action vector.

$$\pm\frac{i\Re M_m \vec{V}_m \pm \dfrac{1}{2}\hbar\left(\overline{\sigma} + i\gamma_4\right)}{\left(Mcr_m / \alpha\right) - \hbar}$$

This represents a particle action vector, presumably the result of the integration of a vector Lagrangian for the particle over all possible paths, from the initial event to its current position. As a result, this it is the most probable, or classical action. When the function is evaluated, at the locus of such a particle, the arrived at, equation of motion, will be the action for that particle and the locus of its eigenvalue for the function.

If the velocity and r_m goes to zero the function becomes:

$$\pm\gamma_4 \frac{\Re Mc}{\hbar}$$

Since $\Re$ is the radius of the universe this is the "maximum" action a particle existing at that location in the universe, can have, Note that at the Compton radius $r = \text{\textbf{r}}$ the function is infinite. But since it is an observation point there is no physical significance.

Focusing on the denominator and presuming $Mcr_m > \hbar$ and the velocity is zero we have:

$$= \frac{i \Re M \vec{V}_m}{\hbar}\left(\frac{1}{Mc^2}\frac{Q^2}{r_m} \right) \rightarrow \frac{i\Re}{\hbar}\left(\frac{1}{c}\frac{Q^2}{r_m} \right)\gamma_4$$

Which makes it the action of the electric energy potential induced from the particle at the observation point. And goes to α when $\Re \rightarrow r$.

The action vector function thus represents a maximum value when $r = 0$ and the minimum when $r \rightarrow \Re$. At a given observation point an extremum is diminished by $\Re/r$, making its contribution proportional to its observed cosmic age.

<center>Appendix IV</center>

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<center>**Appendix VI**</center>

<center>THE FULL EQUATION</center>

Although the proposed general expression Shown in Eq.(4) was originally developed by the following methodology, it is probably not the best approach. Presume:

$$\left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial\left(X^2\right)^2}\right)\vec{\Theta}_1 + \left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial\left(Y^2\right)^2}\right)\vec{\Theta}_1 + \left(\frac{\partial \vec{\psi}_1}{\partial\left(Z^2\right)}\right)\vec{\Theta}_1 - \left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial\left(\Re^2\right)^2}\right)\vec{\Theta}_1 = K\vec{\Theta}_1 \quad (6.1)$$

Which is the sum of the second derivatives if the function with respect to the square of the expanding four dimensional light cone.

Using the Dirac Clifford Algebra this can be separated into two first order derivatives:

$$\gamma_1\left(\frac{\partial \vec{\Theta}_1}{\partial\left(X^2\right)}\right) + \gamma_2\left(\frac{\partial \vec{\Theta}_1}{\partial\left(Y^2\right)}\right) + \gamma_3\left(\frac{\partial \vec{\Theta}_1}{\partial\left(Z^2\right)}\right) + \gamma_4\left(\frac{\partial \vec{\Theta}_1}{\partial\left(\Re^2\right)}\right) = K\vec{\Theta}_1 \quad (6.2)$$

of which we presume our first Eq. (4) is separable By means of a constant to yield:

$$\frac{\partial}{\partial\left(\Re^2\right)}\vec{\Theta}_1 = K\,\vec{\Theta}_1 \quad (6.3)$$

Noting that (6.2) the partial derivative with respect to a displacement of the expanding light cone at the edge of the universe. Since the max of X,Y,Z are just equal the radius, $\Re$, and ∂X is the same for either the edge of the universe, or a local frame $\partial X = \partial x$, thus we can have:

$$\partial\left(X^2\right) = 2X\partial x = 2\Re\partial x$$
$$\partial\left(Y^2\right) = 2Y\partial y = 2\Re\partial y$$
$$\partial\left(Z^2\right) = 2Z\partial z = 2\Re\partial z \quad , \quad (6.4)$$
$$\partial\left(\Re^2\right) = 2\Re\partial x = 2\Re\partial\left(ct\right)$$

And thus arrive at the Dirac expression for the function.

$$\gamma_1\left(\frac{\partial \vec{\Theta}_1}{2R\partial x}\right) + \gamma_2\left(\frac{\partial \vec{\Theta}_1}{2R\partial y}\right) + \gamma_3\left(\frac{\partial \vec{\Theta}_1}{2R\partial z}\right) + \gamma_4\left(\frac{\partial \vec{\Theta}_1}{2\Re\partial\left(ct\right)}\right) = K\vec{\Theta}_1, \quad (6.5)$$

Which is just the field free Dirac expression.

Squaring this we also have:

$$-\left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial x^2}\right)-\left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial y^2}\right)-\left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial z^2}\right)+\left(\frac{\partial \vec{\psi}_1^{\,2}}{\partial (ct)^2}\right)=K\vec{\psi}_1 \qquad (6.6)$$

Which is the field free Klein-Gordon expression.