Causality is inconsistent with quantum field theory

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Abstract

It is shown that the usual quantum field theoretical argument for the vanishing of the commutator (VC) for spacelike separated fields implying causality is not tenable. For VC to be tenable negative energy antiparticles traveling forward in time must exist and negative energy particles traveling backward in time are not allowed. Hence VC denies the existence of positive energy antiparticles.

For as long as quantum field theory has been our current theory governing fundamental physics, it has been accepted without question that causality has been proven by the vanishing of the field commutator (VC) whenever the field operators are spacelike separated. In standard notation where x and y each represent a spacetime four-vector and, as usual, bold letters stand for three-vectors, the fields are spacelike separated if $(x - y)^2 < 0$, where $(x - y)^2 = (x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$. Given the complex scalar (spin zero) fields, $\varphi(x)$ and $\varphi^\dagger(y)$, the quantum field theory commutator expressed by $<0|[\varphi(x), \varphi^\dagger(y)]|0>$ can be given in terms of field propagators. In what follows I shall be concerned with two directions of time and will introduce arrows to indicate in which direction I take time to be “flowing.” I shall also be interested in the sign of the energy, i.e. whether we are looking at propagators with positive or negative energy.

Usually the commutators are expressed as fundamental concepts and the propagators later are shown to be equal to them—every propagator can be shown to be equal to i times an appropriate commutator. The question then becomes one of historical significance. Since one usually begins with classical quantum physics wherein $[\mathbf{x}, \mathbf{p}]=i$ connotes the usual commutation relation between position and momentum, the canonical derivation uses 2nd quantization and promotes the quantum wave functions to field operators. While this is certainly logical it will turn out that it leads to an inconsistency when one compares appropriate commutators and propagators. This fact is often overlooked in textbooks and treatises of quantum field theory.[1] The oversight may simply be due to the historic fact that commutators were taken as fundamental in the definitions of quantum fields while propagators were seen as secondary constructs. In fact they are equal so it should be the case that they be applied in the calculation of amplitudes and probabilities in a consistent manner.

But putting that aside for the moment, let us consider several commutation relations of interest here.[2] If we use the method of 2nd quantization we can define these field operators for a scalar spin *0* charged boson in terms of relevant creation and annihilation operators as follows:

$$\varphi_{\uparrow}{}^{+}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{a_p} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-iE_p x^0}\theta(x^0), \tag{1}$$

$$\varphi_{\uparrow}{}^{-}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{b_p}{}^{\dagger} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \boldsymbol{e}^{iE_p x^0}\theta(x^0), \tag{2}$$

$$\varphi_{\uparrow}{}^{\dagger+}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{b_p} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-iE_p x^0}\theta(x^0), \tag{3}$$

$$\varphi_{\uparrow}{}^{\dagger-}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{a_p}{}^{\dagger} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{iE_p x^0}\theta(x^0), \tag{4}$$

The superscripts are to remind us of the sign of the energy in the field under consideration, so a + superscript means positive energy, a – superscript means negative energy and † stands for Hermitian conjugate as usual. I have also designated these fields with a subscript ↑, to mean time flowing towards the future. $\theta(x^0)$ is the unit step function equal to zero if its argument is negative and one otherwise. In a similar manner I define the same fields with time flowing toward the past with a subscript ↓. From symmetry, I also replace ***p*** with –***p*** in eqns. (5) through (8).

$$\varphi_{\downarrow}{}^{+}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{a_p} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{iE_p x^0}\theta(-x^0), \tag{5}$$

$$\varphi_{\downarrow}{}^{-}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{b_p}{}^{\dagger} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \boldsymbol{e}^{-iE_p x^0}\theta(-x^0), \tag{6}$$

$$\varphi_{\downarrow}{}^{\dagger+}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{b_p} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{iE_p x^0}\theta(-x^0), \tag{7}$$

$$\varphi_{\downarrow}{}^{\dagger-}(x) = \int d^3\boldsymbol{p}\left((2\pi)^3 2E_p\right)^{-1/2} \boldsymbol{a_p}{}^{\dagger} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-iE_p x^0}\theta(-x^0), \tag{8}$$

where the annihilation operators $\boldsymbol{a_p}$, $\boldsymbol{b_p}$, and creation operators $\boldsymbol{a_p}^{\dagger}$, $\boldsymbol{b_p}^{\dagger}$, satisfy the following 10 usual commutator relations:

$$[\, \boldsymbol{a_p} , \boldsymbol{a_q}^{\dagger}] = [\, \boldsymbol{b_p} , \boldsymbol{b_q}^{\dagger}] = \delta^3(\boldsymbol{p} - \boldsymbol{q}), \tag{9}$$

$$[\boldsymbol{a_p} , \boldsymbol{b_q}^{\dagger}]=[\boldsymbol{a_p} , \boldsymbol{a_q}]=[\boldsymbol{b_p} , \boldsymbol{b_q}]=[\boldsymbol{b_p} , \boldsymbol{a_q}^{\dagger}]=[\boldsymbol{b_p}^{\dagger} , \boldsymbol{b_q}^{\dagger}]=[\boldsymbol{b_p}^{\dagger} , \boldsymbol{a_q}^{\dagger}]=[\boldsymbol{a_p}^{\dagger} , \boldsymbol{a_q}^{\dagger}]= [\boldsymbol{a_p} , \boldsymbol{b_q}]= \boldsymbol{0}. \tag{10}$$

In (1) through (8) I use the usual notation:

$$p^2 = E^2 - \boldsymbol{p}^2,\ px = Ex^0 - \boldsymbol{p}\cdot\boldsymbol{x},\ d^4p = dE\,d^3\boldsymbol{p}, \text{ and } E_p = +\sqrt{\boldsymbol{p}^2 + m^2}\,. \tag{11}$$

Commutators

It follows from eqns. (1) through (10) that the only possible non-zero commutation relations for time running forward, ↑, or backward, ↓, are:

$$i < 0\left|[\varphi_\uparrow{}^{+}(x), \varphi_\uparrow{}^{\dagger-}(y)]\right|0 >= (2\pi)^{-3}\int d^3\boldsymbol{p}\; e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,(e^{-iE_pt}/2E_p\,)\theta(t), \tag{12}$$

$$i < 0\left|[\varphi_\uparrow{}^{-}(x), \varphi_\uparrow{}^{\dagger+}(y)]\right|0 >= -(2\pi)^{-3}\int d^3\boldsymbol{p}\; e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,(e^{iE_pt}/2E_p\,)\theta(t), \tag{13}$$

$$i < 0\left|[\varphi_\downarrow{}^{+}(x), \varphi_\downarrow{}^{\dagger-}(y)]\right|0 >= (2\pi)^{-3}\int d^3\boldsymbol{p}\; e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}(e^{iE_pt}/2E_p\,)\,\theta(-t), \tag{14}$$

$$i < 0\left|[\varphi_\downarrow{}^{-}(x), \varphi_\downarrow{}^{\dagger+}(y)]\right|0 >= -(2\pi)^{-3}\int d^3\boldsymbol{p}\; e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}(e^{-iE_pt}/2E_p\,)\,\theta(-t). \tag{15}$$

In the above equations (12) through (15) and for the remainder of the paper, I have substituted the variable t=$x^0 - y^0$. In each case we are commuting a positive energy field operator with a negative energy field operator. All other commutators of field operators specified by (1) through (8) vanish because of (10) and will be of no concern here since, as we will see, a commutator of field operators equals some form of propagator; consequently the vanishing of any commutation of fields means there is no propagator for the process possible. This statement is then taken to mean that there can be no effect produced at one spacetime point, *x*, due to a measurement of a field operator at another spacetime point, *y*. In fact, it is this argument that has led to one accepting that when $(x - y)^2 < 0$ there can be no propagation to a spacetime point *x* outside the light cone of spacetime point *y* and vice versa; hence no violation of causality can occur.

In the usual theory one then constructs the following superpositions of field operators where ↕ means all fields up or down but no mixing of up and down:

$$\varphi_\updownarrow(x) = \{\varphi_\updownarrow{}^{+}(x) + \varphi_\updownarrow{}^{-}(x)\},\ \ \varphi_\updownarrow{}^{\dagger}(x) = \{\varphi_\updownarrow{}^{\dagger+}(x) + \varphi_\updownarrow{}^{\dagger-}(x)\}. \tag{16}$$

The superscripts + and – also refer to particle numbering. The field operator $\varphi_\updownarrow(x)$ can be seen to be a superposition of momentum space integrals over annihilation operators, ($\boldsymbol{a_p}$) destroying positive energy particles of momentum $\boldsymbol{p}$ and creation operators ($\boldsymbol{b_p}^{\dagger}$) making negative energy antiparticles with the same momentum $\boldsymbol{p}$, while $\varphi_{\updownarrow}{}^{\dagger}(x)$ superposes and integrates over annihilation operators ($\boldsymbol{b_p}$) destroying positive energy antiparticles of momentum $\boldsymbol{p}$ and creation operators ($\boldsymbol{a_p}{}^{\dagger}$) producing negative energy particles of the same momentum $\boldsymbol{p}$. Hence the field $\varphi_{\updownarrow}(x)$ stands for the process whereby the field loses a particle and gains an antiparticle with no net change in energy and thus loses a net charge (annihilation field) and $\varphi_{\updownarrow}{}^{\dagger}(x)$ stands for the opposite process whereby the field suffers no net change in energy, loses an antiparticle, gains a particle and thus gains a net charge (creation process). If you consider that antiparticles have the opposite charge of particles you can see that these fields complement each other and also see that indeed they are Hermitian conjugates of each other.

One should notice that the direction of time is immaterial here but will become of interest in what follows. Down arrow propagation works just as well as up arrow propagation. However you can't superimpose or mix a down arrow field and an up arrow field.

Consequently, it is easy to show from (11) through (15) that:

$$< 0\left|[\varphi_{\updownarrow}(x),\ \varphi_{\updownarrow}{}^{\dagger}(y)]\right|0 > =$$
$$< 0\left|[\varphi_{\updownarrow}{}^{+}(x),\ \varphi_{\updownarrow}{}^{\dagger -}(y)]\right|0 > + < 0\left|[\varphi_{\updownarrow}{}^{-}(x),\ \varphi_{\updownarrow}{}^{\dagger +}(y)]\right|0 >. \tag{17}$$

Feynman propagators

It is useful to introduce the quantum field theoretical Feynman propagators for these fields. I see these propagators as being more fundamental than the fields themselves. What should be obvious is that no mention of the fields is even necessary and no mention of antiparticles is necessary either, although this last consideration may seem less obvious. Feynman's propagator approach is therefore, I believe, superior to the commutation of fields approach for this reason: Feynman derives antiparticles from the particle propagators. Whereas in the field commutation derivation, antiparticles must be postulated and then accepted. Furthermore the antiparticle must have negative energies propagating forward through time and when considering time running negatively, particles must have positive energy.

Feynman introduces:

$$G_{\alpha}(x) = (2\pi)^{-3} \int d^3\boldsymbol{p}\ e^{i\boldsymbol{p}\cdot\boldsymbol{x}} g_{\alpha}(E_p) \tag{18}$$

where the *sub-propagator* g_{α} is given by,

$$g_\alpha(E_p) = i(2\pi)^{-1} \oint dE\, e^{-iEt}/(E - E_p \pm i\varepsilon)(E + E_p \mp i\varepsilon). \tag{19}$$

The subscript α depends on where we place the $i\varepsilon$ in the integrand poles. As is easy to see the integrand has two poles in the energy plane at $E = \pm E_p \mp i\varepsilon = \pm\sqrt{(\boldsymbol{p}^2 + m^2)} \mp i\varepsilon$, thus allowing four possible pole placements depending on the signs in front of E_p and $i\varepsilon$. In computing g_α, as is usual, one performs the integration and then lets $\varepsilon \to 0$. Depending on the placement of the poles with finite ε we find four possible sub-propagators in the limit $\varepsilon \to 0$ given quite simply, again using $t = (x^0 - y^0)$:

$$g_{fl} = (e^{-iE_p t}/2E_p)\Theta(t), \tag{20}$$

$$g_{fu} = (e^{iE_p t}/2E_p)\Theta(-t), \tag{21}$$

$$g_{\sim fl} = -(e^{iE_p t}/2E_p)\Theta(t), \tag{22}$$

$$g_{\sim fu} = -(e^{-iE_p t}/2E_p)\Theta(-t). \tag{23}$$

For simplicity in (18) through (23) I use the sub-index notation *fl* to stand for the Feynman sub-propagator computed by closing the integration path in the lower half of the complex energy plane (see Fig. 1) enclosing the positive energy pole, $(E_p - i\varepsilon)$, *fu* to stand for closing the path in the upper half plane (see Fig. 2) enclosing the negative energy pole, $(-E_p + i\varepsilon)$, and *~fl* (Fig. 1) and *~fu* (Fig. 2) correspondingly, what I call *anti-Feynman* sub-propagators, representing closing the path in the lower or upper plane respectively but encircling the opposite sign energy poles of *fl*, $(-E_p - i\varepsilon)$, and of *fu*, $(E_p + i\varepsilon)$, resp.

Pole pushing and Dipping

Feynman realized[3] that one can push the two poles in (19) onto a diagonal alignment so that the poles are placed at $E = E_p - i\varepsilon$ and $E = -E_p + i\varepsilon$ or as we can see one could align the poles along the anti-diagonal so that one has $E = -E_p - i\varepsilon$ and $E = E_p + i\varepsilon$. I call calculating the resulting sub-propagator with the first diagonal pole placement, *DiP* and the second anti-diagonal pole placement *anti-DiP*. With *DiP* and carrying through the momentum space integration as indicated in (18) one arrives at the Feynman propagators, G_{fl} and G_{fu}. While using *anti-DiP* one gets the anti-Feynman propagators, $G_{\sim fl}$ and $G_{\sim fu}$. In each case, with *DiP* or *anti-DiP*, closing the integration path appropriately in either the upper or lower half plane, one gets the residue from one pole only. Feynman only uses *DiP* in all calculations and never *anti-DiP* which is the standard usage in calculating Feynman diagrams in quantum field theory texts to date.[4]

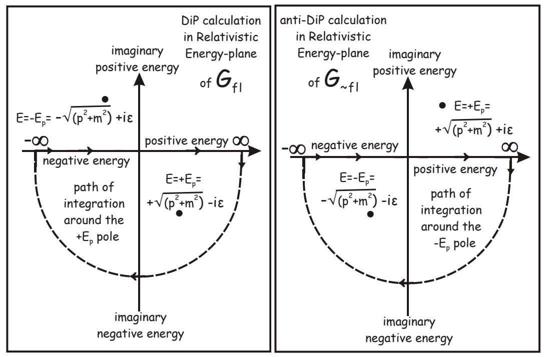

Fig. 1 Feynman’s *DiP* and *anti-Dip* with (*t > 0*).

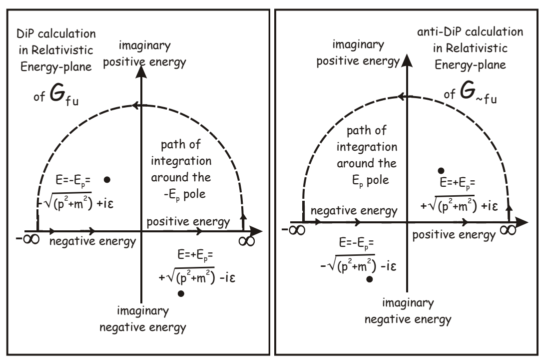

Fig. 2 Feynman’s *DiP* and *anti-Dip* with (*t < 0*).

Pole pushing and Papping

On the other hand, if one pushes both poles into the negative imaginary (lower) half plane, and considers $t = (x^0 - y^0) > 0$, simply by closing the path of integration in the lower half plane and thus enclosing both $(-E_p - i\varepsilon)$ and $(E_p - i\varepsilon)$ poles, one calculates the retarded sub-propagator, g_{ret} given by:

$$g_{ret} = g_{fl} + g_{\sim fl} = \left(\{e^{-iE_pt} - e^{iE_pt}\}/2E_p\right)\Theta(t), \tag{24}$$

I call this calculating the propagator with parallel pole placement in the lower half plane , *PaP*. If on the other hand, one pushes both poles into the positive imaginary (upper) half plane with $t = (x^0 - y^0) < 0$, by closing the path of integration in the upper half plane thus enclosing both $(-E_p + i\varepsilon)$ and $(E_p + i\varepsilon)$ poles one gets the advanced sub-propagator, g_{adv} given by:

$$g_{adv} = g_{fu} + g_{\sim fu} = \left(\{e^{iE_pt} - e^{-iE_pt}\}/2E_p\right)\Theta(-t). \tag{25}$$

I call this calculating the propagator with parallel pole placement in the upper half plane, *anti-PaP*. Using *PaP* or *anti-PaP* and carrying through the momentum space integration as indicated in (18) one arrives at the retarded G_{ret} or the advanced propagator G_{adv} resp.

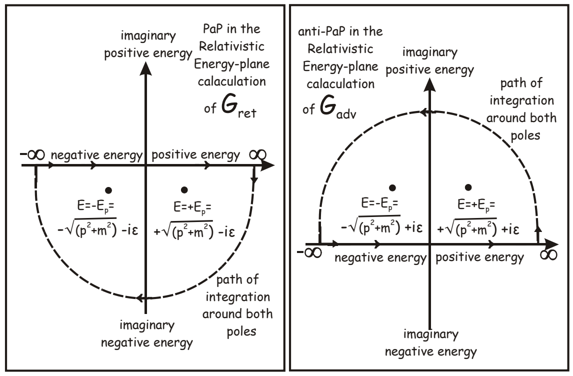

Fig. 3. *PaP* and *anti-PaP*.

Relation of commutators with propagators

One can also easily calculate all of the Feynman propagators G_{fl} , G_{fu} , the anti-Feynman propagators, $G_{\sim fl}$, $G_{\sim fu}$, and the retarded and advanced propagators G_{ret} , G_{adv} , from the commutation relations (11) through (15) and (17). We find:

$$i < 0\left|[\varphi_\uparrow{}^{+}(x),\ \varphi_\uparrow{}^{\dagger-}(y)]\right|0 > \ = G_{fl} = (2\pi)^{-3}\int d^3\boldsymbol{p}\ e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,(e^{-iE_pt}/2E_p\,)\Theta(t), \tag{26}$$

$$i < 0\left|\varphi_\uparrow{}^{-}(x),\ \varphi_\uparrow{}^{\dagger+}(y)\right|0 > \ = G_{\sim fl}\ =\ -(2\pi)^{-3}\int d^3\boldsymbol{p}\ e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,(e^{iE_pt}/2E_p\,)\Theta(t), \tag{27}$$

$$i < 0\left|[\varphi_\downarrow{}^{+}(x),\ \varphi_\downarrow{}^{\dagger-}(y)]\right|0 > \ = G_{fu} = (2\pi)^{-3}\int d^3\boldsymbol{p}\ e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}(e^{iE_pt}/2E_p\,)\,\Theta(-t), \tag{28}$$

$$i < 0\left|[\varphi_\downarrow{}^{-}(x),\ \varphi_\downarrow{}^{\dagger+}(y)]\right|0 > \ = G_{\sim fu} = -(2\pi)^{-3}\int d^3\boldsymbol{p}\ e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}(e^{-iE_pt}/2E_p\,)\,\Theta(-t), \tag{29}$$

$$i < 0\left|\left[\varphi_\uparrow(x),\ \varphi_\uparrow{}^{\dagger}(y)\right]\right|0 >\ =\ G_{ret}\ =\ G_{fl} + G_{\sim fl}, \tag{30}$$

$$i < 0\left|[\varphi_\downarrow(x),\ \varphi_\downarrow{}^{\dagger}(y)]\right|0 >\ =\ G_{adv}\ =\ G_{fu} + G_{\sim fu}. \tag{31}$$

The usual reason people use (30) and (31) as proof of causality when $(x - y)^2 < 0$, is that both G_{ret} and G_{adv} vanish, i.e., $<0\left|[\varphi_\updownarrow(x),\ \varphi_\updownarrow{}^{\dagger}(y)]\right|0> = 0$. This is easy to see using *PaP*. You can see that g_{ret} and g_{adv} both vanish because you can Lorentz transform the spacelike interval $(x - y)$ to a coordinate system where $t = (x^0 - y^0) = 0$. In so doing it is easy to see that $G_{ret} = G_{adv} = 0$, since from (24) with $t = 0$, we find $g_{fl} +\ g_{\sim fl} = 0$ and from (25) with $t = 0$, we find $g_{fu} +\ g_{\sim fu} = 0$.

It is also easy to see that using *PaP* with $t < 0$, and enclosing the integration path in the upper half plane that $G_{ret} = 0$ simply because the integration path encloses no poles. A similar argument holds for proving $G_{adv} = 0$ and enclosing the path in the lower half plane when $t > 0$.

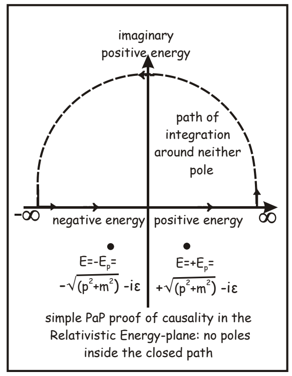

Fig. 4 Simple *PaP* proof of causality.

Thus whenever the fields are spacelike separated the commutators G_{ret} and G_{adv} indeed both vanish.[5] Although this appears perfectly reasonable, the use of VC to argue for causality inconsistently applies the Feynman and anti-Feynman propagators in the commutators defined in (26) through (29). One uses both Feynman and anti-Feynman sub-propagators to calculate *PaP* (and *anti-PaP*) and therefore assumes the superpositions of them valid. But this prescription is invalid for a good reason—G_{ret} mixes negative energy particles (labeled as antiparticles using commutators) propagating <u>forward</u> in time $G_{\sim fl}$, (27),with positive energy particles propagating <u>forward</u> in time G_{fl} , (26) and conversely G_{adv} mixes negative energy particles G_{fu} , (28) with positive energy particles $G_{\sim fu}$ (29) both propagating <u>backward</u> in time. Hence the anti-Feynman propagators $G_{\sim fl}$ and $G_{\sim fu}$ are clearly not valid and should, therefore, not be used in any commutation relations because they predict field operators yielding negative energy particles forward in time or field operators yielding positive energy particles going backward in time.

With *PaP* and *anti-PaP* one must have the residues from both Feynman and anti-Feynman poles contributing to the retarded and advanced propagators, resp. Feynman certainly noticed this since he never used anti-Feynman propagators in his landmark papers.[6] In fact he argued for positive energy particles convincingly in *Elementary Particles and the Laws of Physics*.[7] Consequently we can safely infer that since if it is not wrong it is at least inconsistent to use anti-Feynman sub-propagators, $g_{\sim fl}$ and $g_{\sim fu}$ in quantum field theory. *PaP* is invalid because from (24) you have negative frequencies (energies) coming from $g_{\sim fl}$ contributing to the propagator, G_{ret} and similarly, from (25) using *anti-PaP* you have positive frequencies (energies) coming from $g_{\sim fu}$ contributing to the propagator, G_{adv} regardless of the sign of $(x - y)^2$.

Hence using the commutator (30) or *PaP* , which amounts to the same argument, to get G_{ret} results in a superposition of propagators for both negative energy (anti)-particles and positive energy particles traveling <u>forward</u> in time. Similarly using the commutator (31) or *anti-PaP* to get G_{adv} results in a superposition of propagators for both negative energy particles and positive energy (anti)-particles traveling backward in time. In brief VC or *PaP* or *anti-PaP* implies both positive and negative energy particles traveling in the same direction through time. However this denies the existence of antiparticles (negative energy particles traveling backward in time) derivable form the Feynman propagators without using anti-Feynman propagators.

Certainly the math is correct in VC and *PaP* or *anti-PaP*, but the interpretation is clearly inconsistent or wrong. I therefore conclude that the proof of causality based on VC is wrong simply because the condition is not physically relevant (we don't have anti-Feynman propagators) even though you do get a causality condition from using them. But, I repeat, you don't get antiparticles (as Feynman regarded them—negative energy particles traveling backward in time), since in either the retarded G_{ret} or advanced G_{adv} case you can never get negative energy particles traveling counter in time to positive energy particles; hence the antimatter condition cannot arise.

Thus VC should not be used as a proof of causality since for $t > 0$ it involves *PaP* and not *DiP* as I believe Feynman inferred.

Feynman's reason for antiparticles—they *DiP*

Using just the propagators G_{fl} and G_{fu} Feynman derived antiparticles in a concise manner simply by considering the fact that neither propagator vanishes outside the lightcone. Referring to the spacetime region outside the lightcone as the *elsewhere* or referring to spacelike events, wherein $(x - y)^2 < 0$, one finds that such propagation between events x and y can be viewed in opposite time order. Let us refer to the event y as the creation of a positive energy particle with a charge of +1 and event x as the annihilation of the same particle as witnessed by observer one. Hence supposing the particle carries a mass m; this propagator describes the creation of positive energy m at y and the destruction of positive energy m at x. However since the events are spacelike separated, the propagation of this "particle" from y to x with positive energy as seen by observer one can be seen by another moving observer two as a particle moving backward in time from x to y with negative energy. In brief at y we have a net loss of energy, $-m$, and charge, -1, so the spacetime region around y carries both negative energy and negative charge in comparison with its previous zero energy and charge status before event y occurred. Conversely the spacetime region around event x would show a gain of positive charge +1 and a similar gain of energy m.

While the 1st observer has no problem identifying the propagator for this process as a *DiP* calculation of G_{fl} describing a positive energy particle going forward in time from y to x as seen in the left side of Fig 1, the 2nd observer would regard the same process as a *DiP* calculation of G_{fu} describing a negative energy particle with negative charge -1 moving backward in time from x to y as seen on the left side of Fig 2.

Hence we realize that while observer one sees the time of event y^0 as occurring before the time of event x^0, $(x^0 - y^0) > 0$, observer two sees the time of event y^0 as occurring after the time of event x^0, $(y^0 - x^0) > 0$. So observer two, seeing that the region around x now carries a net positive charge of +1 and a positive energy m, would reason that some kind of "particle" with charge -1 and energy $-m$ traveled from x to y thus accounting for the negative energy $–m$ and charge at y. Thus we have the discovery of antiparticles without resorting to introducing them via extra creation and annihilation operators in the field description, ala the antiparticle creation operator $\boldsymbol{b_p}^{\dagger}$ and destruction operator $\boldsymbol{b_p}$.

Hence the reason for antiparticles arises from elsewhere propagation as Feynman put it; "one man's virtual particle is another man's virtual antiparticle." Once we accept that elsewhere propagation must occur we can reason appropriately using only propagators G_{fl} to describe particles and G_{fu} to describe antiparticles even when the propagation is contained within the lightcone of either particle or antiparticle. In brief, the Feynman prescription is universal—we need not only consider elsewhere propagation. There is no need to invoke the anti-Feynman propagators $G_{\sim fl}$ and $G_{\sim fu}$. Using them introduces an inconsistency, e.g., $G_{\sim fl}$ indicates a negative energy particle traveling forward in time, $G_{\sim fu}$ indicates a positive energy particle traveling backward in time, and each is computed from the commutator of the antiparticle creation operator $\boldsymbol{b_p}^{\dagger}$ and destruction operator $\boldsymbol{b_p}$ as shown in (12). In the Feynman scheme there is no need to include the creation $\boldsymbol{b_p}^{\dagger}$ and destruction $\boldsymbol{b_p}$ operators.

Counter arguments

One can object to my conclusion based on various historical and accepted field theoretical grounds. Let me consider a few. On p. 7 of "The Conceptual Basis Of Quantum Field Theory" by Gerard 't Hooft where in describing the use of the Green's function for calculating propagators, he says: "Our choice can be indicated by shifting the pole by an infinitesimal imaginary number, after which we choose the contour C to be along the real axis of all integrands." Then he says that this prescription results in causality.[8] I agree. However, he doesn't mention that he is shifting two poles so that they are in *PaP* condition. Of course you get causality and VC but I contend you don't have antiparticles since you don't have elsewhere propagation and you must have the anti-Feynman propagators $G_{\sim fl}$ and $G_{\sim fu}$. He further writes:

"This Green function, called the *forward* Green function, gives our expressions the desired causality structure: There are obviously no effects that propagate backwards in time, or indeed faster than light. The converse choice, . . . gives us the backward solution. However, in the quantized theory, we will often be interested in yet another choice, the *Feynman propagator* . . ."

In Weinberg's book. "The quantum theory of fields," the question of what the VC can mean using the Hamiltonian evaluated at two different spacelike separated points x and y, where $(x - y)^2 < 0$, in chapters 3 and 5 is considered.[9] He uses the fact that $[H(x), H(y)] = 0$ where H is the Hamiltonian, for spacelike fields. He writes on the last paragraph of p. 198, and I paraphrase:

"The . . .[commutator] conditions [based on commutation of fields] are plausible for photon fields (ala Bohr and Rosenfeld), however we are dealing with fields like the Dirac field that do not seem measurable . . . The point of view taken here is that . . . $[\psi(x),\psi(y)]_{\pm} = 0$ (anti-commutation or commutation) is needed for Lorentz invariance of the S-Matrix, without any ancillary assumptions about measurability or causality."

I have not disputed this. In fact I have used Weinberg's observation. If you look through his book, he is very careful to not, or to hardly, mention causality at all. He instead uses the term "causal fields" throughout but that always means VC. On pp. 202-205, he recognizes in eq. (5.2.6), the Feynman propagator, what he calls the $\boldsymbol{\Delta}_{+}(x)$ function, fails to vanish. I am not disagreeing with Weinberg here. I am only pointing out that the approach that uses the VC is inconsistent with the Feynman propagator point of view which seems to be valid. What surprises me, and I really do not understand Weinberg here, is that by essentially denying the use of the Feynman propagator scheme that I outline in the paper, he still argues that antiparticles exist.

Hence from Weinberg you apparently do have antiparticles in the KG (Klein-Gordon) scalar field case and yet you exclude propagation into the elsewhere. I really don't understand this. Oh, to be clear, elsewhere propagation is to me the same thing as (virtual) tachyons which is the same thing as my invalidation point about VC (vanishing of the commutator). I think all you really need for an antiparticle, and by implication elsewhere propagation, is for the particle field to carry a charge—some form of interaction in the Lagrangian like *iV*.[10]

So in brief there is objection to tachyons in the KG case, however, this is counter to Feynman's point of view.[11] He consider tachyons, i.e., elsewhere propagation (that Weinberg seems to deny) in the last part of his positron paper as I recall. He didn't use the term tachyon of course—just propagation outside the lightcone (into the elsewhere).

One may also object on the grounds that the Feynman propagator is a *c*-number and therefore not a measurable operator like a quantum field operator. Because the violations of causality occur in the intermediate virtual states where one can also find other weird "particles" emerging, it is natural to simply assume Feynman's method of calculating is just a mathematical tool and only the results of measurement are important. Here we raise the question just what is measurable in quantum field theory? It is usually supposed that VC implies measurability of the fields in the commutator. The Feynman prescription of determining amplitudes from the propagators that when squared appropriately to give probabilities and eventually cross sections, seems to me to be measureable. So I don't see throwing Feynman's propagators out the window. One also needs to consider that the propagators are equal to the commutators (multiplied by *i*).

One can also examine the question of the spin-statistics connection[12] raised by Pauli and his use of the commutator to make his point about spin ½ fields having anti-commuting fields while spin *0,1,* etc, must have commuting spacelike separated fields. Pauli's D function is in effect calculated from integrating $g_{fl} + g_{\sim fl}$ using (1). He also considers another function that he calls D_1 that is the same as integration of the linear combination $g_{fl} - g_{\sim fl}$. Pauli recognizes that D_1 leads to trouble and simply throws it out of consideration in his proofs. He also never considers the time reversal sub-propagators $g_{fu} + g_{\sim fu}$. He wrote:

"The justification for our postulate lies in the fact that measurement at two space points with a spacelike distance can never disturb each other, since no signals can be transmitted with velocities greater than that of light. Theories which would make use of the D_1 function in their quantization would be very different from the known theories in their consequences.[13].

"Hence we come to the result (Italics Pauli's.): *For integral spin the quantization according to the exclusion principle is not possible. For this reason it is essential, that the use of the D_1 function in place of the D function be, for general reasons, discarded*." [14]

By denying D_1 one in effect denies the separability of $g_{fl} + g_{\sim fl}$ into two functions g_{fl} and $g_{\sim fl}$. For with both D_1 and D we can determine these Feynman and anti-Feynman propagators (Green's functions) by simply adding or subtracting D_1 and D. Hence Pauli throws the baby (anti-causality) out with the bathwater (separation of the Feynman and anti-Feynman propagators).

Conclusion

If you insist on VC as proof of causality, whereby nothing propagates into the elsewhere when $(x - y)^2 < 0$, you then must also insist that antimatter cannot exist and particles going forward in time must have both positive and negative energies.[15] This conclusion runs counter to the usual 2nd quantization method of quantum field theory. That is my point—the two are not consistent. Since you can't have it both ways (*PaP* and *DiP*) and since we don't have negative energy particles going forward in time and we do have antiparticles (negative energy particles going backward in time[16]), you must only use *DiP* consistently. Thus the usual VC for spacelike separated fields cannot be used even though it is mathematically true. Simply put, VC gives G_{ret} or G_{adv} constructed from anti-Feynman propagators which are invalid in quantum field theory. G_{ret} implies negative energy particles traveling forward in time and G_{adv} implies positive energy particles traveling backward in time resulting in an unstable universe without the appearance of antiparticles (negative energy particles traveling backward in time). It appears that one must give up causality to gain antimatter and a stable universe.[17]

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[1] There many such textbooks. I will only mention Weinberg, Steven. *The Quantum Theory of Fields*. NY: Cambridge University Press, 1995, 2005.

[2] A good source book can be found on line by Robert D. Klauber. http://www.quantumfieldtheory.info/ http://www.quantumfieldtheory.info/WhCh050x.pdf.

[3] Feynman, R. P. "The Theory of Positrons." *Phys. Rev.* 76 (1949), p. 757.

[4] See for example, Zee, Anthony. *Quantum Field Theory in a Nutshell*. Second Edition. NJ: Princeton University Press, 2010 among many others.

[5] In fact with $(x - y)^2 < 0$, you can show by Lorentz transforming to different observers that you can find $(x^0 - y^0)$ to be positive, negative, or zero and thus with *PaP* you can prove the spacelike commutator vanishes very simply by closing in the half plane that has no poles included (the closed path integral vanishes via the calculus of residues).

[6] Feynman, Richard P. and Steven Weinberg. *Elementary Particles and the Laws of Physics: The 1986 Dirac Memorial Lectures.* New York: Cambridge University Press, 1987. p. 7. Also see Feynman, R. P. "The Theory of Positrons." *Phys. Rev.* 76 (1949), p. 757.

[7] Feynman, Richard P. and Steven Weinberg. *Ibid*. p. 7.

[8] 't Hooft, Gerard. "The Conceptual Basis Of Quantum Field Theory" *Handbook of the Philosophy of Science*, (2004). Also see (http://www.phys.uu.nl/~thooft/) for a complete list of publications.

[9] Weinberg ibid.

[10] See http://wiki.physics.fsu.edu/wiki/index.php/Klein-Gordon_equation for more on negative energy states and antiparticles.

[11] Feynman, R. P. *ibid*.

[12] Pauli, W. "The Connection Between Spin and Statistics." *Phys. Rev.* 58, (1940), pp. 717-22. Also see Sudarshan, E. C. G. The Fundamental Theorem on the Connection between Spin and Statistics, in Proc. Nobel Symposium 8; Elementary Particle Theory, Relativistic Groups and Analyticity, Nils Svartholm (ed.), Almqvist and Wiksell, Stockholm (1968), pp.379-386; and see Sudarshan, E. C. G. and Ian Duck Toward an Understanding of the Spin-Statistics Theorem; Am. J. Phys. 66(4), 284 (1998).

[13] See Pauli, W. p.721. *Ibid*.

[14] See Pauli, W. p.722. *Ibid*.

[15] We cannot have negative energy particles going forward in time because they would act as an energy sink eventually swallowing up all positive energy particles. Similarly we cannot have positive energy particles traveling backward in time for the same reason. Also see Sudarshan, E. C. G., The Nature of Faster than Light Particles and Their Interactions, Arkiv. für Physik 39, 40 (1969).

[16] Of course in this paper I am using the term "going backward in time" as a calculation tool. Whether particles actually do this is not subject to any experimental proof to date. All one needs to do is remember that a negative energy particle moving backward in time with a negative charge would be seen by observers as a positive energy particle moving forward in time with a positive charge and vice versa. Also see Sudarshan, E. C. G. and O. M. P. Bilaniuk, Causality and Spacelike Signals;, Nature 223, 386 (1969).

[17] Please see my book, Wolf, F. A. *Time-loops and Space-twists: How God created the universe* (San Francisco, CA: Hampton Roads, Red Wheel Weiser, 2011) for a popular discussion of this topic and other concepts from quantum field theory.