Earth's charge and the charges of the Van Allen belts

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ABSTRACT

In this paper three separate charges are distinguished for the Earth and its magnetosphere. First, it is assumed, that the Earth and its nearest atmosphere bear a net negative charge Q_E. Secondly, a positive charge Q_i and a negative charge Q_o are proposed for the inner and outer Van Allen belt, respectively. Thirdly, a belt with net zero charge (electron slot) will be assumed to be present between both charged belts.

According to the three tori model, recently developed for pulsars and black holes, equilibrium may exist between the charges Q_E, Q_i and Q_o. Three expressions for the Coulomb electric field at different distances from Earth's centre are derived from the same model. Using available data, values for the three charges are deduced for the solar minimum and maximum, respectively. An averaged charge Q_E of about – 1 C is extracted for the Earth. Some other features of the model are discussed, among them the flow of charge during the change from solar minimum to maximum.

Furthermore, it is shown that the magnitude of Earth's magnetic field cannot be explained by the motion of the charges Q_E, Q_i and Q_o. In order to obtain a better explanation, the so-called Wilson-Blackett law is discussed. In addition, a large toroidal electric current in the Earth is proposed.

1. INTRODUCTION

It is generally assumed, that the naked Earth bears a large negative electric charge, Q_s, generating a vertical electric field at its surface. In the fair-weather area the magnitude of this electric field is about – 100 V/m, corresponding to a charge $Q_s = 4\pi\varepsilon_0 r_s^2 E = -\,4.5\times10^5$ C (r_s = 6371 km) at Earth's surface (see, e.g., Uman [1]). However, an almost equal amount of positive charge, is distributed throughout Earth's nearest atmosphere. In this study it is attempted to deduce Earth's residual charge Q_E, up to an altitude of about 70 km.

Apart from Q_E, it is assumed that the two Van Allen belts [2] also bear a net electric charge: a positive charge Q_i for the inner torus and a negative charge Q_o for the outer torus. The two described belts are separated by a region with zero net charge, the so-called "electron slot". The magnetic field around the Earth is important for the orientation of the Van Allen belts, but in this paper we mainly investigate the electric interactions between the charges Q_E, Q_i and Q_o.

Starting from the three tori model, recently developed for pulsars and black holes [3, 4], equilibrium between the charges Q_E, Q_i and Q_o appears to be possible. From the same model three different expressions for the Coulomb electric field, depending on the distance from Earth's centre, have been derived. The deduced Coulomb electric field at the plasmapause is put equal to the so-called co-rotation field. Values for the charges Q_E, Q_i and Q_o can then be calculated for the solar maximum or minimum, respectively. The results of the idealized model are discussed.

In addition, the contributions to Earth's magnetic field caused by the proposed charges Q_E, Q_i and Q_o are calculated. Since the latter contributions appear to be almost negligible, a previously proposed gravitational explanation of Earth's magnetic field is considered (see, e.g., refs. [5–9] and references therein). Especially, the so-called Wilson-

Blackett law is discussed. In order to obtain agreement between observed and predicted magnetic fields, the existence of a large toroidal current in the Earth is assumed.

2. ELECTRIC FIELDS IN THE MAGNETOSPHERE

The three tori model has been developed for the explanation of quasi-periodic oscillations (QPOs) of pulsars, black holes and white dwarfs [3, 4]. In this work this model will be applied to the Earth and its magnetosphere. Apart from Earth's residual negative charge Q_E, it will be assumed that the Earth is surrounded by three circular tori in the equatorial plane. First, an *inner* torus with radius r_i, containing a total positive charge Q_i. Secondly, an *outer* torus with radius r_o, containing a total negative charge Q_o (Q_o and Q_E should have the same sign). Thirdly, a third uncharged torus with radius r_m, containing a total mass m_m (the subscript m stems from *middle*) is assumed to be present between the two other tori. Thus, it is assumed that $r_i < r_m < r_o$. The idealized model is displayed in figure 1.

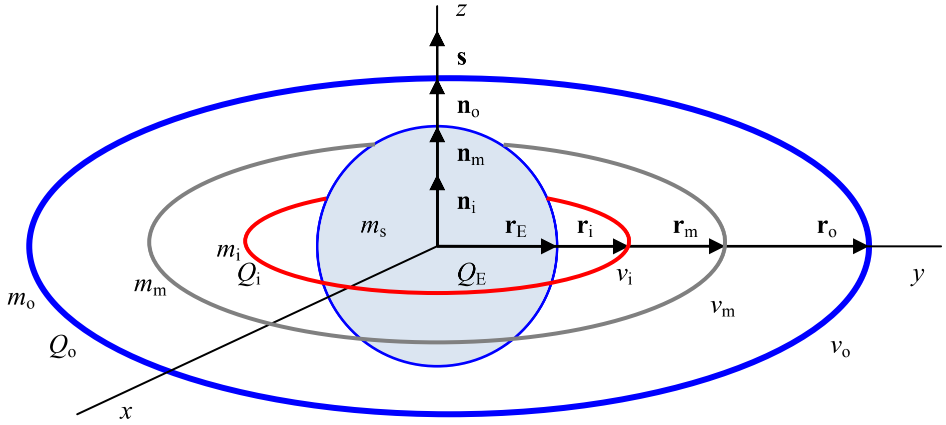

Figure 1. Idealized model of Earth's charge and charges in the magnetosphere. A net negative charge Q_E (blue) is adopted for the Earth and its nearest atmosphere. In addition, three circular tori around the Earth are assumed: an inner torus with charge Q_i (red), radius r_i and mass m_i, an outer torus with charge Q_o (blue), radius r_o and mass m_o and a torus with charge $Q_m = 0$ (grey), radius r_m and mass m_m between the other tori. The unit vector of Earth's rotation axis $\mathbf{s} \equiv \mathbf{\Omega}_s/\Omega_s$ and the unit vectors along the rotation axes of the three tori ($\mathbf{n}_i$, $\mathbf{n}_m$ and $\mathbf{n}_o$) are all assumed to coincide. In addition, the frequencies of charge elements dQ_i and dQ_o in the tori are denoted by ν_i and ν_o, whereas the frequency of a mass element dm_m is denoted by ν_m.

By using Coulomb's law only, it can be shown, that equilibrium between Earth's charge Q_E and the charges Q_o and Q_i in the tori is possible [3, 4]. The following relations then apply

$$Q_E = -g(x)Q_i \quad \text{and} \quad Q_E = x^2 f(x) Q_o, \tag{2.1}$$

where the parameter x is defined by $x \equiv r_i/r_o$. It is noticed that the expressions of (2.1) are most easily deduced for a point charge Q_E. It can be shown, however, that these relations remain valid for the electric interaction between a sphere with homogeneous charge density and total charge Q_E, charge Q_i and charge Q_o for any value of $r_i \geq r_E$ and $r_o > r_i$ (r_E is Earth's naked radius r_s, plus its nearest atmosphere). The functions $g(x)$ and $f(x)$ are defined by

$$g(x) \equiv \frac{2}{\pi}\left\{\frac{E(x)}{1-x^2}\right\} \quad \text{and} \quad f(x) \equiv \frac{-2}{\pi x}\left\{K(x) - \frac{E(x)}{1-x^2}\right\}, \tag{2.2}$$

where $K(x)$ and $E(x)$ are elliptic integrals of the first and second kind, respectively.

Following the formalism of refs. [3, 4], the total equatorial *Coulomb electric field*, $E_{\mathrm{Coul}}(r)$, due to the charges Q_{E}, Q_{i} and Q_{o}, at radius r in the interval $r_{\mathrm{E}} \le r < r_{\mathrm{i}}$ can be calculated to be

$$E_{\mathrm{Coul}}(r) = \frac{k}{r^2}\left[Q_{\mathrm{E}} - \frac{r^2}{r_{\mathrm{i}}^2}\left\{ f(r/r_{\mathrm{i}})Q_{\mathrm{i}} + x^2 f(r/r_{\mathrm{o}})Q_{\mathrm{o}} \right\} \right], \tag{2.3}$$

where $k = 1/(4\pi\varepsilon_0) = 10^{-7}c^2 = 8.9876\times10^9$ N.m^2.C^{-2} is the Coulomb constant (ε_0 is the vacuum permittivity). It is noted that SI units are used throughout this work. The quantities $f(r/r_{\mathrm{i}})$ and $f(r/r_{\mathrm{o}})$ are analogously defined to function $f(x)$ in (2.2). Note that function $f(r/r_{\mathrm{i}})$ displays a singularity near $r = r_{\mathrm{i}}$. Substitution of (2.1) into (2.3) yields

$$E_{\mathrm{Coul}}(r) = \frac{kQ_{\mathrm{E}}}{r^2}\left[1 + \frac{r^2}{r_{\mathrm{i}}^2}\left\{ \frac{f(r/r_{\mathrm{i}})}{g(x)} - \frac{f(r/r_{\mathrm{o}})}{f(x)} \right\} \right]. \tag{2.4}$$

When the radii r, r_{i} and r_{o} are known, the quantities $f(r/r_{\mathrm{i}})$ and $f(r/r_{\mathrm{o}})$, $g(x)$ and $f(x)$ can be calculated (see refs. [3, 4]). Substitution of the value for $E_{\mathrm{Coul}}(r)$ and the other parameters into (2.4) then would yield the corresponding value for the charge Q_{E}.

In addition, the total equatorial *Coulomb electric field*, $E_{\mathrm{Coul}}(r)$ for a radius r in the interval $r_{\mathrm{i}} < r < r_{\mathrm{o}}$ can be calculated to be

$$E_{\mathrm{Coul}}(r) = \frac{kQ_{\mathrm{E}}}{r^2}\left[1 - \frac{r^2}{r_{\mathrm{i}}^2}\left\{ \frac{g(r_{\mathrm{i}}/r)}{g(x)} + \frac{f(r/r_{\mathrm{o}})}{f(x)} \right\} \right], \tag{2.5}$$

where the quantity $g(r_{\mathrm{i}}/r)$ is analogously defined to function $g(x)$ in (2.2). The functions $g(r_{\mathrm{i}}/r)$ and $f(r/r_{\mathrm{o}})$ can be calculated from chosen r and known r_{i} and r_{o}. Singularities occur for $g(r_{\mathrm{i}}/r)$ near $r = r_{\mathrm{i}}$ and for $f(r/r_{\mathrm{o}})$ near $r = r_{\mathrm{o}}$.

Finally, the total equatorial *Coulomb electric field*, $E_{\mathrm{Coul}}(r)$, for a radius $r > r_{\mathrm{o}}$ can be calculated to be

$$E_{\mathrm{Coul}}(r) = \frac{kQ_{\mathrm{E}}}{r^2}\left\{ 1 - \frac{g(r_{\mathrm{i}}/r)}{g(x)} + \frac{g(r_{\mathrm{o}}/r)}{x^2 f(x)} \right\}, \tag{2.6}$$

where the quantity $g(r_{\mathrm{o}}/r)$ is also analogously defined to function $g(x)$ in (2.2). It is noticed that $g(r_{\mathrm{o}}/r)$ displays a singularity near $r = r_{\mathrm{o}}$.

In presence of a magnetic field transformation properties of the electric field become important. For example, the transformation of an electric field from a rotating to a nonrotating frame of reference is given by

$$\mathbf{E}' = \mathbf{E} + \mathbf{v}\times\mathbf{B}, \tag{2.7}$$

where the field $\mathbf{E}'$ is measured in the rotating frame. The velocity $\mathbf{v}$ and the fields $\mathbf{E}$ and $\mathbf{B}$ are given in the nonrotating frame of reference. If no electric field is measured in the rotating frame ($\mathbf{E}' = 0$), an electric field

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} \tag{2.8}$$

will exist in the nonrotating frame.

It is generally assumed that the plasma in the magnetosphere, subjected to Earth's magnetic induction field **B**, moves nearly rigidly with the Earth. The so-called *co-rotational electric field* $\mathbf{E}_{\text{cor}}(r)$ can be calculated from (2.8), resulting into (see, e.g., Volland [10])

$$\mathbf{E}_{\text{cor}}(r) = -\mathbf{v}\times\mathbf{B} = -(\mathbf{\Omega}_{\text{s}}\times\mathbf{r})\times\mathbf{B} = -\Omega_{\text{s}}(\mathbf{s}\times\mathbf{r})\times\mathbf{B}_0\frac{r_{\text{s}}^3}{r^3}, \tag{2.9}$$

where $\mathbf{v} = \mathbf{\Omega}_{\text{s}}\times\mathbf{r}$ is the speed of the co-rotating plasma in the equatorial plane. The field **B** at distance r is approximated by the field of an ideal magnetic dipole, so that $\mathbf{B} = \mathbf{B}_0\, r_{\text{s}}^3/r^3$ ($\mathbf{B}_0$ is Earth's magnetic equatorial field at distance r_{s}). For convenience' sake, it has been assumed that the unit vectors of the rotation axes of the Earth $\mathbf{s} \equiv \mathbf{\Omega}_{\text{s}}/\Omega_{\text{s}}$ and the circular orbit of the rotating plasma coincide. When r is expressed in units r_{s} (so $r = L\, r_{\text{s}}$), relation (2.9) can be rewritten as

$$E_{\text{cor}}(r) = -\frac{\Omega_{\text{s}} r_{\text{s}}^3 B_0}{r^2} = -\frac{\Omega_{\text{s}} r_{\text{s}} B_0}{L^2} = -\frac{13.9}{L^2}\,\text{mV/m} \tag{2.10}$$

for $\Omega_{\text{s}} = 7.292\times10^{-5}$ rad.s^{-1}, $B_0 = 3\times10^{-5}$ T and $r_{\text{s}} = 6371$ km. It is noticed that the electric field of (2.10) could have been generated by a net hypothetical charge from the Earth of magnitude $Q_{\text{cor}} \equiv -k^{-1}\Omega_{\text{s}}\, r_{\text{s}}^3\, B_0 = -62.9$ C.

Apart from the proposed equatorial Coulomb electric fields of (2.4), (2.5) and (2.6), other contributions to the total electric field are usually assumed. For example, an additional electric field in the magnetosphere arises from the solar wind, the so-called *convection electric field*, $E_{\text{conv}}(r)$. According to the model of Volland [10], the radial electric convection field in the equatorial plane, $E_{\text{conv}}(r)$, can be written as

$$E_{\text{conv}}(r) = -\frac{2A(k_{\text{p}})L}{r_{\text{s}}}\sin\varphi, \tag{2.11}$$

where $\varphi = 0^{\circ}$ points to the midnight, $\varphi = 90^{\circ}$ towards dawn, and so on. The multiplier $A(k_{\text{p}})$ depends on the k_{p} index, an empirical quantity that quantifies the level of geomagnetic activity. Maynard and Chen [11] deduced the following expression for $A(k_{\text{p}})$

$$A(k_{\text{p}}) = \frac{45}{\left(1-0.159k_{\text{p}}+0.0093k_{\text{p}}^{\,2}\right)^3}\,\text{V}. \tag{2.12}$$

Note that for a constant value of $A(k_{\text{p}})$ integration of $E_{\text{conv}}(r)$ of (2.11) from $\varphi = 0^{\circ}$ to $\varphi = 360^{\circ}$ yields a zero result.

In this work $E_{\text{Coul}}(r)$ of (2.6) will be put equal to $E_{\text{cor}}(r)$ of (2.10) for the radius $r = r_{\text{pp}}$ of the plasmapause. Substitution of all necessary parameters into (2.6) then yields a value for the charge Q_{E} in the case of the solar maximum and minimum. In table 1 values for the solar minimum and maximum of radii r_{i}, r_{m} and r_{o}, extracted from observations of Vette [12], are summarized. The values for r_{i}, r_{m} and r_{o} are estimated from the AE-8 max/min equatorial radial profiles of figures 45 and 40 (equatorial omnidirectional electron fluxes at electron energies of 3 MeV versus L have been considered). The values for $r_{\text{pp}} = L_{\text{pp}}\, r_{\text{s}}$ are related to k_{p} by an expression given by O'Brien and Moldwin [13]

$$L_{\text{pp}} = -0.43k_{\text{p}} + 5.9. \tag{2.13}$$

We estimate that $L_{\text{pp}} = 4.5$ and 5.5 for the solar maximum and minimum, respectively, corresponding to k_p values of 3.3 and 0.9. From the data the ratios r_i/r_{pp}, r_o/r_{pp} and $x \equiv r_i/r_o$ can be calculated, whereas the quantities $g(r_i/r_{pp})$, $g(r_o/r_{pp})$, $g(x)$ and $f(x)$ in (2.6) can be obtained from their respective series expansions (compare with tables 1 in refs. [3, 4]). Likewise, the other functions f and g can be obtained. The values of $E_{cor}(r_{pp})$ can be calculated from (2.10) from the chosen values of L_{pp}. Subsequently, the value of Q_E can be calculated from (2.6) and those of Q_i and Q_o from (2.1), respectively. All results have been given in table 1.

Table 1. Radii r_i, r_m and r_o are extracted from data from Vette [12], whereas r_{pp} is estimated from (2.13). From these data the ratios r_i/r_{pp}, r_o/r_{pp} and $x \equiv r_i/r_o$, and quantities $g(r_i/r_{pp})$, $g(r_o/r_{pp})$, $g(x)$ and $f(x)$ in (2.6) are calculated. Other functions f and g can be obtained in the same way. The quantity $E_{cor}(r_{pp})$ is calculated from (2.10). Q_E is then obtained from (2.6) and Q_i and Q_o from (2.1).

radius (r_s)	ratio	f	g	$E(r)$ (mV/m)	Q (C)
solar maximum					
r_i 1.55	r_i/r_{pp} 0.344	$f(r_s/r_i)$ 0.587	$g(r_i/r_{pp})$ 1.100	$E_{Coul}(r_E)$ [a] – 0.251	Q_i + 1.00
r_m 2.4					Q_m 0
r_o 3.7	r_o/r_{pp} 0.822	$f(r_s/r_o)$ 0.147	$g(r_o/r_{pp})$ 2.468		Q_o – 25.5
r_{pp} 4.5				$E_{cor}(r_{pp})$ [b] – 0.688	
	$x \equiv r_i/r_o$ 0.419	$f(x)$ 0.260	$g(x)$ 1.158		Q_E – 1.16
solar minimum					
r_i 1.55	r_i/r_{pp} 0.282	$f(r_s/r_i)$ 0.587	$g(r_i/r_{pp})$ 1.064	$E_{Coul}(r_E)$ [a] – 0.202	Q_i + 0.83
r_m 2.4					Q_m 0
r_o 4.2	r_o/r_{pp} 0.764	$f(r_s/r_o)$ 0.127	$g(r_o/r_{pp})$ 1.997		Q_o – 31.5
r_{pp} 5.5				$E_{cor}(r_{pp})$ [b] – 0.461	
	$x \equiv r_i/r_o$ 0.369	$f(x)$ 0.217	$g(x) =$ 1.117		Q_E – 0.93

[a] Calculated from (2.4). [b] Calculated from (2.10).

For comparison, choosing $k_p = 3.3$, $\varphi = 90^o$ and $r = r_{pp} = 4.5\,r_s$, combination of (2.11) and (2.12) leads to a value $E_{conv}(r_{pp}) = -0.32$ mV/m. Compare this result with that for $E_{cor}(r_{pp}) = -0.688$ mV/m in table 1. According to (2.11), however, $E_{conv}(r_{pp})$ only distorts a circular orbit, but averages to zero over a full revolution around the Earth.

It is stressed that the charges Q_E, Q_i and Q_o in table 1 have been obtained by equating the electric fields $E_{Coul}(r)$ of (2.6) and $E_{cor}(r)$ of (2.10) for $r = r_{pp}$. Extrapolation of this equalization up to $r = r_s$ would result into a predicted field of $E_{Coul}(r_s) = E_{cor}(r_s) = -13.9$ mV/m and a hypothetical charge $Q_{cor}(r_s) = -62.9$ C (see (2.10)). As an example, the latter charge can be compared with the charge $Q_E = -1.16$ C for the solar maximum, calculated from (2.6) and given in table 1. Alternatively, for $r_E \approx r_s$ substitution of $Q_E = -1.16$ C and the other necessary parameters from table 1 into (2.4) results into the following value for $E_{Coul}(r_E)$

$$E_{Coul}(r_E) = -0.258(\text{from}\,Q_E) - 0.054(\text{from}\,Q_i) + 0.061(\text{from}\,Q_o) = -0.251\,\text{mV/m}. \quad (2.14)$$

This result shows that $E_{Coul}(r_E)$ is dominated by the contribution from charge Q_E, whereas the contributions from the charges Q_i and Q_o are an order of magnitude smaller and nearly compensate each other.

In principle, predicted electric fields at Earth's surface can be compared with observations. For example, an average value of about – 100 V/m has been obtained from fair-weather observations (the negative sign indicates that the electric field vector is directed downward). The corresponding charge Q_s at Earth's surface then amounts to $Q_s = 4\pi\varepsilon_0 r_s^2 E = -4.5\times10^5$ C (see e.g., Uman [1]). However, a nearly equal amount of positive charge is distributed in Earth's nearest atmosphere, resulting in a much lower residual electric field at higher altitudes. As an example, Volland [10, section 2.3] gave an approximate empirical expression for the fair-weather electric field

$$E(z) = -\{93.8\exp(-4.527z) + 44.4\exp(-0.375z) + 11.8\exp(-0.121z)\}, \quad (2.15)$$

where $E(z)$ is the electric field in V/m and z is the altitude in km. This equation is valid at mid-latitudes below about 60 km altitude. For z = 60 km the first two terms on the right hand side of (2.15) can be neglected and a value $E(z)$ = – 8.3 mV/m results. The latter value is smaller than the value of $E_{cor}(r_s)$ = – 13.9 mV/m obtained from (2.10). More recent observations, however, indicate, that the electric field at higher altitudes may still be smaller. From data given by Rycroft *et al*. [14] (see their figure 15b) an extrapolated value of about – 0.2 mV/m can be extracted for the fair-weather electric field at an altitude of 70 km above the Earth. The latter value is in fair agreement with calculated values for $E_{Coul}(r_E)$ calculated from (2.4). For the solar maximum and minimum these calculated values for $E_{Coul}(r_E)$ are about – 0.251 mV/m and – 0.202 mV/m, respectively (see table 1). In order to obtain a value for the charge Q_E, we also could equalize $E_{Coul}(r_E)$ of (2.4) and the observed electric field at $r = r_E$, instead of equating $E_{Coul}(r)$ of (2.6) and $E_{cor}(r)$ of (2.10) for $r = r_{pp}$. The latter choice, however, seems to be more reliable.

Table 2. Calculated values for $E_{cor}(r)$ from (2.10) and $E_{Coul}(r)$ from (2.4), (2.5) and (2.6) for the solar maximum as a function of r, expressed in units of L, in the interval $r_s < r \le r = 6.0\ r_s$.

L	$E_{cor}(r)$ (mV/m)	$E_{Coul}(r)$ (mV/m)	L	$E_{cor}(r)$ (mV/m)	$E_{Coul}(r)$ (mV/m)	L	$E_{cor}(r)$ (mV/m)	$E_{Coul}(r)$ (mV/m)
1	– 13.9	– 0.251	2.8	– 1.78	0.404	4.5	– 0.688	– 0.688
1.2	– 9.68	– 0.202	3.0	– 1.55	0.546	4.6	– 0.659	– 0.613
1.4	– 7.11	– 0.303	3.2	– 1.36	0.808	4.8	– 0.605	– 0.501
1.55	– 5.80	0	3.4	– 1.21	1.43	5.0	– 0.557	– 0.422
1.6	– 5.44	0.971	3.6	– 1.08	4.60	5.2	– 0.515	– 0.363
1.8	– 4.30	0.262	3.7	– 1.02	0	5.4	– 0.478	– 0.318
2.0	– 3.48	0.215	3.8	– 0.965	– 5.09	5.6	– 0.444	– 0.282
2.2	– 2.88	0.225	4.0	– 0.871	– 1.77	5.8	– 0.414	– 0.252
2.4	– 2.42	0.259	4.2	– 0.790	– 1.09	6.0	– 0.387	– 0.227
2.6	– 2.06	0.308	4.4	– 0.720	– 0.784			

Using the result Q_E = – 1.16 C for the solar maximum and other data from table 1, it is possible to calculate values for $E_{Coul}(r)$ for all values of $r \ge r_E$ from (2.4), (2.5) and (2.6) for the intervals $r_E \le r < r_i$, $r_i < r < r_o$ and $r > r_o$, respectively. The results are given in table 2 and are displayed in figure 2. In deducing the three expressions for $E_{Coul}(r)$, it has been assumed that the charges Q_i, Q_o and Q_E are in equilibrium for the values L = 1.55 ($r_i = 1.55\ r_s$) and L = 3.7 ($r_o = 3.7\ r_s$). In these cases the conditions $E_{Coul}(r_i) = 0$ and $E_{Coul}(r_o) = 0$ follow directly from the applied formalism [3, 4].

It is noticed that the adopted equilibrium situations at L = 1.55 and L = 3.7 in our model are *unstable*. As an illustration, an electron located at a radius r *slightly* different from $r_o = 3.7\, r_s$ will be subjected to strong repulsion from the nearby toroidal negative charge Q_o. Thus, the relative large values of $E_{Coul}(r)$ for values slightly different from L = 1.55 and L = 3.7 may explain the large values of the observed equatorial omnidirectional electron fluxes close to these L-values (see figure 2). Moreover, between these two L-values the equatorial omnidirectional electron flux reaches a minimum, at L = 2.4 in our examples (see Vette [12]). Thus, the occurrence of an electron slot may also be explained by our model.

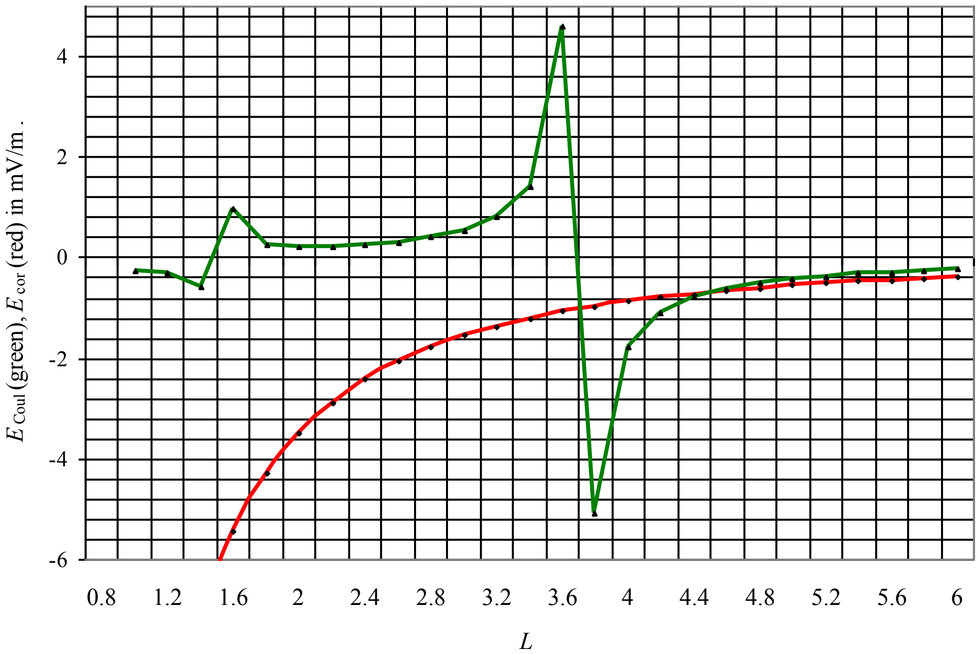

Figure 2. The co-rotational field $E_{cor}(r)$ from (2.10) is shown as a function of L (curve in red). In addition, the Coulomb field $E_{Coul}(r)$, calculated from equations (2.4), (2.5) and (2.6), respectively, is given (curve in green). It has been assumed for the solar maximum that both fields are equal at L = 4.5. See text for further comment.

Comparison of the curves for the field $E_{cor}(r)$ from (2.10) and the fields $E_{Coul}(r)$ from equations (2.4), (2,5) or (2.6) shows, that the field predicted by (2.6) nearly coincides for values of $r \geq r_{pp}$. For values of $r < r_{pp}$ our model predicts electric fields that substantially deviate from those predicted by the co-rotational electric field. For $r = r_E$ the value of $E_{Coul}(r_E)$ may, however, be in better agreement with observations (see, e.g., refs. [10, 14]. It is noticed that recent observations of the distribution of He^+ by Sandel *et al.* [15] show that plasma in the range $2 < L < 4$ most frequently rotates at a rate that is roughly 85–90 % of the co-rotation rate $\mathbf{\Omega}_s$. As a result, the field $E_{cor}(r)$ from (2.10) has to be multiplied by a factor 0.85–0.90. In that case, the calculation of Q_E then should start from combination of $E_{Coul}(r_{pp})$ of (2.6) and a somewhat lower value of $E_{cor}(r_{pp})$.

It is stressed that the adopted three tori model with a net charge Q_E for the Earth and charges Q_i and Q_o for the inner and outer Van Allen belts is only a first approximation. For example, the directions of the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_o$ along the rotation axes of the inner and outer torus do not coincide with the direction of Earth's rotation unit vector $\mathbf{s} \equiv \mathbf{\Omega}_s/\Omega_s$, but differ by an angle of about 11.5º. Moreover, the charges Q_i and Q_o are not confined to toroidal circles in the equatorial plane, but the belts extend to a latitude of about 60º from the magnetic equator. The presented model, however, predicts, that the orbit of the inner torus with charge Q_i is stable for angles smaller than $\Delta = 90º - \theta_0$ between the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_o$ (see refs. [3, 4] for a detailed calculation of Δ or θ_0

for an arbitrarily chosen value of $x = x_0$). As an example, for $x = 0.419$ in the solar maximum $\Delta = 31.1^{\circ}$, whereas Δ approaches to 35.26° in the limiting case $x \rightarrow 0$.

The results of table 1 show, that the values of Q_o are more than an order of magnitude larger than Q_E or Q_i. This result is directly predicted by relation (2.1): the smaller the ratio x, the greater Q_o compared with Q_E and Q_i. In addition, going from solar maximum to solar minimum the different charges change as follows: $\Delta Q_o \equiv Q_o(\text{max}) - Q_o(\text{min}) = +6.0$ C, $\Delta Q_i \equiv Q_i(\text{max}) - Q_i(\text{min}) = +0.17$ C and $\Delta Q_E \equiv Q_E(\text{max}) - Q_E(\text{min}) = -0.23$ C. These figures may suggest an outward flow of some negative charge from Earth's equator to the inner and outer torus during the transition from solar maximum to minimum. Alternatively, some positive charge may flow inward from the outer torus to Earth's equator. By the Lorentz force both the outward and the inward flow of charge at Earth's equator will generate toroidal currents of the same direction inside the Earth.

3. EARTH'S MAGNETIC FIELD

In this section the consequences of the proposed Earth's net charge Q_E and the charges Q_i and Q_o of the inner and outer Van Allen belts on Earth's magnetic field are investigated. See figure 1 as an illustration of our model. If the charge Q_E would be homogeneously be distributed over Earth's volume and its nearest atmosphere, the following relation exists between Earth's magnetic dipole moment $\mathbf{M}(Q_E)$ and its angular momentum $\mathbf{S}$ (see, e.g., ref. [3])

$$\mathbf{M}(Q_{\mathrm{E}}) = \frac{Q_{\mathrm{E}}}{2m_{\mathrm{s}}}\mathbf{S}. \tag{3.1}$$

The magnetic induction field at the poles at distance r_E from the centre, $\mathbf{B}_p(Q_E)$, is then given by (see, e.g., refs. [3, 8])

$$\mathbf{B}_{\mathrm{p}}(Q_{\mathrm{E}}) = \frac{\mu_0}{4\pi}\frac{2\mathbf{M}(Q_{\mathrm{E}})}{r_E^3} = \frac{\mu_0}{4\pi}\frac{Q_{\mathrm{E}}\mathbf{S}}{m_{\mathrm{s}}r_E^3} \approx \frac{\mu_0}{4\pi}\frac{2f_{\mathrm{s}}Q_{\mathrm{E}}\mathbf{\Omega}_{\mathrm{s}}}{5r_{\mathrm{s}}}, \tag{3.2}$$

where $\mu_0 = 4\pi\times10^{-7}$ A^{-1}.V.m^{-1}.s is the vacuum permeability. The angular momentum is given by $\mathbf{S} = 2/5 f_s m_s r_s^2 \mathbf{\Omega}_s$, where f_s is a dimensionless factor depending on the mass density of the spherically assumed Earth. For a homogeneous mass density $f_s = 1$, whereas $f_s = 0.827$ for the Earth. The radius r_E on the right hand side of (3.2) has been approximated by r_s. Note that the directions of $\mathbf{B}_p(Q_E)$ and $\mathbf{\Omega}_s$ are predicted to be parallel.

As an example, the value $Q_E = -1.16$ C for the solar maximum yields a value of $\mathbf{B}_p(Q_E) = -4.4\times10^{-19}$ $\mathbf{s}$ T from (3.2), extremely small compared with the observed value of $\mathbf{B}_p(\text{tot}) = -6.1\times10^{-5}$ $\mathbf{s}$ T ($\mathbf{s} \equiv \mathbf{\Omega}_s/\Omega_s$). For convenience' sake, the directions of $\mathbf{B}_p(\text{tot})$ and $\mathbf{s}$ are assumed to be parallel throughout this paper.

In order to explain the large discrepancy between $\mathbf{B}_p(\text{tot})$ and $\mathbf{B}_p(Q_E)$ from (3.2), it has previously been proposed that the basic magnetic induction field for rotating celestial bodies like the Earth, $\mathbf{B}_p(\text{gm})$, is from gravitational origin (see, e.g., refs. [5–9] and references therein). Starting from the theory of general relativity, the so-called Wilson-Blackett law can be deduced [6, 7] from this approach

$$\mathbf{M}(\text{gm}) = -\tfrac{1}{2}\beta\left(\frac{G}{k}\right)^{1/2}\mathbf{S}, \tag{3.3}$$

where $\mathbf{M}(\text{gm})$ is the gravitomagnetic dipole moment. The dimensionless constant β of unity does not follow from theory; it is taken equal to $\beta = +1$ (for a discussion see ref. [7]). From the Coulomb constant $k = 1/(4\pi\varepsilon_0) = 8.9876\times10^9$ N.m^2.C^{-2} and the gravitational constant $G = 6.674\times10^{-11}$ m^3. kg^{-1}. s^{-2} the remarkable gyromagnetic constant $1/2(G/k)^{½} = 4.309\times10^{-11}$ C. kg^{-1} can be calculated. Combination of (3.1) and (3.3) shows that for an electron with elementary charge $e = -1.602\times10^{-19}$ C and mass $m_e = 9.109\times10^{-31}$ kg the dimensionless ratio $(M(\mathrm{gm})/S)/(M(\mathrm{em})/S)$ for $\beta = +1$ is equal to

$$\frac{M(\mathrm{gm})/S}{M(\mathrm{em})/S} = \left(\frac{G}{k}\right)^{½}\left(\frac{m_\mathrm{e}}{e}\right) = 4.900\times10^{-22}. \tag{3.4}$$

Therefore, predicted magnetic fields from gravitomagnetic origin are usually extremely small and difficult to isolate from fields due to electric charges. From (3.3) the following gravitomagnetic field $\mathbf{B}_\mathrm{p}(\mathrm{gm})$ can be deduced for $\beta = +1$

$$\mathbf{B}_\mathrm{p}(\mathrm{gm}) = \frac{\mu_0}{4\pi}\frac{2\mathbf{M}(\mathrm{gm})}{r_E^3} = -\frac{\mu_0}{4\pi}\left(\frac{G}{k}\right)^{½}\frac{\mathbf{S}}{r_E^3} \approx -\frac{\mu_0}{4\pi}\left(\frac{G}{k}\right)^{½}\frac{2f_\mathrm{s} m_\mathrm{s}\mathbf{\Omega}_\mathrm{s}}{5r_\mathrm{s}}. \tag{3.5}$$

The gravitomagnetic field $\mathbf{B}_\mathrm{p}(\mathrm{gm})$ is identified as a magnetic induction field, resembling magnetic induction fields from electromagnetic origin, like $\mathbf{B}_\mathrm{p}(Q_\mathrm{E})$ from (3.2). Substitution of Earth's mass $m_\mathrm{s} = 5.977\times10^{24}$ kg and the other parameters into (3.5) yields a value of $\mathbf{B}_\mathrm{p}(\mathrm{gm}) = -\ 1.95\times10^{-4}\ \mathbf{s}$ T.

It is assumed that the total magnetic induction field, $\mathbf{B}$(tot), consisting of the sum of the fields from electromagnetic origin, $\mathbf{B}_\mathrm{p}$(em), and $\mathbf{B}_\mathrm{p}$(gm), differs from $\mathbf{B}_\mathrm{p}$(gm) by a factor β^*

$$\mathbf{B}_\mathrm{p}(\mathrm{tot}) = \mathbf{B}_\mathrm{p}(\mathrm{em}) + \mathbf{B}_\mathrm{p}(\mathrm{gm}) = \beta^*\mathbf{B}_\mathrm{p}(\mathrm{gm}). \tag{3.6}$$

When the total field $\mathbf{B}$(tot) is only due to gravitomagnetic origin, the dimensionless factor β^* reduces to $\beta^* = +1$. From the calculated value for $\mathbf{B}_\mathrm{p}$(gm) and the observed value for $\mathbf{B}_\mathrm{p}$(tot) a value $\beta^* = +0.31$ for the Earth is obtained.

Since charges may move in different ways in rotating bodies, one can hardly expect that β^* is a constant. Indeed, different results for β^* have been found for about fourteen rotating bodies: metallic cylinders in the laboratory, moons, planets, stars and the Galaxy [6, ch. 1]. On the other hand, from a linear regression analysis of this series an almost linear relationship between the observed magnetic moment $|\mathbf{M}|$ and the angular momentum $|\mathbf{S}|$ was obtained. This result is in fair agreement with the prediction of (3.3) ($|\mathbf{M}|$ and $|\mathbf{S}|$ vary over an interval of *sixty decades!*). From this analysis an average value of $|\beta^*| = 0.076$ was obtained. Although this result is distinctly different from the gravitomagnetic prediction $\beta^* = 1$, the correct order of magnitude of β^* for so many, *strongly different*, rotating bodies is amazing. For pulsars a separate analysis was given [9]. Since magnetic fields from electric origin may affect the total field, the reported results may reflect the validity of the gravitomagnetic hypothesis.

Analogously to pulsars and black holes, an expression of the total electromagnetic field $\mathbf{B}_\mathrm{p}$(em) from the charges Q_E, Q_i and Q_o has previously been calculated [3, 4]. First, the contribution $\mathbf{B}_\mathrm{p}(Q_\mathrm{E})$ generated by the charge Q_E at Earth's pole at radius r_E can be calculated (see (3.2)). For convenience' sake, radius r_E will be put equal to Earth's radius r_s. In addition, the rotational frequency at radius r_E will be approximated by Earth's rotational frequency v_s. The second contribution $\mathbf{B}_\mathrm{p}(Q_\mathrm{i})$ arises from the torus with charge Q_i moving with a rotational frequency v_i in the circular torus of radius r_i (see figure 1). The third contribution $\mathbf{B}_\mathrm{p}(Q_\mathrm{o})$ is generated by the total charge Q_o moving with a rotational frequency v_o in the circular torus of radius r_o. Combination of the gravitomagnetic contribution $\mathbf{B}_\mathrm{p}$(gm) of (3.5) with the three contributions to $\mathbf{B}_\mathrm{p}$(em) leads to the following expression for the parameter β^* (see refs. [3, 4])

$$\beta^* = 1 + \beta^*_{\text{current}} - Q_E{}' - \tfrac{5}{2} Q_i{}' \frac{\nu_i}{\nu_s} \frac{r_i^2/r_s^2}{\left(1 + r_i^2/r_s^2\right)^{3/2}} - \tfrac{5}{2} Q_o{}' \frac{\nu_o}{\nu_s} \frac{r_o^2/r_s^2}{\left(1 + r_o^2/r_s^2\right)^{3/2}}, \tag{3.7}$$

where Q_E' is defined by the dimensionless quantity $Q_E' \equiv (G^{1/2} m_s)^{-1} k^{1/2} Q_E$, Q_i' by $Q_i' \equiv (G^{1/2} m_s)^{-1} k^{1/2} Q_i$ and so on. In the case of co-rotation of both tori with the Earth, all rotational frequencies are equal, so that $\nu_s = \nu_i = \nu_o$. The term β^*_{current} in (3.7) has been added to account for a possible contribution from toroidal currents in the Earth. Calculation of the last three terms on the right hand side of (3.7) from data, e.g., for the solar maximum shows, that they all are negligible compared with unity value. Calculation then shows, that $\beta^*_{\text{current}} \approx \beta^* - 1 = -0.69$. Thus, the proposed magnetic field from gravitational origin may be reduced by a toroidal electric current inside the Earth.

4. CONCLUSIONS

It is generally assumed that the naked Earth bears a large negative charge Q_s of about -4.5×10^5 C (see e.g., Uman [1]). In this work it is attempted to calculate Earth's net charge, Q_E, including the charge of its nearest atmosphere. An estimate for Q_E in the solar minimum and maximum has been deduced from the so-called three tori model [3, 4]. An averaged charge of about -1.0 C for Q_E can be obtained from our table 1. To our knowledge this result is the first estimate of Earth's net charge Q_E. Thus, the obtained value of Q_E appears to be many orders of magnitude smaller than the charge Q_s at Earth's surface.

In addition, the three tori model may explain the existence of an inner and outer Van Allen belt, with charge Q_i and Q_o, respectively, separated by a belt with net zero charge, the so-called electron slot. Present work illustrates, that the three tori model, recently developed for the explanation of QPOs of pulsars, black holes and white dwarfs, may also be applied to the Earth and its magnetosphere.

From the deduced average value of net charge $Q_E = -1.0$ C an averaged poloidal magnetic field of $\mathbf{B}_p(Q_E) = -3.8\times10^{-19}$ $\mathbf{s}$ T is calculated from (3.2), extremely small compared with the observed value of $\mathbf{B}_p(\text{tot}) = -6.1\times10^{-5}$ $\mathbf{s}$ T ($\mathbf{s} \equiv \mathbf{\Omega}_s/\Omega_s$). A much better agreement with the observed value is obtained from a previously proposed gravitomagnetic theory [5–9]. The Wilson-Blackett law following from this approach yields a value of $\mathbf{B}_p(\text{gm}) = -1.95\times10^{-4}$ $\mathbf{s}$ T. The difference with the observational value may be attributed to a large toroidal current inside the Earth.

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