Tornado Genesis

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Abstract

It is inferred from the phenomenological facts of tornado that tornado itself should have a driving force generating the updraft in the vortex of tornado. The driving force is supposed to be originated from the electric interaction between a storm cloud and a crustal conducting body underground. The electric interaction in tornado is considered to be similar to a lightning discharge in respect that the electric potential is reduced between a storm cloud and the ground, but the electric discharging process in tornado should be much slower than the lightning discharge and result in the atmospheric vortex formation of tornado. The mechanism of tornado formation is suggested, in which Townsend avalanche process is assumed inside the tornado vortex with a dangling conducting channel that is embedded in the funnel cloud of tornado. The strong updraft inside the vortex of tornado is produced by momentum diffusion process when the positively ionized air molecules move upward under the influence of electric field inside the vortex. With the mechanism of tornado formation, phenomenological expectations are followed.

1. Introduction

Natural phenomena, especially natural disasters, such as tropical cyclones, earthquakes, tsunamis, volcanoes, tornadoes, etc., directly affect our daily life on this planet. The more we can understand about those of disasters scientifically, the better way we can find out to protect ourselves.

For tropical cyclones and earthquakes, for example, a possible scientific connection of ionosphere and lithosphere of the earth has been investigated [1]. Although it cannot be explained clearly, the scientific connections of the natural disasters to the sun's activities have been searched since the natural disasters are frequently accompanied with the earth's ionospheric variations, which is mainly affected by the sun's activities that is related with sunspot number, solar cycle, etc.

Recently, a scientific speculation -- solar magnetic storms is not just a phenomenological coincidence but correlated to earthquakes [2] -- has been came up from the fact that strong earthquakes occur frequently with solar magnetic storms in the earth's ionosphere.

Besides the solar effects on the earth, there is an interesting fact about solar cycle and the orbital periods of solar planets. It has been known that solar magnetic polarity is reversed when sunspot number is reached to a maximum; therefore, the variation of sunspot number has a close relation to the variation of solar magnetic filed. The interesting fact is a possible relation among the periods of solar cycle (~11 years), the orbital period of Jupiter (~22 years), the period of Jupiter-Saturn conjunction or opposition (~10 years), and the orbital periods of other planets in solar system. It should be an interesting scientific inquisition whether solar cycle has a relation to the periods of solar planets, especially Jupiter and Saturn. However, if basic theories in physics are not enough to explain natural phenomena, there should be a limit confronted in the scientific connections besides a statistical correlation.

The basic theories in physics, such as classical theory, relativities, and quantum physics, should be understood in a comprehensive way; this means, those theories should be connected in a logical way -- no matter what, in one way or another -- as long as they have a common fundamental base. The fundamental theory [3] was suggested to have the logical connection, and Mass-Charge interaction was derived from the theory and investigated in macroscopic phenomena at around the earth [4]. For the Mass-Charge interaction, primitive-virtual negative (PVN) charge was introduced for the mass object. It was estimated that PVN charge of the earth is $Q_v \sim -6\times10^5$ C; hence, the PVN charge of the earth contributes to the natural DC electric field on the earth, which is estimated as $E \sim 130$ V/m on the earth's surface (sea level) [4].

Besides, it was supposed that inside the earth is also electrically connected to the outer space through the atmosphere. Hence, the natural electric fields on the earth's surface have diurnal variations due to the solar PVN charge effect and the corresponding charge distributions in the ionosphere and underground of the earth. Moreover, it depends on local atmospheric conditions, such as atmospheric temperature, pressure, wind speed, moisture, aerosol density, etc., which affect atmospheric electrical conductivity.

The origin of the earth's geomagnetic field was also discussed with an alternative model. In the model, two magnetic dipole moments were introduced -- one is pointing at the earth's magnetic north pole but 3 orders smaller than the other; the other is pointing at the earth's magnetic south pole. From the outer core to the Moho region inside the earth, gravity is ignorable but geomagnetic pressure is dominant. As mentioned in the model, if geomagnetic pressure under the earth's crust is dominant and, thus, it makes the mechanical force balance with the downward gravitational pressure on the crust, the occurrence of earthquakes and plate tectonics of the earth would be affected directly by the solar magnetic storms in the earth's ionosphere.

One of peculiar phenomena on the earth's surface is tornado: Although there is no clear cut definition, tornado is known as *a violently rotating column of air extending between, and in contact with, a cloud and the surface of the earth*. If tornado occurs over water, it is called waterspout; if it occurs on land, it is called landspout (non-supercell tornado). However, a general mechanism of tornado touchdown is still unknown: Although it has been known that storm clouds, especially supercell clouds, spawn the tornadoes (supercell tornadoes), the tornado formation itself, for instance, tornado touchdown mechanism, cyclonic rotation, etc., cannot be explained clearly or, at least, qualitatively in a logical way with only the thermodynamics in atmospheric science. Moreover, it seems not appropriate to assume blindly that the supercell tornado is different from the waterspout and landspout in its formation mechanism because all of them seem to be apparently same natural phenomena [5].

2. Phenomenological Facts

Being compared with a tropical cyclone, tornado is highly localized and small in its physical size, but it can generate powerful winds in the vortex, which results in lots of damage along its path. In the phenomenological point of view, tornadoes occur frequently with supercell thunderstorm clouds [6] on land; first, a funnel is appeared coming down from the cloud base, and when it touchdown the ground (in many cases), the vortex of tornado appears to be formed. In many cases, especially if the tornado strength is weak, the funnel is just standing without moving for a short time and dissipated quickly with showing a rope shaped funnel pendent from the cloud base. However, tornado can travel along its path on the ground and creates lots of damage along the path. The strength of tornado is estimated for the maximum wind speed that is rated by Enhanced Fujita scale (EF scale) [7] on the base of tornado damage.

There have been many stories known as tornado oddities [8]; however, some of them have been confirmed as facts. Sometimes, straws or toothpicks are found being penetrated into the hardwood in tornado debris. Although it is strange and difficult to understand how that is possible, that was proved as phenomenological facts through experimentation using tornado gun and the wind velocities were also estimated for each specific cases [9]. Another strange thing is chicken plucking; but it has not been answered yet clearly. Besides the tornado oddities in its debris, phenomenological features of tornado also show peculiarities in its rapidness of outbreak and dissipation, cyclonic propensity, moving path, etc.

2.1 Storm Clouds

Many tornadoes occur with supercell clouds[1], but some tornadoes occur with non-supercell clouds, too. Furthermore, tropical cyclones spawn many tornadoes on the paths after they come into the land [17]. However, the storm cloud does not always spawn tornado, and a specific atmospheric condition of the storm cloud – supercell clouds -- also cannot be generalized as the necessary condition for the tornado. Therefore, it is better to say that the storm cloud (cumulonimbus cloud) is one of necessary ingredients for tornado, in general. A thunderstorm cloud develops quickly and grows vertically rather than spread horizontally that extends from near ground up to the troposphere or even further, and it contains lots of water.

[1] The supercell has a long-lived, intense, rotating updraft that sustains the storm in its mature stage for up to six hours or more [11].

2.2 Cyclonic Rotation

The earth is rotating once in every 24 hours from West to East; hence, the kinematical motion in a coordinate system fixed on the earth's surface (non-inertial frame) is affected by the inertial forces, such as centrifugal force, transverse force, and Coriolis force [10].

On a moving object on the earth's surface (rotating coordinate system) the Coriolis force is appeared as $\vec{F}_C = -2m(\vec{\Omega} \times \vec{\upsilon}_r)$ in which m is mass of the object, Ω is the angular velocity of the earth's rotation, and υ_r is velocity of the object in the rotating coordinate system. In general, the criterion of Coriolis effect is expressed by the Rossby number given as $R_0 = U/(\Omega L)$, in which L/U is called as the characteristic time of translation. For example, since the Coriolis force is proportional to the velocity (υ_r), the transversal momentum of the object acquired for a time t is proportional to the path length $(l \sim \upsilon_r t)$ of the object if the angular velocity $\Omega << 1$. Hence, the Coriolis effect is dependent on the path length that the mass object travels without being affected by any counter force against the Coriolis force.

In the atmosphere, the pressure gradient force (PGF) makes air to move (wind); however, the direction of wind is not determined only by the PGF due to the Coriolis force that makes the wind to deflect in perpendicular direction – to the right in Northern Hemisphere and to the left in Southern Hemisphere. Since the earth's angular velocity is so small as $\Omega \sim 7.27 \times 10^{-5}/\text{sec}$, the Coriolis effect is also small; thus, it can be observed easily in a large-scale air movement as in global wind patterns and tropical cyclones. Typical hurricanes (tropical cyclones) are 300 miles wide with velocity of 100-150 miles/hr. On the other hand, Dust devil [12] has a small size of vortex as tornado$(R_0 >> 1)$ with maximum velocity less than 100 miles/hr (≤ F0 tornado), but the ratio of cyclonic and anti-cyclonic patterns is almost 50-50%.

If the major driving force of tornado is come from the twisting clash of surrounding air movements (PGF) that results to form the violently rotating column of air (vortex of tornado), the probability for rotating direction of tornado should be equal (or close) in both directions -- cyclonic and anti-cyclonic directions -- because tornado is highly localized. According to the statistics, the rotating direction of tornado is in counterclockwise in the northern hemisphere and in clockwise in the southern hemisphere (99%) [13]. From this statistical fact, it can be speculated that the mechanism of tornado formation is different from the typical tropical cyclone or dust devil.

However, Rossby number is the criterion not for the intensity of Coriolis force but for the Coriolis effect representing the amount of angular deviation from the original direction or the ratio of transversal momentum acquired by Coriolis force to the original momentum.

When the vortex of tornado is formed, Coriolis force must be big enough to overcome the wind shear effect from a turbulent surrounding air movement. To be consistent with the cyclonic propensity of tornado, it can be questioned whether a driving force is inside the vortex of tornado or not.

2.3 Vortex Size

The vortex size of tornado can be estimated from cyclostrophic balance of horizontal pressure gradient and centrifugal forces. In the cyclostrophic balance, the vortex size has a correlation with the swirling wind velocity around the vortex. Then, tornado intensity also should be correlated to the vortex size of tornado, but it has been known that tornado intensity (maximum wind speed) has not much correlation with only the vortex size [14].

In the dissipation stage of tornado, the funnel is shrunk, detached from the ground, and dangled from the cloud base like a rope before it disappears. In many cases, if the tornado strength is weakening, the funnel is appeared as like the trunk of an elephant. If the vortex formation in tornado is come from PGF (pressure gradient force) in the atmosphere, the size of vortex in the dissipation stage should not be shrunken in the view of the cyclostrophic balance.

2.4 Traveling Path and Direction

Tornado can travel with its funnel cloud that is attached or close to the ground, and it has been known that the traveling direction of tornado is in the same direction as its parent cloud system moves. Even though it is not always true, tornadoes in tornado alley, USA, for example, move to north-east or south-east (from south-west to north-east, in general) [14].

In considering that the physical size of tornado system is much smaller than the parent storm cloud system and that some tornadoes move much faster than the parent clouds, it is so hard to think that the parent cloud system affects the tornado traveling and vice versa. It is better to suppose that a pre-existing condition affects both, the parent cloud system and the tornado, in their movements.

2.5 Geological Significance

From April 3 to 4 in 1974, one storm system spawned 148 tornadoes spanned 13 states in U.S.A. (the super outbreak of tornadoes). Considering that the meteorological prediction of tornado is successful with ~25% or less even at now, the meteorological thermodynamics is definitely not enough to predict the tornado outbreak. There should be more critical conditions for the tornado outbreak than the atmospheric proper conditions for tornado. There is another strange fact; on the same day for three straight years, May 20 in 1916, 1917, and 1918, tornadoes made the passages through the township of Codell, Kansas [15]. By accident or inevitable consequence it is not easy to figure out the coincidences; however, there should be a scientific reason that we don't know or have not confirmed yet.

There are significant geological dependencies among tornado occurrence, lightning, and agricultural area in the world. Not only is tornado occurred in the lightning area, but it is also concentrated on the agricultural area in the world [16]. Although a storm cloud, frequently accompanied by lightning, is needed for tornado occurrence, definitely it is not a sufficient condition because the strong lightning activities in central Africa are not followed by the tornado occurrences. There should be another ingredient factor that might be found in the ground conditions and lightning activities in the agricultural area.

It has been known that the lightning activity is increased when tropical cyclone approaches land and later the tropical cyclone spawns tornadoes [17]. The number of CG (cloud-to-ground) lightning strikes is increased before tornado touchdown and decreased as tornado is touching down; positive CG lightning strikes are also increased but decreased rapidly after tornado touchdown. Besides, the lightning spots on ground are concentrated on the spot (or nearby) of tornado touchdown and the tornado path [18].

There is a strong correlation between the tornado occurrences and lightning activities in global distributions, in which both cases are quite rare at deep sea, Arctic region, and Antarctica region. The necessary ingredients of lightning activities should be a storm cloud and charge separation in the storm cloud. Tornado also occurs with a storm cloud, but the storm cloud does not always spawn tornado. On the other hand, it has been reported that the frequency of positive lightning is anomalously increased before or after tornado occurrences [19]. Probably, there should be a connection between tornado and CG lightning activity. From these facts, another ingredient is suggested for tornado formation, which should be related with lightning activity on land and, especially, electrical properties of the ground.

In the North American Continent of United States, the geological significance of tornado occurrences was studied by S. Mori [20] with suggesting, so called, “Fuel Cell” Model. However, the detail mechanism how tornadoes occur on the ground surface over the petroleum/natural gas deposits has not been substantiated yet.

3. Conditions for Tornado Formation

From the phenomenological features of tornado in Section (2) it is inferred that there should be a driving force that is not related to the atmospheric thermodynamics but correlated to the lightning in the atmosphere.

Considered that lightning activity in the atmosphere is a natural process to minimize the electric potential energy between a storm cloud and the ground, it is supposed that tornado is also a similar natural process as the lightning discharge in the atmosphere that reduce the electric potential energy. However, the electrical discharging mechanism of tornado should be different from the lightning process if tornado lifetime is compared with the duration of lightning discharge. In addition, the electric interaction, presumably, inside the tornado vortex should result in the atmospheric vortex formation and the strong winds inside the vortex.

The driving force of tornado should be in the vortex itself in which the driving force generates a strong updraft; then, PGF (pressure gradient force) is produced for surrounding air. If the origin of the tornado formation can be found in the atmospheric thermodynamics, only possible way to produce the strong updraft should be a thermal convection in the atmosphere with intense lightning [21]; however, it has been known that the thermal convection mechanism is not possible for the tornado formation.

Then, the impetus to generate a strong updraft in the vortex of tornado should be originated from electromagnetic force or gravitational force. However, for this temporal and dynamic situation of tornado, the possibility of gravitational anomaly is excluded because it is so small, if any, being compared with electromagnetic interaction.

3.1 Electric Equilibrium in Atmosphere and Underground

As mentioned before, electric charge distributions of inside the earth, atmosphere, and ionosphere keep being changed for an electric equilibrium state. Owing to the atmospheric electrical conductivity, the conduction current density between the earth's atmosphere and the ground surface has been known as; for example, $j_z \sim 10^{-12}\ \mathrm{A \cdot m^{-2}}$ under fair weather conditions in which the positive direction of current density is earthward. If the electrical conductivity in air nearby the ground surface is changed, correspondingly the current density is also changed. If the natural DC electric field intensity just above the ground surface is E_z, the air-earth conduction current density is given as $j_z = \sigma E_z$. As long as $E_z > 0$ and $\sigma > 0$, there should be excess negative charges (negatively ionized particles or electrons) being generated just above the ground surface, while some positively ionized particles are neutralized again on the ground surface.

In damp and cloudy weather conditions, the electrical conductivity in the atmosphere is smaller than in fair weather because of the dielectric property of moisture in the air. If the electric potential between a cloud and the ground is higher than the dielectric breakdown voltage, electric discharges (lightning) occur, which is transferring a considerable amount of charges for millisecond order of time interval ($\sim 10^0 - 10^2$ C for $\sim 10^0 - 10^2$ msec).

Although the detail mechanism of lightning discharge is still not clear, it has been known as following [22]: Conducting passages– lightning channels – are built by stepped-leaders from inside the cloud and a streamer from the ground, in which electric charges are getting converged into the both ends of the conducting passages. As coming down from the cloud, the stepped-leader meets the streamer raised up from the ground with the other polarity. Then, the electric discharge is occurred when the electric potential difference on both ends of the conducting passages is higher than the breakdown voltage in air.

Therefore, in CG (cloud-to-ground) lightning process the charge accumulation is a necessary condition in the cloud and the counter part (the earth's surface). If it is assumed that the electrical conductivity in cloud is relatively more uniform than the ground, the non-uniformity of electrical conductivity in the ground (the earth's surface) should be an important factor to initiate the charge accumulation on both sides. As a reference, the frequency ratio of CG to IC (intra-cloud) lightning in land is variable and dependent on geological situations [23].

Lightning between a storm cloud and the ground is a sudden electric discharge to transfer electric charges ($-$CG, $+$CG, $\pm$bipolar, etc.) to the ground and/or to the cloud, which makes both sides to reduce excess charges. In lightning activities of tropical cyclone, it is known that the ratio of positive CG to negative CG lightning frequencies is distinctively different at sea and in land; however, the ratio of negative CG to positive CG in land is about 90% in general, and that the positive CG occurs frequently in winter season and mountain area. Also, it has been reported that pre-dominant positive lightning, negative lightning, and polarity changes of the lightning are preceded tornado formation in supercell thunderstorm. For the prevailing CG lightning activity, there should be a reason in general.

3.2 Charge Polarization in Storm Clouds and Electrostatic Induction

There have been many theories to explain cloud polarization, which is based on the microphysical process inside the cloud. Nonetheless, it seems to be still in a debate among scholars. The lightning activities on the jovian planets and, recently, in the atmosphere on Venus and inside a dust devil on the surface of Mars have been detected in spite that the atmospheric conditions of those planets are very different from the one of the Earth.

Alternatively, it can be speculated that the cloud polarization is produced by external charge source and the electrostatic induction between the storm cloud and the ground as following: For the formation of a thunderstorm cloud (cumulonimbus), there should be an updraft (convection in low pressure region) and convergence of air (winds) at the surface toward the updraft region in the developing stage (towering cumulus stage) that is taken about 10 min. Since the converging air at the surface sweep ground surface, the electrical conductivity (EC) in the air is increased due to the triboelectricity in the windy condition. This means, the converging air in the developing stage contains excess negative charges; the excess negative charges are converged into the updraft region and accumulated into the cumulus cloud. Meanwhile, the induced counter electric charge distribution should be expected on the ground against the localized cloud charge accumulation because the EC of earth (surface and underground) is much higher than in the atmosphere. Then, negative charges should be induced to the bottom of cumulus cloud, while positive charges should go upward in the cloud.

3.3 Crustal Electrical Conductivity Anomaly

Telluric current is an electric current observed in the earth's crust and mantle. The cause of telluric currents has been known to be either electromagnetic induction by the time-varying geomagnetic field or water movement across the permanent geomagnetic field [24]. However, the electrostatic induction by the PVN charge effect of mass also should be included as one of fundamental causes of telluric currents, for instance, in tidal phenomena or in any case that gravity is being changed on the earth's surface. Moreover, the electrostatic induction by the PVN charge can affect the behavior of deep telluric currents due to the property of electrostatic interaction.

If the non-uniform electrical conductivity under the ground is ascribed to an irregular distribution of conducting materials, the electric disturbance like a thunderstorm in the atmosphere affects the electric potential distribution under the ground. On the other hand, if an electric disturbance occurs under the ground by any reason (for instance, underground discharge, mechanical movement, geothermal activity, etc.), the electric potential disturbance in the atmosphere should be accompanied.

In the study of Conductive Structures in the North-Western United States and South-west Canada by H. Porath (1969), *it was believed that the anomaly is associated with a narrow, high-conductive* *body which concentrates currents induced in a large three-dimensional region of the upper crust* [25]. In that telluric current is induced also by a lightning strike, the underground electrical condition can be surmised with lightning activities (frequency and the ratio of +CG to –CG). For instance, the geophysical characteristics in the central area of USA should be correlated to the anomalous behavior in electrical conductivity; thus, the anomalous electrical conductivity in the area should be related to the non-uniform electrical conductivity underground.

It has been known that the underground electrical conductivity or resistivity is significantly dependent on frequencies of EM waves, water content of rocks, mineral deposits, and etc. [26].

4. Tornadogenesis

From the phenomenological features of tornado, it can be inferred that the driving force of tornado should be in the vortex of itself and have an origin of electric interaction. Moreover, from the facts that lightning activities and geological significance are also connected to the tornado occurrences in the world, the ingredients for tornado touchdown can be summarized as the storm cloud and the distribution of DC electrical conductivity under the ground.

Through the earth's surface, presumably, the earth is in an electric equilibrium state between the atmosphere and inside the earth, and it is supposed that induced negative charges (negative ions and electrons) are distributed beneath the crust (Moho), which generates main geomagnetic field of the earth [4]. However, there can be many conductive passages extended from the negative charge distribution up to near the earth's surface like branches of a tree and isolated twigs. If the electrostatic equilibrium is disturbed by other than electrostatic force, such as mechanical force (PGF) or thermal convection in the atmosphere, the equilibrium is restored slowly with conduction currents through the earth's surface or sometimes violently and rapidly by electric discharging process, such as a lightning.

In facts, the charge distributions in the atmosphere and inside the earth (from Moho to the surface of the earth) keep being changed during day and night; thus, an electrostatic situation never be made. Nevertheless, an electrostatic interaction (approximately) can be supposed between atmosphere and inside the earth, at least, during the time scale of tornado events and/or lightning activities.

If the electrostatic induction mechanism is supposed among storm clouds, the earth's surface, and the earth's crust (underground), a theoretical speculation for the tornado touchdown is described as below.

4.1 Electrostatic Induction between Storm Cloud and Crustal Conducting Body

Once positive charges are induced at the ground in the region underneath a storm cloud and negative charges are induced to the cloud base, the charge accumulation process is continued on both sides with the storm cloud growing; however, the charge distribution at the ground may not be uniform.

If the increased electric potential between the storm cloud and the ground exceeds the threshold voltage (breakdown voltage) of lightning under the low conductivity of humid air, a series of ±CG lightning are followed. For instance, in negative CG lightning, the density of negative charges in the cloud is getting lower; however, the excess electrons bombarded on the ground are conducted to surrounding medium reducing electric potential between the cloud base and the ground. But, still the cloud base and the ground are unstable in the view of electric equilibrium.

If an underground conducting body is being reached near the ground surface but electrically blocked with a very low EC (electrical conductivity) layer between the conducting body and the ground surface, the charge distribution in the conducting body is also affected if the EC of the underground conducting body is much higher than in the ground surface. Especially if the storm cloud is the long-standing supercell cloud and the electrical capacitance on the ground surface is not large enough to follow the cloud charge increment, the electrostatic induction can be initiated in the conducting body.

Just after a negative CG lightning to the ground surface above the underground conducting body, the ground surface is transiently close to electric neutral state due to the excess electrons transported from the cloud while the electric potential of the underground conducting body to the ground surface and to the cloud base is increased. Then, if the electrical charge capacitance in the conducting body is large enough, the underground conducting body takes over the electrostatic induction process against the cloud charges.

Once the underground conducting body takes over the electrostatic induction process, the charge distribution on the ground surface is also affected and the electric potential between the conducting body and the cloud base is rapidly increased due to the high EC of the underground conducting body. In this unstable stage, some negative charges can be induced on the ground surface that is right above the underground conducting body because EC in the cloud is much lower than in the ground; then, against the negative charges, in turn, positive charges are induced in the cloud base; positive CG lightning is initiated on the ground surface right above the underground conducting body. After each positive CG lightning negative charges should be induced again, if possible, to the ground surface. Eventually the negative charge induction from surrounding area will be not enough to initiate another positive lightning due to the temporal depletion of negative charges in the area; instead, some negative CG lightning can be initiated in the surrounding area.

4.2 Dangling Conducting Channel and Townsend Avalanche Process

Now, if the induced negative charges on the ground surface are not enough to shield the electric field produced from the underground conducting body, positive ions nearby the ground surface and negative ions in the atmosphere start moving according to the electric field direction. At the same time, negative charges in the cloud move to the cloud base that is right above the underground conducting body. Hence, the electric field in the atmosphere is getting converged into the space between the cloud base and the ground surface right above the underground conducting body. Since the positive ion number density is dominant at PBL (planetary boundary layer) in the atmosphere, the direction of wind is also getting converged into the space between the cloud base and the ground surface.

As like the stepped-leaders in lightning process [29], a conducting channel (stepped ladder) is supposed to be coming down and getting close to the ground surface right above the underground conducting body. In this stage, it is supposed that the streamer (positive polarity), which is coming up from the ground surface and meets the stepped-leader that came down from the cloud in lightning process, is not functioning as in the lightning process, even if it is created, because the electric field source (accumulated positive charges) is situated underground (upper crust) and electrically insulated without a direct connection to the ground surface. Furthermore, at the ground surface that is directly below the conducting channel some negative charges can be induced instead of positive charges.

Hence, a lightning process is not initiated (defunct) because the conducting channel (streamer) for the lightning process is not formed. However, the conducting channel that came down from the cloud is being staggered and dangled in the atmosphere but close to the ground surface.

Meanwhile, due to the high electric field in forefront of the conducting channel, some electrons can be detached out from the conducting channel and accelerated down; then, some of them acquire the kinetic energy high enough to initiate Townsend avalanche process [30]. Here, the electric field is assumed high enough to initiate the avalanche process.

Air molecules are ionized in the avalanche process, then, the positively ionized molecules (positive ions) are accelerated in upward direction by the electric field that is highly localized between the underground conducting body and the cloud base. However, the velocity of positive ions is reached to a terminal velocity due to the viscosity in the atmosphere, while the momentum of positive ions is diffused to surrounding neutral air molecules. Meanwhile, electrons keep coming down through the conducting channel from the cloud base and maintain the avalanche process. Owing to the PGF (pressure gradient force) in the atmosphere produced by the upward moving ionized and neutral air molecules, neutral air molecules keep coming into the evacuated volume, being ionized, and moving upward. At the same time, negative ions, mainly electrons move to downward and initiate another avalanche multiplication, if possible. Atmospheric updraft is formed by the ionized and neutral air molecules under the influence of electric field.

4.3 Vortex Formation and Funnel Cloud

Since Townsend avalanche process occurs near the ground surface (near ground level) at the beginning, atmospheric PGF (pressure gradient force) is occurred near the ground surface in which the updraft starts; thus, surrounding air is attracted into the region, near the ground surface of the updraft.

Owing to the confinement of air influx at near the ground surface and the cylindrical geometry, the closer to the region where the updraft starts, the higher velocity of the influx is expected. Correspondingly, the nearer to the updraft region, the stronger Coriolis force is expected. Although the amount of influx is small at the beginning, the radial velocity near the updraft region can be high enough so that Coriolis force may overcome any possible surrounding wind shear. The effect of Coriolis force at near the updraft region is getting bigger with time elapse; soon, the centrifugal force will be dominant in the cyclostrophic balance. The cyclonic vortex formation of tornado starts near the updraft region and near the ground surface, and it develops outward in radial direction and upward in axial direction in cylindrical coordinate system.

However, the Rossby number is not appropriate as a criterion for the formation of tornado vortex: The formation of tornado vortex is initiated by the localized high velocity in the updraft and correspondingly a strong Coriolis force near the updraft region, not by the influx from surrounding atmosphere that has accumulated Coriolis effect as explained in Section 2.2.

Because of the electrical interaction among the positive charges, the charge distribution of ionized air molecules in the vortex is diffused out in radial direction; the swirling momentum of the ionized molecules is transferred to neutral air. It makes stronger swirling velocity and more pressure drop inside the vortex. However, the charge distribution inside the vortex is maintained by electric fields (external and internal) and magnetic fields created by the swirling and upward moving positive ions.

The upward momentum of positively ionized air molecules is diffused to surrounding neutral air molecules with the influx from nearby the ground. The convection effect from the influx makes the diffusion process be constricted toward inner vortex, which is close to the diffusion source distribution; then, the upward velocity of ionized molecules can be even increased because the terminal velocity ($u_\tau \equiv \mu E,\ \mu:$ mobility) of the ionized air molecules is relative to the velocity of surrounding medium under the electric field between the cloud base and the ground.

Due to the cyclostrophic balance the pressure drop occurs inside the vortex and an inner edge of the vortex is formed that is separating from major airflow in tornado. The physical properties of vortex inside the inner edge should be different from outside; however, a thermal conduction is still expected from outside, which makes a thermal convection (updraft) on the inner edge of vortex.

When the vortex formation is reached to the cloud base, a funnel cloud (visible condensation cloud) is appeared and coming down from the cloud base along the centerline of the vortex. Soon, it reaches or close to the ground; tornado vortex has been formed including the dangling conducting channel inside the funnel. Here, the vortex of tornado is like a hollow cylinder with the funnel embedded at the center of the vortex. Inside the vortex, positively ionized molecules are moving up with swirling and the maximum swirling velocity is made at the inner edge of vortex. In the mean time, electrons are moving downward through the funnel and the vortex.

Since the positively ionized air molecules are moving upward with swirling, there are two components of magnetic fields produced by the motion of the ionized molecules; one is axial component produced by the swirling motion, and the other is horizontal component produced by the upward moving. By virtue of the pinch effect [31] from the horizontal component of magnetic field and the electrostatic force from the conducting channel inside the vortex, the upward moving of positive ions is being stayed close to the funnel and getting higher, in which the mechanical force balance is supposed among the electrostatic force and Lorentz forces from the axial and horizontal components of magnetic fields in addition to the cyclostrophic balance of horizontal pressure difference and centrifugal force. However, for the movement of electrons the cyclostrophic balance can be ignored due to the small inertial mass of electron.

The electric field intensity between the ground area attached from the swirling upward column of air (tornado vortex) and the cloud base is getting weaker because of the bombardment of electrons; hence, the column of tornado vortex attached on the ground area is moving to high electric field area along the path of tornado, in which the path is a projection on the ground surface from the underground conducting body.

5. Phenomenological Expectations

The major driving force of tornado is the strong electric field that has been induced between the conducting channel which came down from the cloud (negative polarity) and the underground conducting body (positive polarity). Since the column of tornado vortex contains a bundle of ionized air molecules moving upward and electrons moving downward, a variation of magnetic field should be measured when a tornado is passing nearby. Before and after tornado touchdown, the variation of DC electric field should be distinguished a lot since the electric field is concentrated inside the vortex after tornado touchdown is made. As a reference, there have been some measurements of electric field and/or magnetic field accompanied with tornado touchdowns [27][36][37].

On the other hand, since negative charges (electrons) keep coming down to the ground, the variation of telluric currents should be measured, and the directional variation of telluric currents should be correlated with the tornado moving on its passage.

The amount of influx (neutral air coming into the tornado vortex) should be the same as the amount of updraft (positively ionized and neutral air molecules), and the amount should be dependent on the efficiency of molecular ionization through the avalanche process, the electric field inside the vortex, and viscosity in air; hence, the speed of influx air is directly connected to the driving force of tornado and its effective area on the ground.

Meanwhile, tangential velocity inside the vortex can be estimated roughly by cyclostrophic balance if the vortex size and the pressure difference are known. The horizontal pressure difference should be related to the amount of influx air and, thus, to the driving force of tornado. Therefore, the intensity of tornado should be proportional to the upward electric field intensity that is producing the driving force of tornado inside the vortex.

5.1 The Vortex of Tornado

Since the ionization process (Townsend avalanche) is highly concentrated near ground level, major influx of surrounding air should be from near ground level, too. Moreover, as shown in Fig. (1), it is supposed that the ionized air molecules are distributed nearby the inner radius $\left(R_i\right)$ of the vortex owing to the pinch effect and the electrostatic force from the conducting channel inside the vortex. The size of inner radius is expected to be dependent on the electric field intensity and the geometry of underground field source.

As a simple approximation, it can be assumed that the influx of air is uniform in a vertical range $(0 < z \le \Delta h)$ and the updraft is only in the near the vortex center of tornado $(R_i \le r \le R_i + \Delta r)$ in which positively ionized and neutral air molecules move upward in $z-$direction (cylindrical coordinates). As shown in Fig. (1), the positively ionized air molecules are confined near the inner edge of the vortex, but the electrons are relatively free from the mechanical forces – centrifugal force and PGF – to cross the edge of the vortex. In Fig. (1) and Fig. (2), it is assumed that $\Delta r / R_i < 10^{-1}$ and $\Delta h \sim 10^0$ -10^1 m.

Since the driving force of tornado is supposed at around inner edge of the vortex, the variations of pressure and tangential velocity should be sensitive for the variation of the driving force. For example, if the intensity of tornado is changed, the pressure and the tangential velocity at the inner edge of vortex should be changed first. The driving force of tornado can be defined as $F_z \sim \int_V n_p q_p E_z \, dV$, in which n_p is positive ion number density; q_p, ion charge; E_z, electric field in z-component in the volume $V\left(R_i < r < R_i + \Delta r\,;\, 0 \le z < z_c\right)$. The height limit, z_c, in the volume can be the height of the cloud base or the height at which the electric field is zero. Here, the gravitational force or thermal convection effect is supposed to be very small compared with the electric force inside the vortex.

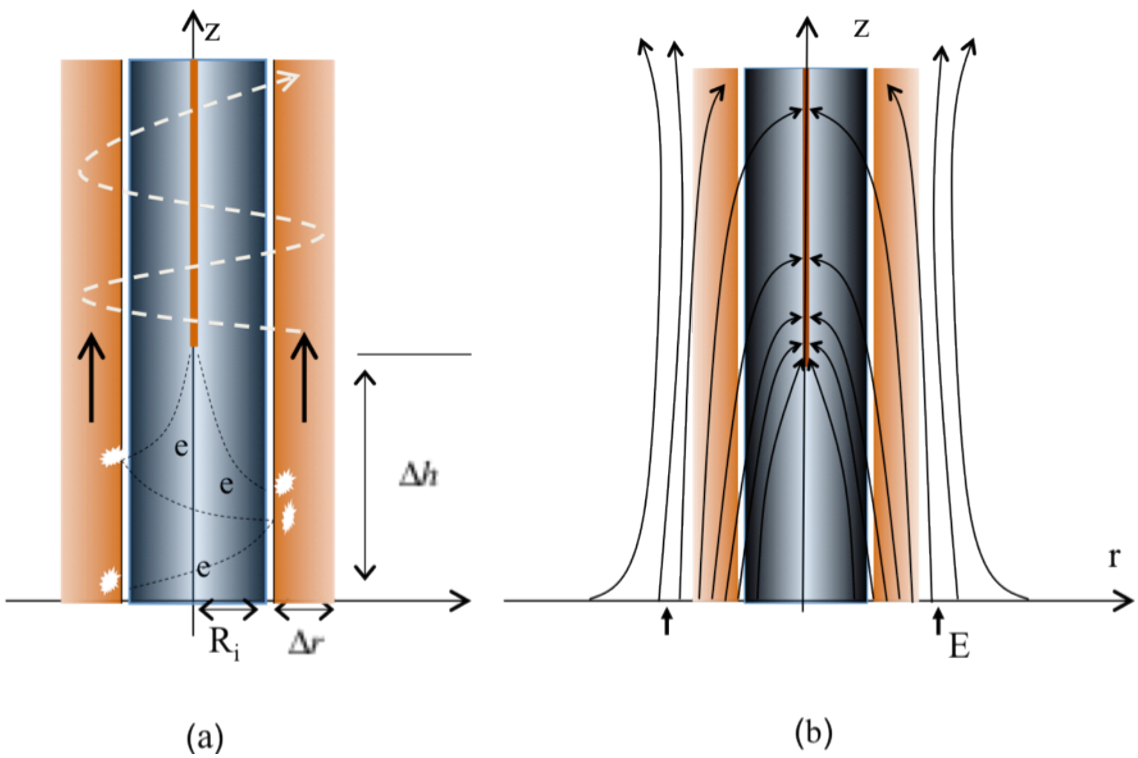

Fig. 1: (a) Townsend avalanche process and the swirling upward motion of the positively ionized air molecules inside the tornado vortex. (b) Electric fields inside the vortex with the dangling negative conducting channel (some negative charges are induced at the ground surface below the funnel cloud).

Let's think a spiral vortex in 2-D with assuming that airflow is an inviscid steady flow (viscosity, $\nu = 0$) and the density of air is constant $(\rho = c)$. Hence, energy conservation is valid, $\nabla \cdot \vec{V} = 0$ from mass conservation, and the vorticity $(\vec{\varsigma} = \nabla \times \vec{V})$ is zero since the energy is conserved along the streamlines of the inviscid flow. The spiral stream velocity is given as $V_r(r) = -Q/(2\pi r)$ and $V_\theta(r) = \kappa_0/r$, in which Q is the influx rate with dimension $[L^2t^{-1}]$ in 2-D, and κ_0 is vortex strength given as $\kappa_0 = rV_\theta$ at $r = R_0\ (R_0 \equiv R_i + \Delta r)$. The pressure and energy density per unit volume along the spiral streamlines are given as $p(r) = p_\infty - \frac{\rho}{2r^2}C$ and $E(r) = \frac{\rho}{2r^2}C$ with the constant C that is defined as $C \equiv (\kappa_0^2 + Q^2/4\pi^2)$[32].

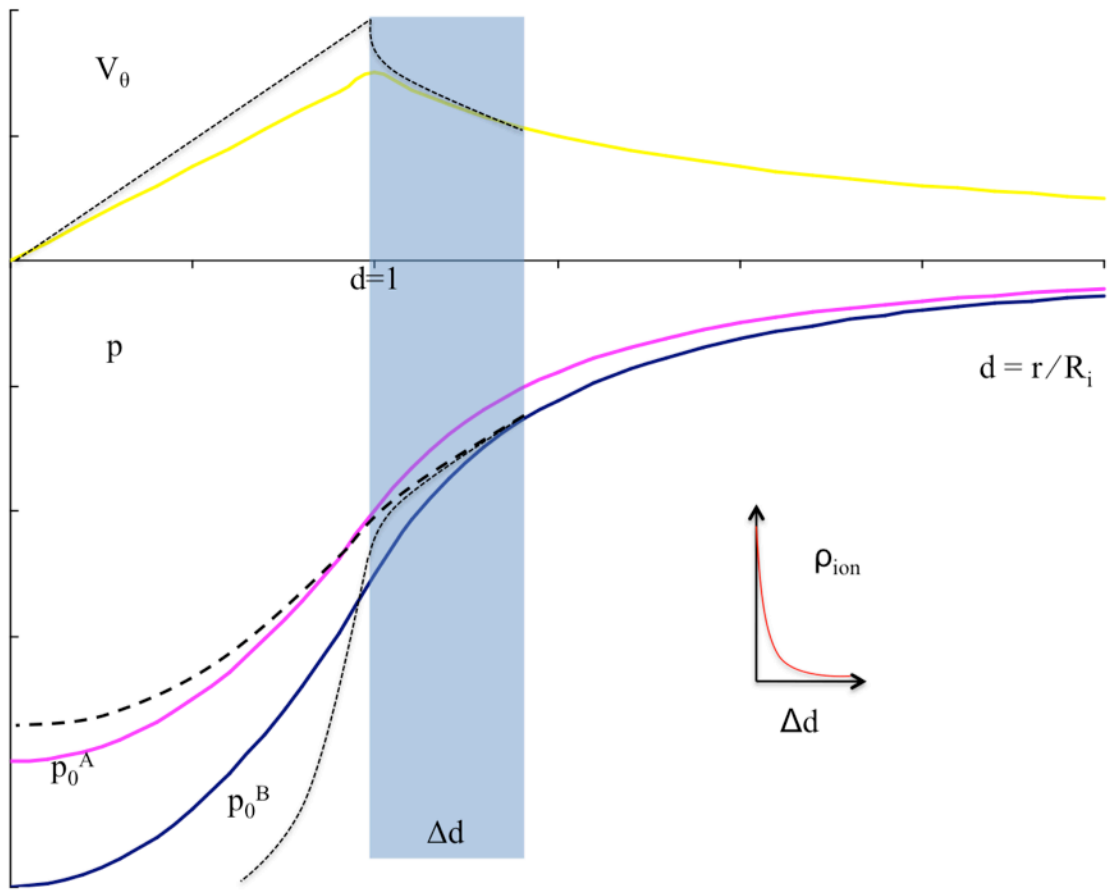

Fig. 2: Tangential velocity and pressure profiles at around the vortex of tornado (Scales are relative).

However, the viscous effects cannot be ignored nearby the vortex center of tornado due to the production of possible turbulence in the avalanche process. Thus, let's say; the streamlines go into a ring shaped turbulent vortex $(R_i \le r \le R_i + \Delta r)$, in which the updraft formation of tornado is made.

For the core region of tornado $(0 \le r \le R_i)$, a rigid vortex is supposed to combine the incoming turbulent vortex, which is similar to the Rankine's combined vortex [32] except the turbulent vortex embedded in the middle. The velocity in the rigid vortex is assumed as $V_r(r) = 0$ and $V_\theta(r) = r\omega_{\mathrm{r}}$ with vorticity $\varsigma = 2\omega_{\mathrm{r}}$. The pressure and energy density per unit volume in the rigid vortex motion are given as $p(r) = p_0 + \frac{\rho}{2}(r\omega_{\mathrm{r}})^2$ and $E(r) = \frac{\rho}{2}(r\omega_{\mathrm{r}})^2$, in which p_0 is the reference pressure at the vortex center $(r = 0)$.

In general, the kinematic energy and momentum cannot be conserved in the turbulent vortex because of the viscous effect and the possible turbulences; nevertheless, if a steady state is assumed, the mass conservation should be valid, in which $\nabla \cdot \vec{V} = 0$ if air density $\rho = \mathrm{c}$ (constant).

In the steady state, the dynamic pressure in radial direction that is $\frac{\rho}{2r^2}\left(\frac{Q^2}{4\pi^2}\right)$ in the spiral vortex should be converted to the upward $z-$direction in the region, $R_i \le r \le R_0$ $(R_0 \equiv R_i + \Delta r)$. Hence, the pressure drop in the region $(R_i \le r \le R_0)$ should be more than one from the Rankine's combined vortex model as shown in Fig. (2). Furthermore, since the pressure profiles are supposed to be connected to each other smoothly across the boundaries, the pressure profile in rigid vortex should be connected to the pressure profile in turbulent vortex at $r = R_i$ as shown with thick dashed line. In Fig. (2), $p^A(R_i)$ is the pressure profile from Rankine's combined vortex model given as

$$p^A(R_i) = p_\infty - \frac{\rho}{2R_i^2}\kappa_0^2. \tag{1}$$

On the other hand, $p^B(R_i)$ shows a limit profile if the dynamic pressure in radial direction (spiral vortex) is not disappeared in the region, $R_i < r \le R_0$;

$$p^B(R_i) = p_\infty - \frac{\rho}{2R_i^2}\left(\kappa_0^2 + \Delta^2\right) \tag{2}$$

in which $\Delta^2 = \left(Q^2/4\pi^2\right)$. Meanwhile, the pressure in the rigid vortex at $r = R_i$ is

$$p(R_i) = p_0 + \frac{\rho}{2}\left(R_i^2\omega_{\mathrm{r}}^2\right). \tag{3}$$

If the pressure and tangential velocity in the turbulent vortex are supposed to be continuous on the boundary of rigid vortex, the pressure at the center of the rigid vortex can be evaluated for two limiting cases as followings.

$$p_0{}^A = p_\infty - \rho\left(\frac{\kappa_0^2}{R_i^2}\right) \qquad \left(R_i^2\omega_{\mathrm{r}} = \kappa_0\right) \tag{4}$$

and

$$p_0{}^B = p_\infty - \rho\left(\frac{\kappa_0^2 + \Delta^2}{R_i^2}\right) \qquad \left(R_i^2\omega_{\mathrm{r}} = \sqrt{\kappa_0^2 + \Delta^2}\right). \tag{5}$$

If the energy density in the turbulent vortex region (2-D) is conserved along the streamline, at least, until the streamlines meet the driving force at $r \le R_i + \Delta r$, the tangential velocity is getting increased along the streamlines. Anyhow, the radial velocity in the streamlines should be vanished in the region of $(R_i \le r \le R_i + \Delta r)$.

If the charge distribution of ionized air molecules is electrically diffused out, the tangential velocity of the ionized air molecules is also diffused to neutral air molecules (kinematic energy gain); then, it results in a pressure drop in the region due to the cyclostrophic balance. The thin dotted line in Fig. (2) shows the example for the possible pressure drop and the increased tangential velocities in the region.

If the average updraft velocity per unit volume of air is u_z in the region of $R_i \le r \le R_i + \Delta r$ (near the ground), pressure (p) should be dependent on the u_z and R_i as $p = p(r; u_z, R_i)$ with a condition as $p(r;0,R_i) - p_\infty = 0$. As long as the airflow is in a steady state and the density of air is constant $\left(\nabla \cdot \vec{V} = 0\right)$, the updraft velocity is related to the incoming radial velocity V_r as $Q = 2\pi R_i(\Delta r/\Delta h)u_z$. Hence, for $r > R_0$;

$$V_r(r) = -R_i\left(\frac{\Delta r}{\Delta h}\right)\frac{u_z}{r}$$

$$V_\theta(r) = \sqrt{\frac{2}{\rho}\left|p(r;u_z,R_i)\right| - \left(\frac{\Delta r}{\Delta h}\right)^2\left(\frac{R_i^2}{r^2}\right)u_z^2} \qquad \left(p_\infty \equiv 0,\, p(r,u_z,R_i) \le 0\right) \tag{6}$$

in which Δr and Δh are the perpendicular limits in the direction of updraft and incoming airflows as shown in Fig. (1), respectively. $V_\theta^{\max}$ is given as

$$V_\theta^{\max}(R_i) \sim \sqrt{\frac{2}{\rho}\left|p(R_i;u_z,R_i)\right|} \tag{7}$$

in which $V_r(R_i) = 0$ at $r = R_i$.

The energy conservation in the rigid vortex cannot be assumed in tornado; instead, if some kinetic energy loss is supposed through the edge of vortex at $r \sim R_i$ even though the energy loss is compensated by downward airflow at the center of the vortex and by the thermal conduction from outside, the pressure drop at the center of the vortex should be smaller than p_0^A in Eqn. (4) that is estimated from classical Rankine's combined vortex model. It is shown in Fig. (2) that the pressure drop at the center of rigid vortex (thick dashed line) appears to be smaller than p_0^A when the profile is extrapolated from $r = R_i$ with assuming that the pressure profiles should be connected smoothly.

Hence, the maximum tangential velocity should be less than $V_\theta^{\max} = \sqrt{|\Delta p|/\rho}$ in the classical Rankine's combined vortex model, in which Δp is the pressure drop measured at the center of the vortex.

However, there should be the electric charge diffusion in radial direction; then, the diffusion of tangential momentum results in some kinetic energy gain nearby the inner edge of tornado vortex. As shown in Fig. (2), if the kinetic energy gain is considerable, a sudden increase of the tangential velocity should be appeared (thin dotted line); thus, the pressure drop (thin dotted line) at the center of vortex can be bigger than p_0^A or even p_0^B.

The pressure and tangential velocity profiles of tornado and the relation between them can be variable as shown in Fig. (2), and also it should be very sensitive to the driving force of tornado that is inside the tornado vortex not outside (PGF) [33].

5.2 Updraft Formation

The upward momentum of ionized air molecules under the strong electric field and the diffused momentum of neutral air molecules form the updraft in tornado. Therefore, besides the molecular ionization process (Townsend avalanche) in the atmosphere, the time dependence of the momentum diffusion process is another critical condition in the updraft formation of tornado. Being considered of the cyclonic propensity of tornado and the rapidness of touchdown, the formation of a localized strong updraft is necessary and the formation time of the updraft should be in a short time period after the ionization process starts.

In the updraft region $\left(R_i \le r \le R_i + \Delta r\right)$ in Fig. (2), the positively ionized air molecules are accelerated by the strong electric field; soon, the velocity is reached to a terminal velocity (u_τ) due to the viscous effect in air. The terminal velocity can be estimated by the ion mobility in the air, and the dissipated energy is transferred to surrounding neutral air molecules in their kinetic energies in $z-$direction (momentum diffusion) and thermal energies; and some of them can be ionized and/or excited through the collisions among air molecules.

Since the kinematic viscosity in air is so small as $\nu \sim 10^{-5}\ \mathrm{m^2/s}$ at $\sim 300\,\mathrm{K}$, the momentum diffusion in air is highly localized to near the momentum source. For example, if a diffusion source is supplied by a laminar flow in y-direction at boundary surface (yz plane) and the source is being diffused in positive x-direction, a characteristic length is given as $\delta \sim \sqrt{\nu t}$, in which the characteristic length is the distance from the source position $\left(x = 0\right)$ to a point where the diffusion has not been occurred yet [32].

Especially, if the convection effect is included due to the influx of neutral air near the ground $\left(z \le \Delta h \text{ in Fig. } 1\right)$, it can be supposed that all the diffused momentum is within the source distribution as shown in Eqn. (A17) and Figures in Appendix. Although the terminal velocity (u_τ) of ionized air molecules is not constant with time because u_τ is relative to the velocity of surrounding neutral air molecules, let's make a simple approximation to estimate the critical condition for the updraft formation of tornado as following: Let's assume that the density of air is constant; the electric field is constant in the space between the ground surface and the cloud base as in the case of a parallel plate capacitor with a constant voltage applied; the collisions among air molecules (neutral and ionized) are elastic, which ignores any molecular internal excitation; and the momentum diffusion to neutral air molecules is within the source distribution.

In the 2-D region $\left(R_i \le r \le R_i + \Delta r\right)$ in Fig. (2), the transferred kinetic energy to neutral air molecules from an ion charge q_{ion} has the relation as

$$\Delta p_z(r,z) + U(r,0) = \frac{1}{2} m_{\text{ion}} u_{\text{ion}}^2(r,z) + \left\langle \Delta T_{\text{neu}} \right\rangle_{\Delta V} \Delta V + U(r,z) \tag{8}$$

in which the molecular ion $\left(m_{\text{ion}}\right)$ has velocity u_{ion} in z-direction.
In Eqn. (8), $\Delta p_z(r,z) = p_z(r,0) - p(r,z)$, $U(r,z) = q_{\text{ion}} E(H - z) + m_{\text{ion}} g z$ in which E is electric field intensity, H is the height from ground $(z = 0)$ up to the cloud base, m_{ion} is ion mass $\left(m_{\text{ion}} \approx m_{\text{mol}}\right)$, g is gravitational constant, and ΔV is neutral air volume involved in the energy transferred. $\left\langle \Delta T_{\text{neu}} \right\rangle_{\Delta V}$ is the increment of average kinetic energy density of neutral air in ΔV. The pressure difference Δp_z is expected to be very small because the pressure drops sharply in the region $\left(R_i \le r \le R_i + \Delta r\right)$; hence, $q_{\text{ion}} E\, z >> \Delta p_z - m_{\text{ion}} g z$.

Once the velocity of molecular ion is reached to the terminal velocity in z-direction given as $u_\tau = \mu E$ (μ: mobility) instantaneously[2], $q_{\text{ion}} E\, z \sim \frac{1}{2} m_{\text{ion}} u_\tau^2 + \Delta V \cdot \left\langle \Delta T_{\text{neu}} \right\rangle_{\Delta V}$ in the approximation. Then, the increment of kinetic energy, $\left\langle \frac{d\left(\Delta T_{\text{neu}}\right)}{dz} \right\rangle_{\Delta V} \cdot (\Delta V) \sim q_{\text{ion}} E$ (constant in time), which is the same rate of momentum transferred per unit time as $\Delta(\text{momentum})_z / \Delta t \cong q_{\text{ion}} E$. Then, the energy loss rate to surrounding neutral air molecules in ΔV should be as $\Delta(\text{Kinetic Energy})/\Delta t \cong q_{\text{ion}} E\, u_\tau$.

Now, if net charge Q is in a steady state with the velocity u_τ and, thus, net current $I = Q\, u_\tau$ in the region $\left(R_i \le r \le R_i + \Delta r\right)$, the ΔV can be set as $\Delta V = 2\pi R_i \Delta r$ $(\Delta z \equiv 1)$. If the velocity of neutral air is reached to $\left\langle u_{\text{neu}} \right\rangle_z = U < u_\tau$ in z-direction, the elapsed time is estimated as $t \sim \Delta V \left(\rho_{\text{m}}^{\text{neu}}\, U\right) / (Q\, E)$. Since $U < u_\tau$, $t < \left(\mu \rho_{\text{m}}^{\text{neu}}\right) / \left\langle \rho_{\text{ion}} \right\rangle$, in which $\rho_{\text{m}}^{\text{neu}}$ is neutral air mass density $\left(\rho_{\text{m}}^{\text{neu}} \sim \rho_{\text{air}}\right)$, and $\left\langle \rho_{\text{ion}} \right\rangle$ is average charge density in the volume ΔV. Here, the critical time t_{crit} can be defined as

$$t_{\text{crit}} \equiv \mu \rho_{\text{air}} / \left\langle \rho_{\text{ion}} \right\rangle_{\Delta V}. \tag{9}$$

In the respect of energy conservation, there should be an increment of thermal energy in the volume ΔV. To make it simple, approximately, if only kinetics is considered among air molecules in the volume in which all particles are identical and collisions among particles are elastic, the thermal

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[2] characteristic time, $\tau = \mu \cdot m_{\text{ion}} / q_{\text{ion}} \sim 10^{-11}$ sec.

energy increment in the volume until the average velocity of neutral molecules reaches to U can be estimated as $\langle \Delta(\text{Therm}) \rangle_{\Delta V} \sim \rho_{\text{air}} U \cdot u_\tau - \frac{1}{2}\rho_{\text{air}} U^2 = \frac{1}{2}\rho_{\text{air}} U^2 \left[\frac{2u_\tau}{U} - 1\right]$, in which $0 < U < u_\tau$. The thermal energy increment is bigger than the kinetic energy increment in z-direction and it is dominant when U is much smaller than u_τ.

Since the thermal energy increment is not small, the temperature is increased inside the vortex; thus, thermal convection effect is also expected. Furthermore, the thermal conduction toward the center of the vortex is dominant at inner edge of the vortex due to the geometry; however, owing to the convection effect from surrounding atmosphere (influx), the thermal conduction effect toward the center of the vortex is bigger in the region of $z < \Delta h$ than upper region, and it can make the funnel cloud evaporated from the region.

However, there have been many observations of luminous phenomena in tornadoes [34]. As a matter of fact, those luminous phenomena can be possible if the driving force of tornado is changed but the corresponding pressure inside the vortex and/or the vortex size of tornado is not followed promptly. In that case, the ion charge density inside the vortex can be increased, or electric potential energy of electrons detached from the dangling conducting channel can be increased; then, a glow discharge can be appeared that produces luminous lights, and also it can be developed to an arc discharge with the dangling conducting channel at the center of vortex.

In the order of estimation, the critical time in Eqn. (9) should be small enough in the comparison with the time scale in the vortex formation of tornado. If the time scale is less than $\sim 10^2$ sec including the touchdown of funnel cloud, practically the updraft formation of neutral air molecules should be made in $\sim 10^0 - 10^1$ seconds; hence, $t_{\text{crit}} \sim 10^0 - 10^1$ sec., and correspondingly $\langle \rho_{\text{ion}} \rangle$ has a low limit as

$$\langle \rho_{\text{ion}} \rangle_{\text{crit}} \geq \mu \rho_{\text{air}} \cdot \left(10^0 - 10^1\right) \approx \mu \cdot \left(10^0 - 10^1\right) \left[\text{C/m}^3\right]. \tag{10}$$

For example, if the estimated current is 10A in a tornado vortex, then current density, $\rho_{\text{ion}} u_\tau = \rho_{\text{ion}} \mu E \sim \frac{10}{2\pi R_i \Delta r}\left(\text{C/m}^2 \cdot \text{sec}\right)$. If the mobility of molecular ions[3] $\mu \sim 10^{-4}\left(\text{m}^2/\text{V} \cdot \text{sec}\right)$, $\rho_{\text{air}} \sim 1.2 \left(\text{kg/m}^3\right)$, $R_i \sim 50$ m, $\Delta r \sim 2$m, and $E \sim 10^6 (\text{V/m})$; then, $\rho_{\text{ion}} \sim 1.6 \times 10^{-4} \left(\text{C/m}^3\right)$, and without including thermal convection effect, the updraft velocity can be reached to ~ 100 (m/sec) in a few seconds.

5.3 Change of Vortex Size and Intensity of Tornado

In Townsend avalanche process, in general, the number of ionized molecules (or amount of current) is very sensitive for the applied electric field. By the same token, for a small variation of electric field between the cloud base and the ground surface, the variation of charge amount is considerable

[3] $\mu \sim 1.9 \times 10^{-4}\left(\text{m}^2/\text{V} \cdot \text{sec}\right)$ in STP of air.

in the vortex of tornado. Hence, the small variation of electric field results in the sensitive variations of the amount of updraft in tornado vortex.

Since the driving force of tornado is supposed to be inside the vortex of tornado, the variation of vortex size should be dependent on the variation of the driving force – the number of ionized molecules and the electric field between the cloud base and the ground.

If the electric field E is getting weaker in Fig. (1), the updraft velocity gets lower. Then, pressure p in Fig. (2) gets higher momentary in the region $\left(R_i \le r \le R_i + \Delta r\right)$ due to the influx of air. If the pressure gets higher, the inner radius of the vortex $\left(R_i\right)$ is reduced and the Δr is increased relatively owing to the cyclostrophic balance and the flux conservation. On the other hand, if the electric field is getting stronger and the influx of air does not follow up promptly, the radius R_i is increased and the Δr is decreased relatively. However, the intensity of tornado should be dependent on the electric field intensity E, the radius of vortex R_i, and Δr in general.

In the dissipation stage of tornado, the electric field intensity gets weaker; the tornado intensity gets weaker; then, owing to the cyclostrophic balance, the vortex size get smaller from the bottom of tornado; the funnel cloud is detached from the ground and pushed up with getting thinner; finally, tornado is disappeared. On the other hand, if the electric field intensity gets stronger, tornado intensity gets stronger; then, the vortex size gets bigger [35].

If the variations of vortex size and intensity of tornado is compared with the variations of eye size in a typical tropical cyclone and its intensity, there is a distinguished feature in the dissipation stages of tornado and tropical cyclone, in which the vortex size of tornado is getting smaller; however, the vortex size of tropical cyclone should be getting bigger if the change of the vortex is initiated simply by surrounding horizontal air pressure.

5.4 Electric and Magnetic Fields

Although major impetus of tornado formation is the electric field between the cloud base and the ground surface, there are electric and magnetic forces inside the vortex of tornado, which maintain the tornado shape.

For the upward moving ions (positively ionized air molecules) and downward moving electrons, if a steady state current is assumed approximately (pseudo-steady-state), tangential magnetic field $\left(B_\theta\right)$ should be measured that is circularly symmetry about the tornado vortex ($z-$axis at the center of the vortex in cylindrical coordinates). On the other hand, the upward moving ions and neutral air molecules are spiraling around the vortex in the cyclonic direction; thus, axial magnetic field $\left(B_z\right)$ is also expected.

However, in the Townsend avalanche process, the average horizontal momentum of secondary electrons generated from the collision between an air molecule and a fast electron in the region of $z < \Delta h$ should be close to zero in the coordinate system fixed in the air molecule because the mass of air molecule is much bigger than the electron and, thus, the initial horizontal momentum of electron can be ignored in the avalanche process. Therefore, the axial magnetic field $\left(B_z\right)$ is relatively dominant in the upper region of vortex $(z > \Delta h)$. In which Δh is expected as $\Delta h \sim 10^0 - 10^1$ (m) for weak tornadoes and strong tornadoes, respectively. Fig. (3) is the schematic drawing showing the electric field distribution inside the tornado vortex, where the dangling conducting channel is embedded at the center of the vortex with the distance Δh above the ground.

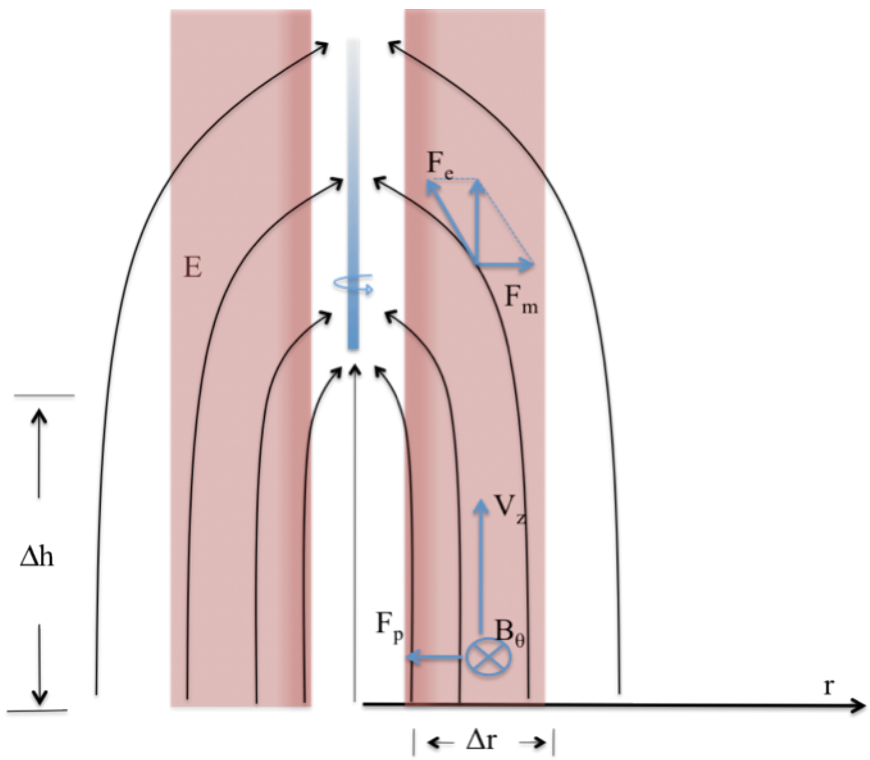

Fig. (3): Electric field distribution inside the vortex and magnetic forces on the ionized molecules

The magnetic field distribution in the vortex of tornado is similar to the one of a solenoid when electric current is applied on it, in which, practically, axial magnetic field (B_z) is inside the solenoid while tangential magnetic field (B_θ) is outside the solenoid. The tangential magnetic field (B_θ) and the upward velocity of ionized molecules produce the pinch effect $\left(F_{\mathrm{p}} \propto V_{\mathrm{z}} B_\theta\right)$ that makes the charge distribution remain in the region of Δr against the electrostatic repulsion among the ionized molecules. On the other hand, the axial magnetic field (B_z) at $z > \Delta h$ and the tangential velocity of ionized molecules (V_θ) produce the magnetic force as $F_{\mathrm{m}} \propto V_\theta B_z$ that makes the ionized molecules keep going upward.

The source electrons for the avalanche multiplication are keep coming down through the dangling conducting channel and expected to rotate in the same cyclonic direction as the positively ionized molecules do in the vortex, which makes the electrons confined in the conducting channel; thus, it makes the conducting channel stable at the center of vortex. Furthermore, it is expected that the electrons be detached easily from the conducting channel on reaching at the end of conducting channel because the strength of axial magnetic field is getting weak; thus, corresponding gyro-radius is getting bigger. However, the height (Δh) is supposed to be variable according to the electric field intensity inside the vortex, density of ionized molecules, and the size of tornado vortex.

The probabilities of charge recombination and the electrical breakdown (arc discharge) in the avalanche process are significantly reduced due to the tornado vortex geometry as shown in Fig. (1) and Fig. (3); hence, the amount of upward current can be kept increasing without the electrical breakdown.

If the electric field is measured inside the tornado vortex, the field intensity should be much smaller than the field intensity expected in a typical thunderstorm situation because the cloud and the ground are already being connected electrically through the tornado vortex. By the same reason, the electric field inside the vortex should be more sensitive than outside for lightning activities occurring outside the vortex [36].

Electric currents accompanying tornado activity had been estimated by measuring magnetic field and earth current in the vicinity of a tornado by Mark Brook in 1967 [37]. The magnetic field deflection was 15 gamma $\left(1 \text{ gamma} = 10^{-9} \text{ tesla}\right)$ that was measured at 9.6 km east from the tornado, and the cloud height H was about 6 km. The duration of the magnetic effects was estimated to be between 5 and 10 minutes. The estimated current was $I \geq 225$ A and the electric energy supplied per unit time was estimated as $W \sim 10^{10}$ (J/s).

It is impressive that when the tornado touched down, the step-like deflections on telluric currents (vertical and horizontal components) and magnetic fields (Z and H components) were appeared even at $\sim$10 km away from the tornado. The vertical deflection of telluric current can be interpreted as the Faraday induction against the horizontal magnetic field increased; the horizontal deflection of telluric current can be ascribed to the induced electric field from suddenly increased negative charges (electrons) at the ground base of tornado; the vertical deflection of magnetic field should be due to the horizontal deflection of telluric current; and the horizontal step-like deflection of magnetic field should be due to the horizontally induced telluric current toward the ground base of tornado in addition to the horizontal magnetic field produced by the current inside the vortex of tornado. The deflection angle of magnetic field (D-component in the magnetic variograph [37]) was almost zero at the time of tornado touchdown and later it was increased and then decreased. It is guessed that the tornado was stationary at the beginning and moved to north later.

For the estimated power $W \sim 10^{10}$ J/s, it was supposed that major charge carriers are electrons as in a typical lightning discharge and the electric field was assumed as $E \sim 7.4 \times 10^{3}$ V/m such as in a lightning situation. However, major charge carriers inside the vortex of tornado are supposed to be ionized molecules, the mobility of which is much smaller than electrons in the air.

The assumed model for the estimations was similar to one in a typical lightning discharge in which ground surface is assumed as a perfectly conducting plane; thus, an image current is needed. For typical lightning discharges it can be understood because the charge distribution only at the ground surface is presumably involved. On the other hand, if the electric field inside the vortex of tornado is produced from the accumulated positive charges in a conducting body that is, supposedly, at upper crust of the earth, simply a line segment current model is appropriate, as a first trial model, that is connected from the cloud base (or deeper than the cloud base) to the conducting body underground. Consequently, the estimated current can be $\sim 10^3\,\mathrm{A}$ for the 15 gamma in which the amount of transported charges is reached to the order of $\sim 10^5$ C for the 5-10 minutes.

Using the tornado geometry as shown in Fig. (1) and the current of $\sim 10^3\,\mathrm{A}$ the power can be estimated as $W = I^2 \cdot H / (2\pi\sigma_{\mathrm{air}} R_i\, \Delta r)$, in which σ_{air} is the average electrical conductivity for the molecular ions in the region of $(R_i < r < R_i + \Delta r)$ and $2\pi R_i \Delta r \sim \Delta S$ when $R_i >> \Delta r$. If the molecular ion charge density in the region $\langle \rho_{\mathrm{ion}} \rangle \geq \mu \rho_{\mathrm{air}}$ in which $\mu \sim 10^{-4}$ $\mathrm{m^2/V \cdot sec}$ and $\rho_{\mathrm{air}} \sim 1$ $\mathrm{kg/m^3}$, $\sigma_{\mathrm{air}} = \rho_{\mathrm{ion}} \mu \geq 10^{-8}$ S/m and the resistivity $1/\sigma_{\mathrm{air}} < 10^8$ $\Omega\cdot\mathrm{m}$. If the average electric field intensity in vertical direction is assumed as $E \sim 10^5 - 10^6 (\mathrm{V/m})$ inside the vortex, correspondingly, $W \sim 10^{11} - 10^{12} (\mathrm{J/s})$ and $\Delta S < 10^5$ -10^6 $\mathrm{m^2}$. If $\Delta r \sim 0.05 R_i$, it is expected that the vortex size of tornado was big. Unfortunately, more detail information – funnel size, path, etc. -- about this historical tornado cannot be found in anywhere.

5.5 Electromagnetic Radiations

Radio atmospheric signals (Sferics), usually known as radio emissions from lightning strokes, are also measured for tornadoes [38]. The sferics by tornado showed a peculiar pattern before and after the tornado touchdown, in which after the tornado touchdown the sferics by lightning is decreased while much higher frequencies was detected.

Since the positively ionized molecules are swirling around the vortex center of tornado, it is natural to expect electromagnetic radiations in VLF region. Besides the radio frequencies from the swirling positively ionized molecules, much higher frequencies are also expected in VHF and UV band regions. The radiative energy loss of electrons (*bremsstrahlung*) in Townsend avalanche process cannot be ignored even though the energy loss is so small comparative to the collisional losses (ionization and excitation) [39]. The average bremsstrahlung photon energy is quite small; therefore, most of all the radiative energy is reabsorbed by water vapor and air molecules in the atmosphere. However, some of bremsstrahlung photons can be measured in a remote detector. Moreover, from the radiative relaxation of excited molecules and from the recombination of ionized molecules UV radiations (for instance, fluorescent light) is also expected.

5.6 Telluric Currents

Since tornado is inferred as an electrical discharging process between a storm cloud and the ground, the variation of telluric currents should be measured from the beginning to the end during all stages of tornado, in which the variation should be dependent on the tornado intensity and its path on the ground. Even before the tornado touchdown the signature of impending tornado can be shown in the variation pattern of telluric currents; thus, a precise forecasting of tornado touchdown can be possible. It is interesting if the variation of telluric currents is compared with the specific variation on a weather radar, for example, hook echo that is a classical hallmark which is one of signatures for impending tornado touch down (tornado warning).

However, the telluric current accompanied by a tornado can be distinguished from lightning activities (CG) in the atmosphere. In CG lightning process the electric discharge occurs between the cloud charges and ground surface charges; thus, the variation of telluric currents should be appeared mainly in a shallow depth under the ground surface. On the other hand, tornado should affect deep telluric currents because the source of electric field is supposed to be at upper crust of the earth.

6. Discussion

There have been many hypothetical theories and thoughts [28][20] insisting that tornado formation is not originated in atmospheric thermodynamics. Although the feasibility and self-consistency of each idea should be another matter, they must have been trying to find a comprehensive explanation for tornadogenesis.

If the formation of tornado vortex is originated in the electric interactions between a storm cloud and a crustal conducting body, it is not so difficult to get, at least, qualitative explanations for tornado oddities, such as sucking up a pond/river water, animal raining, tornado hopping, etc. and also for the multiple vortices of tornado (wedge tornado) and multiple tornadoes at the same time because the main reason can be ascribed to the temporal variations of electric field inside the vortex and/or the irregular electrical conductivities under the ground. However, the geological pattern or formation of the crustal conducting bodies in Tornado Alley, USA is inferred to have been developed with the repeated weather pattern in the area and the earth's rotation for a long time.

It have been known that sometimes waterspouts (non-supercell tornadoes) and dust devils are observed nearby active volcanoes and that the variation of geoelectric and geomagnetic fields is also observed which is accompanied with volcanic and/or seismic activity. From these facts, it can be inferred; main mechanism of volcanic waterspout is also based on the electric interaction as the tornado in land, but the volcanic waterspout is initiated from the disturbance of underground electric charge distribution, which is supposed to be accompanied with volcanic eruption.

In tornado debris, frequently, toothpicks or straws are found embedded in hardwood or electric poles. Although it was proved as a fact through the experimentation [9], the detail mechanism -- how the flimsy tip can be penetrated into the hardwood without being crushed – is not known yet. Although it is supposed to be very rare case, much more weird things had been reported as "*An iron jug was blown inside out... a rooster was blown into a jug, with only its head sticking out of the neck of the container*." and "*an iron water hydrant was found full of splinters*." in Great Bend, Kansas Tornado of 1915 [8][41]. To understand these weird phenomena or, at least, to get a clue for further investigation, the ontological review of fundamental physics theories should be preceded.

The *chicken plucking* is another strange story. Although a possible explanation was found as the defeathering of a chicken is the protective response called "flight molt" [40], still it is not clear yet.

It might be interesting if the physical effect of a biological body, for instance, chicken is investigated in a DC dielectric barrier discharge.

It is supposed that the electric charge distribution at around the earth system is not independent of earth's gravity but changes on the change of gravity. In the respect of the global electrical circuit system, the underground electric charge distribution (below the earth's surface) also should be considered that is affected by and affects the gravity variations and electric charge distribution in the atmosphere. If the ionized particles (airborne dust, aerosols, etc.) are under the influence of strong DC electric fields in the atmosphere, atmospheric convection can be produced as long as the ionized particle number density is high enough. Therefore, it is also interesting if the local atmospheric events, such as dust devil, downburst, etc., are reviewed as the electric interactions with underground electric charge distribution.

The strong electric fields inside the tornado vortex with which Townsend avalanche process can be possible and the existence of underground conducting body that takes over the electrostatic induction process for the negative cloud charges, both are critical conditions for the vortex formation mechanism of tornado suggested in this paper.

Appendix

The upward moving positively ionized air molecules in tornado vortex are supposed to be not in a laminar flow but in a radial distribution from the inner edge of the vortex as shown in Fig. (2). Therefore, instead of the viscosity effect in a flow boundary, the diffusion of upward momentum should be considered with the diffusion source distribution, in general. As long as the boundary conditions and the diffusion source distribution are known, the upward momentum distribution of neutral air molecules can be estimated by using Green function method.

If momentum source distribution is known in the region interested, the equation can be given as $\vec{\nabla}\cdot\left(-D\vec{\nabla}M_i\right)+\vec{\nabla}\cdot\left(\vec{v}M_i\right)+\frac{\partial}{\partial t}M_i\left(\vec{r},t\right)=M_i^{\mathrm{S}}\left(\vec{r},t\right)$, in which D is the diffusivity in air; M_i is the *i*-component of average momentum in unit volume of air; $\vec{\nabla}\cdot\left(\vec{v}M_i\right)$, from convection of M_i; and $M_i^{\mathrm{S}}\left(\vec{r},t\right)$, the diffusible momentum source density per unit time.

If the diffusible source momentum is provided by an electric current of which the charge carriers are electrons, the effect of collisional momentum transfer (momentum diffusion) is very small since the inertial mass ratio of electron to air molecule is about 10^{-4}. Furthermore, for the same amount of currents, the amount of source momentum is also ~ 4 orders smaller than the amount from the current of ionized air molecules. Therefore, if charge carriers are electrons, thermal conduction, instead of diffusion, should be the major effect to surrounding atmosphere.

In fact, the momentum diffusion in tangential component (cylindrical coordinate system) is also expected as mentioned before. If the density of ionized air molecules keep being created nearby inner vortex of tornado, the momentum source itself should be diffused out due to the electrostatic interaction among the ionized air molecules; Since the tangential velocity of the ionized air molecules inner edge of vortex is higher than surrounding neutral air molecules due to the cyclostrophic balance, the diffusion makes a sudden increased tangential velocity and, thus, more pressure drop at inner edge of the vortex.

Because main concern is to find out whether the upward momentum diffusion can generate the strong updraft in tornado vortex or not, let's assume a simple case as following: The density of air $\left(\rho_{air}\right)$ is constant; the density of ionized air molecules and the upward velocity is independent of z; and axial symmetry in the tornado vortex is assumed. Then, upward momentum distribution in tornado vortex (*z*-direction in cylindrical coordinate system) can be described with upward velocity profile. Furthermore, if the flux conservation is considered, $\nabla\cdot\left(\vec{V}\,V_z\right)=\vec{V}\cdot\nabla V_z$ in the convection term because $\nabla\cdot\vec{V}=0$. For a simplicity, if the diffusion in $z-$direction is ignorable, $\partial^2 V_z/\partial z^2\sim 0$; then, the diffusion equation for the upward velocity profile in the vortex of tornado is expected as

$$-D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)+\left(V_r\frac{\partial V_z}{\partial r}+V_z\frac{\partial V_z}{\partial z}\right)+\frac{\partial V_z}{\partial t}=S(r,z,t). \tag{A1}$$

Here, $V_z(r,z,t)$ is the upward velocity distribution; $V_r=V_r(r,z,t)$; $S(r,z,t)=M_z^{\mathrm{S}}/\rho_{air}$. Because of the cyclostrophic balance and influx of air, the velocity diffusion should be confined from the inner edge of tornado vortex to a finite radial distance that depends on the diffusion in radial direction and the convection effect at a specific height in the vortex of tornado.

Green Function

If the convection terms in Eqn. (A1) is considered as another diffusion source and $S=S(r,t)$ that is independent of z, the green function can be described as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right)-\frac{1}{D}\frac{\partial G(r,t\,|\,r',t')}{\partial t}=-\frac{1}{2\pi Dr'}\delta(r-r')\delta(t-t') \tag{A2}$$

in polar coordinate system. Using Laplace transformation for time variable ($t\geq 0$) the Eqn. (A2) is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{G}}{\partial r}\right)-\frac{s}{D}\tilde{G}(r,s\,|\,r',t')=-\frac{e^{-st'}}{2\pi Dr'}\delta(r-r'), \tag{A3}$$

where $\tilde{G}(r,s\,|\,r',t')=\int_0^\infty G(r,t\,|\,r',t')\,e^{-st'}\,dt$; momentum source exists at r' from the time t' in which $0\leq t'\leq t$. If $r\neq r'$, the function $\tilde{G}(r,s\,|\,r',t')$ can be a linear combination of $I_0(kr)$ and $K_0(kr)$ (modified Bessel functions of zero order) as $\tilde{G}=A(r',s,t')I_0(kr)+B(r',s,t')K_0(kr)$ in which $k\equiv\sqrt{s/D}$.

Since the momentum diffusion is negligible at $r<a$ (a: inner edge of tornado vortex), the boundary conditions are given as $\partial\tilde{G}/\partial r=0$ at $r=a$ (Neumann boundary condition) and $\tilde{G}=0$ at $r\to\infty$ for $t\geq 0$, then the corresponding function $\tilde{G}_N$ should be like

$$\tilde{G}_N(r_>\,,\,r_<)=A\big[I_0(kr_<)K_1(ka)+K_0(kr_<)I_1(ka)\big]\cdot K_0(kr_>) \tag{A4}$$

because Green function should be continuous at $r=r'$ and symmetry in exchanging r and r' in which $r_>$ $(r_<)$ is the smaller (larger) of r and r'. The condition of discontinuity at $r=r'$ such as $\left.\frac{d\tilde{G}_N}{dr}\right|_{r'-\varepsilon}^{r'+\varepsilon}\neq 0$ when $\varepsilon\to 0$ in Eqn. (A3) provides the remaining coefficient A. Hence, the Laplace transformed Green function is

$$\tilde{G}_N\left(r_>,r_< \mid s,t'\right)=\frac{e^{-st'}}{2\pi D K_1(ka)}\left[I_0(kr_<)K_1(ka)+K_0(kr_<)I_1(ka)\right]\cdot K_0(kr_>) \tag{A5}$$

with the Neumann boundary condition at $r=a$. Using inverse Laplace transformation, which is given as $G(r,t \mid r',t')=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\tilde{G}(r,s \mid r',t')\,e^{st}\,ds$ where γ is an arbitrary positive real number to make the function $\tilde{G}$ to be converged in complex s-plane, the Green function is expressed as

$$\tilde{G}_N(r,t;r',t')=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} ds\,\frac{e^{s(t-t')}}{2\pi D K_1(ka)}\left[I_0(kr_<)K_1(ka)+K_0(kr_<)I_1(ka)\right]\cdot K_0(kr_>) \tag{A6}$$

where $k\equiv\sqrt{s/D}$. If a closed contour integration is chosen with a branch cut on negative real axis as

$$\begin{array}{ll}(1)\ s=\gamma-i\infty\rightarrow s=\gamma+i\infty\,; & (2)\ s=\rho e^{i\frac{\pi}{2}}\rightarrow s=\rho e^{i\pi}\ \left(|s|=\rho,\ \rho\rightarrow\infty\right);\\ (3)\ s=\rho e^{i\pi}\rightarrow s=\delta e^{i\pi}\ \left(|s|=\delta,\ \delta\rightarrow 0\right); & (4)\ s=\delta e^{i\pi}\rightarrow s=\delta e^{-i\pi};\\ (5)\ s=\delta e^{i\pi}\rightarrow s=\rho e^{i\pi}; & (6)\ s=\rho e^{-i\pi}\rightarrow s=\rho e^{-i\frac{\pi}{2}}\end{array} \tag{A7}$$

the path integration results to zero because there is no pole inside the path. If $\beta\equiv\sqrt{\rho/D}\cdot a$, then, $ka=i\beta$ and $ka=-i\beta$ on the path (3) and path (5), respectively. Hence, the Eqn. (A6) is

$$\begin{aligned}G_N(r,t;r',t')=\frac{-1}{\pi a^2}\int_0^\infty \beta\,d\beta\,\mathrm{e}^{-\beta^2 D(t-t')/a^2}\times\\ \mathrm{Im}\left\{\frac{1}{K_1(i\beta)}\left[I_0\left(i\beta\frac{r_<}{a}\right)K_1(i\beta)+K_0\left(i\beta\frac{r_<}{a}\right)I_1(i\beta)\right]\cdot K_0\left(i\beta\frac{r_>}{a}\right)\right\}\end{aligned} \tag{A8}$$

For $-\pi<\arg s\leq\frac{\pi}{2}$; $I_\nu(s)=e^{-\frac{1}{2}\nu\pi i}J_\nu(is)$ and $K_\nu(s)=\frac{\pi i}{2}e^{\frac{1}{2}\nu\pi i}H_\nu^{(1)}(is)$; $H_\nu^{(1)}(-s)=-e^{-\nu\pi i}H_\nu^{(2)}(s)$ and $J_\nu(-s)=e^{\nu\pi i}J_\nu(s)$. Using these facts, the Green function in Eqn. (A8) with Neumann boundary condition at $r=a$ can be expressed as

$$G_N(r,t;r',t')=\frac{1}{2\pi a^2}\int_0^\infty\frac{R\left(\frac{r}{a},\beta\right)R\left(\frac{r'}{a},\beta\right)}{\left[J_1(\beta)\right]^2+\left[Y_1(\beta)\right]^2}\ e^{-\beta^2 D(t-t')/a^2}\ \beta d\beta, \tag{A9}$$

in which $R(x,\beta)=J_0(\beta x)Y_1(\beta)-Y_0(\beta x)J_1(\beta)$ [42]. Once the Green function is known with the boundary conditions (Neumann B.C.) in the region interested $(a\le r<\infty)$, the diffused momentum distribution can be evaluated. Using Green's identity and the convolution theorem in Laplace transformation, the velocity distribution $V_z(r,t)$ is expressed as

$$V_z(r,t)=\int_{t'=0}^{t}dt'\int_{r'=a}^{\infty}2\pi\, r'dr'G_N(r,r';t-t')S(r',t')-2\pi Da\int_{t'=0}^{t}dt'G_N(r,r';t-t')\left(\frac{\partial V_z}{\partial r'}\right)_{r'=a}, \qquad \text{(A10)}$$

where the Green function is satisfied with Neumann boundary condition since the momentum diffusion is negligible to the region of $0\le r\le a$ and; therefore, $\left.\frac{\partial V_z}{\partial r}\right|_{r=a}=0$.

Laminar flow of momentum source

The Townsend avalanche process should be close to the inner edge of tornado vortex. If the ionized air molecules are confined in the inner edge without diffusing out in radial direction, the momentum source can be assumed like a laminar distribution at $r=a$. With Neumann boundary condition at $r=a$ the diffused velocity distribution,

$$V_z(r,t)=\int_{t'=0}^{t}dt'\int_{r'=a}^{\infty}2\pi\, r'dr'G_N(r,r';t-t')\,S(r',t'), \qquad \text{(A11)}$$

in which $S(r',t')$ can be given as $S(r',t')\sim\delta(r'-a)/(2\pi r')$, but it can have a time dependency since the terminal velocity of ionized air molecules is relative to the upward velocity of surrounding neutral air. The diffusivity D is the same as the coefficient of kinematic viscosity ν[43] that is about $1.5\times10^{-5}(m^2/s)$ at $20\ {}^{o}\mathrm{C}$, for example.

Convection and updraft formation

In the formation of tornado vortex, a localized strong updraft is a principal factor; thus, the time dependence of momentum diffusion is critical. Without including the convection term in the diffusion process, the updraft formation may not be possible in the view of tornado touchdown phenomena because the thermal energy generation inside the vortex cannot be ignored. If the convection term is included as in Eqn. (A1), then, the upward velocity distribution of neutral air can be expressed as

$$V_z(r,z,t)=V_z^{[0]}(r,t)+\int_{t'=0}^{t}dt'\int_{r'=a}^{\infty}2\pi r'dr'G_N(r,r';t-t')(-\vec{V}\cdot\nabla)V_z(r',z,t'), \qquad \text{(A12)}$$

in which $V_z^{[0]}(r,t)$ is the solution without including the convection term, and the influx velocity $\vec{V}=(V_r,V_z)$ with the condition of $\nabla\cdot\vec{V}=0$. It is not easy to evaluate the iteration process in cylindrical coordinate system, and the velocity in the convection term is not defined clearly.

To see the convection effects let's choose an extremely simple case as $V_r=-Q_0/r$ ($Q_0\geq 0$: constant) and $r_0=a$. Then, the differential equation satisfied by the Laplace transformed Green function is

$$\frac{\partial^2\tilde{G}}{\partial r^2}+\frac{(1+2\nu)}{r}\frac{\partial\tilde{G}}{\partial r}-k^2\tilde{G}(r,s\,|\,r',t')=-\frac{e^{-st'}}{2\pi Dr'}\delta(r-r'), \tag{A13}$$

in which $2\nu\equiv Q/D\geq 0$ and $k^2\equiv s/D$. If $r\neq r'$, the homogeneous solution of Eqn. (A13) is $\tilde{G}\sim r^{-\nu}\times\{I_\nu(kr),K_\nu(kr)\}$. Since the momentum diffusion is ignored at $r<a$ and the mass conservation should be valid as $\nabla\cdot\vec{V}=0$ at $r=a$, the convection boundary condition is needed as $\left[\partial\tilde{G}_C/\partial r+2\nu\tilde{G}_C/r\right]_{r=a}=0$.

$\tilde{G}_C(r,s\,|\,r',t')=\alpha\, r_<^{-\nu}r_>^{-\nu}\left[AI_\nu(kr_<)-BK_\nu(kr_<)\right]K_\nu(kr_>)$. Here, $r_>(r_<)$ is the smaller (larger) of r and r'; A and B are constants that is satisfied with the convection b. c. as following;

$$A=\left[\nu K_\nu(ka)+ka\left.\frac{dK_\nu(x)}{dx}\right|_{x=ka}\right]\text{ and }B=\left[\nu I_\nu(ka)+ka\left.\frac{dI_\nu(x)}{dx}\right|_{x=ka}\right].$$

From the discontinuity condition of $\left.\frac{d\tilde{G}_C}{dr}\right|_{r=r'-\varepsilon}^{r=r'+\varepsilon}\neq 0$ the remaining constant α is determined as $\alpha=\frac{r'^{2\nu}}{2\pi D}\frac{e^{-st'}}{A}$. Hence, the fundamental solution of Eqn. (A13) is

$$\tilde{G}_C(r,s\,|\,r',t')=\frac{e^{-st'}}{2\pi D}\left(\frac{r'^\nu}{r^\nu}\right)\left[I_\nu(kr_<)-\left(\frac{B}{A}\right)K_\nu(kr_<)\right]K_\nu(kr_>). \tag{A14}$$

Then, Green function $G_C(r,t\,|\,r',t')$ is expressed with the inverse Laplace transformation of $\tilde{G}_C$ as following:

$$G_C(r,t\,|\,r',t')=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}ds\;\frac{e^{s(t-t')}}{2\pi D}\left(\frac{r'^\nu}{r^\nu}\right)\left[I_\nu(kr_<)-\left(\frac{B}{A}\right)K_\nu(kr_<)\right]K_\nu(kr_>). \tag{A15}$$

With the contour integration in complex s-plane as in the path (A7)

$$G_C(r,t\,|\,r',t') = \frac{1}{2\pi a^2}\left(\frac{r'}{r}\right)^{\nu}\int_0^{\infty}\beta\, d\beta\; e^{-\frac{\beta^2 D}{a^2}(t-t')} \times$$

$$\mathrm{Im}\left[\frac{J_\nu\left(\beta\frac{r_<}{a}\right)\left(\nu H_\nu^{(2)}(\beta)-\frac{\beta}{2}\left[H_{\nu+1}^{(2)}(\beta)-H_{\nu-1}^{(2)}(\beta)\right]\right)-\left[\nu J_\nu(\beta)-\frac{\beta}{2}\left[J_{\nu+1}(\beta)-J_{\nu-1}(\beta)\right]\right]H_\nu^{(2)}\left(\beta\frac{r_<}{a}\right)}{\nu H_\nu^{(2)}(\beta)-\frac{\beta}{2}\left[H_{\nu+1}^{(2)}(\beta)-H_{\nu-1}^{(2)}(\beta)\right]}\right]H_\nu^{(2)}\left(\beta\frac{r_>}{a}\right). \tag{A16}$$

After a little algebra to find the imaginary part in Eqn. (16) the Green function with the convection boundary condition such as $\left[\partial G_C/\partial r + 2\nu G_C/r\right]_{r=a} = 0$ is obtained as following.

$$G_C(r,t\,|\,r',t') = \frac{1}{2\pi a^2}\left(\frac{r'}{r}\right)^{\nu}\int_0^{\infty}\frac{R_\nu\left(\frac{r_<}{a},\beta\right)R_\nu\left(\frac{r_>}{a},\beta\right)}{\left[U_\nu(\beta)\right]^2+\left[W_\nu(\beta)\right]^2}\, e^{-\frac{\beta^2 D}{a^2}(t-t')}\,\beta d\beta, \tag{A17}$$

in which $U_\nu(\beta) \equiv 2\nu J_\nu(\beta) - \beta\left[J_{\nu+1}(\beta)-J_{\nu-1}(\beta)\right]$; $W_\nu(\beta) \equiv 2\nu Y_\nu(\beta) - \beta\left[Y_{\nu+1}(\beta)-Y_{\nu-1}(\beta)\right]$; and $R_\nu(x,\beta) \equiv J_\nu(\beta x)W_\nu(\beta) - Y_\nu(\beta x)U_\nu(\beta)$. As expected, If ν is set to 0 (no convection case) in Eqn (A16), the Green function with Neumann boundary condition, $G_N(r,r';t-t')$ is obtained as in Eqn. (A9). With the convection boundary condition, $\left[\partial G_C/\partial r + 2\nu G_C/r\right]_{r=a} = 0$, the updraft velocity distribution in the vortex is given as

$$\begin{aligned} V_z(r,t) &= \int_{t'=0}^{t} dt' \int_{r'=a}^{\infty} 2\pi r' dr' G_C(r,r';t-t')S(r',t') + 2\pi Da \int_{t'=0}^{t} dt' \left\{ V_z(r',t')\left(\frac{\partial G_C}{\partial r'}\right)_{r'=a} - G_C(r,r';t-t')\left(\frac{\partial V_z}{\partial r'}\right)_{r'=a} \right\} \\ &= \int_{t'=0}^{t} dt' \int_{r'=a}^{\infty} 2\pi r' dr' G_C(r,r';t-t')S(r',t'), \end{aligned}$$

in which the surface term is vanished because $\left[\partial V_z/\partial r + 2\nu V_z/r\right]_{r=a} = 0$ is assumed as before. As shown in Eqn. (A17), the radial distribution of velocity is considerably dependent on the convection effect. For example, following figures shows the convection effect after 300 seconds with a source density as $S(r) = \delta(r-a)/2\pi r$, in which $a = 20$ m (inner edge of the vortex). For a simple computer simulation, the second convection term, $V_z\frac{\partial V_z}{\partial z}$ in Eqn. (A1) is set to zero with assuming that $\frac{\partial V_z}{\partial z} = 0$.

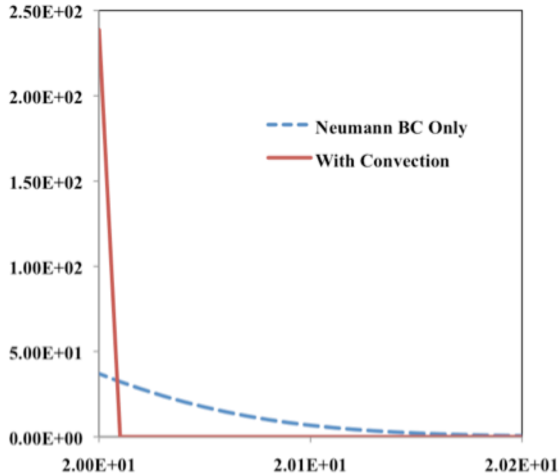

Fig. (a)

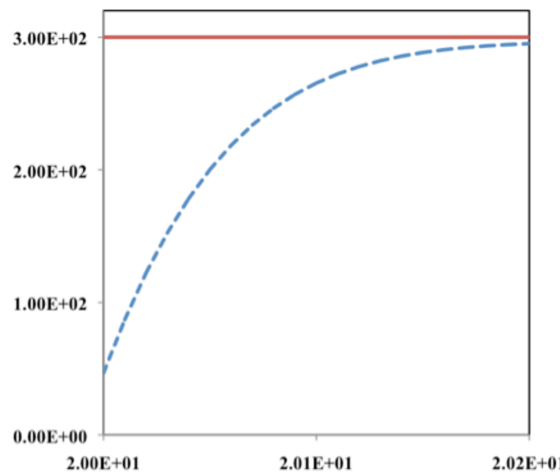

Fig. (b)

The velocity profile, $V_z(r)$ .vs. r, is shown in Fig. (a); if the convection effect is considered in the momentum diffusion process (diffusivity $\nu = 1.5\times10^{-5}$ m^2/s), the velocity profile is compressed toward the source distribution, which is shown by the solid line. On the other hand, the dashed line represents the velocity profile without including the convection effect. Fig. (b) represents accumulated velocity profiles in radial direction for both cases.

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