The Derivation of the Fine Structure Constant

By

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Let me start this paper with a short discussion of a theoretical state of matter known as the Planck conditions. These conditions are well known to physicists. They are sometimes characterized by a temperature of about 10^{32}degree K. The important thing to understand about the Planck conditions is that they are a state of matter for which there is no experimental evidence. Furthermore, the Planck conditions are defined in terms of relationships between some of the accepted constants of nature. This means that the Planck conditions were discovered by a dimensional analysis and not by measurement. As an example, one can form the relationship hc/G. All of the dimensions in this relationship cancel except for mass which appears as a square. So, we take the square root, $(hc/G)^{1/2}$ and call it the Planck mass. The Planck time, 10^{-43} sec., and the Planck Temperature,10^{32} degree K, are also discovered by the same procedures.

Even though the Planck temperature is many orders of magnitude higher than any temperature ever observed in the real universe, the Planck conditions are the starting point for many theoretical works. Does this make sense in light of the above considerations?

What I will do in this paper is to apply these same dimensional procedures to the dimensions that appear in the Gravitational force and in the Electromagnetic force. The result of this analysis will be two new variables which I will call n_3 and n_4. The rest of the paper is an analysis of what this means. I try to show that the Planck conditions are a sort of mathematical waypoint, on the path towards a non zero theoretical coldest temperature. We will also find a new way to compute the inverse fine structure constant, and a formula that extends Einstein's mass energy equation to include electromagnetism and radiation theory.

The electromagnetic force is defined:

$$e'_f = 8.94*10^{18}*(Q_1Q_2/r^2)gcm^3/sec^2coulomb^2$$

This can be interpreted in the following way. Let $Q_1=n_1q$ and $Q_2=n_2q$, where q = $1.6*10^{-19}$coulomb/ecu. Then:

$$e_f = 2.30*10^{-19}(n_1*n_2/r^2)(gcm^3/sec^2ecu^2) \quad \text{where } e = e'q^2$$

Under this interpretation, the electromagnetic force constant e has the dimensions gcm^3/sec^2ecu^2. I am primarily interested in the dimensions, call it:

$$e = 2.30*10^{-19}gcm^3/sec^2ecu^2$$

The gravitational force is defined:

$$G_f = 6.67*10^{-8}\ (m_1*m_2/r^2)\ (cm^3/gsec^2)$$

Again, I am primarily interested in the dimensions, call it:

$$G = 6.67*10^{-8} cm^3/gsec^2$$

Divide G by e $\qquad G/e = G/e*(cm^3/gsec^2)/(gcm^3/sec^2ecu^2)$

$\qquad\qquad\qquad = G/e*(ecu^2/g^2)$

Using dimensional methods, I will create a function whose dimension is the ecu.

Take the square root $\qquad = (G/e)^{1/2}*(ecu/g)$

Multiply by m(g) $\qquad = (G/e)^{1/2}*m(ecu)$

(1) Call it n_3 $\qquad n_3 = (G/e)^{1/2}*m(ecu)$

Now multiply G*e $\qquad = (G*e)*(cm^3/gsec^2)*(gcm^3/sec^2ecu^2)$

$\qquad\qquad\qquad = (G*e)*(cm^6/sec^4ecu^2)$

Invert it $\qquad = 1/(G*e)*(sec^4ecu^2/cm^6)$

Multiply by c^4 $\qquad = c^4/(G*e)*(ecu^2)/(cm^2)$

Take the square root $\qquad = c^2/(G*e)^{1/2}*(ecu/cm)$

Multiply by λ (cm) $\qquad = c^2/(G*e)^{1/2}*\lambda\ (ecu)$

(2) Call it n_4 $\qquad n_4 = c^2/(G*e)^{1/2}*\lambda\ (ecu)$

We have created another function whose dimension is the ecu.
If we now multiply n_3*n_4 we arrive at:

(3) $\qquad n_3*n_4 = mc^2*\lambda/e$

This equation is an extension of Einsteins's famous mass energy relationship. It is important because it connects our concepts of mass, energy, electromagnetism, and radiation all together in one equation. Every student of physics should be aware of it.

However, Noting that $m\lambda = h/c$, if both Einstein and Planck were right, we get:

(4) $n_3 * n_4 = hc/e$

Here $hc/e = 1/\alpha$ and α is the inverse fine structure constant, not adjusted by 2π.

I believe that this equation is a new way to arrive at the inverse fine structure constant. I have never seen n_3 and n_4 any where else in the physics literature. Some people say that this is not truly a derivation. It probably is not overly important whether it qualifies as a derivation or merely a computation. Never the less, some of the implications seem significant.

At $n_3 = n_4$, we have:

$$n_3^2 = n_4^2 = hc/e \text{ or } n_3 = n_4 = (hc/e)^{1/2,}$$

If you plug these values into equations (1) and (2) and solve for m and λ respectively, you will discover that m and λ are the Planck mass and wavelength. This means that $n_3 = n_4 = (hc/e)^{1/2}$ at the Planck scale. See table (1) where all of my analysis is summarized.

Now let $n_3 = a*(hc/e)^{1/2}$ and $n_4 = (1/a)*(hc/e)^{1/2}$ These substitutions always returns the relationship $n_3 * n_4 = hc/e$ for any “a”. This means that “a” can serve as a scaling factor when one wants to move, theoretically, from one temperature scale to another. Since “a” = 1 at the Planck scale, every theoretical state of matter can scale from there. Note also that $n_3/n_4 = a^2 = 1$ at the Planck scale.

The next step is to find theoretical scale factors that lead to a very cold state of matter. There is a term in equation (3) that seems to work. In that equation, c^2/e, links our concepts of mass, energy, electromagnetism and radiation, i.e. $n_3 * n_4/m\lambda = c^2/e$ and so I will simply choose “a” as e/c^2 and 1/a as its reciprocal in n_3 and n_4. However, in the context of scaling factors, c^2/e and its inverse need to be pure or dimensionless numbers. To say that the term c^2/e can be thought of as a dimensionless number, in this very limited context, should not pose any problem. I chose these numbers because they lead to a cold black body temperature of about 10^{-8} degree K. There might be a better choice for a theoretical non-zero coldest state of matter, but this is the one I chose. Where ever the truth lies, it can be describable by some “a” and its inverse, and it will be an “a” that will probably not be to “distant” from the one chosen here. My point is that 10^{-8} degree K is probably much “closer” to some minimum blackbody temperature than the Planck temperature,10^{32} degree K, is to some realistic hottest blackbody temperature.

Many years ago I came to the view that the Planck conditions might be a sort of mathematical waypoint on the road to a theoretical non-zero coldest temperature. That seems to be what the above analysis is telling us. Even if you don’t agree with the limit that I chose, as temperature gets closer to absolute zero, n_3 gets smaller and n_4 gets larger.

I also came to the view that what theoretical physics really needed were more realistic non zero, non infinite limits at both the cold and hot ends of the temperature spectrum. I thought of these limits as limits to blackbody temperatures and not as the smallest or largest fundamental particles that might exist in nature. Once these limits were chosen, but not necessarily agreed to by everyone, we could argue about where the experimental limits might force us to modify the theoretical limits, but the zeros and infinities that plague physics today could then be abolished. Clearly, I have not proven that the Planck conditions are never achieved in the real world. Never the less, I chose 1/k or about 10^{15} degrees K, as a potential limit at the hot end of the temperature spectrum. This is equivalent to a frequency at the high end of the gamma ray spectrum. I do not believe that any one has reported an experimental frequency higher than 1/h or about 10^{26} cycles per second.

I think that the most significant thing that falls out of the analysis found in this paper is that it has produced an equation that can be viewed as an extension of Einstein's famous mass energy relationship. It links our concepts of mass, energy, electromagnetism and radiation.

$$mc^2 = n_3 * n_4 * e/\lambda$$

	COLD	HOT	PLANCK
TIME	$(Gh/c^5)^{1/2}(c^2/e)$	h	$(Gh/c^5)^{1/2}$
t=1/f	5.26382E-04	6.62608E-27	1.35122E-43
FREQUENCY	$(c^5/hG)^{1/2}(e/c^2)$	1/h	$(c^5/hG)^{1/2}$
f=1/t	1.89976E+03	1.50919E+26	7.40073E+42
ENERGY	$(hc^5/G)^{1/2}(e/c^2)$	1	$(hc^5/G)^{1/2}$
E=hf	1.25880E-23	1	4.90378E+16
MASS	$(hc/G)^{1/2}(e/c^2)$	$1/c^2$	$(hc/G)^{1/2}$
$M=hf/c^2$	1.40060E-44	1.11265E-21	5.45621E-05
WAVELENGTH	$(hG/c^3)^{1/2}(c^2/e)$	hc	$(hG/c^3)^{1/2}$
$\lambda=ct$	1.57805E+07	1.98644E-16	4.05084E-33
TEMPERATURE	$k^{-1}(hc^5/G)^{1/2}(e/c^2)$	1/k	$k^{-1}(hc^5/G)^{1/2}$
T=E/k	9.12E-08	7.24291E+15	3.55177E+32
n_3	$(hc/e)^{1/2}(e/c^2)$	$(hc/e)^{1/2}(G/hc^5)^{1/2}$	$(hc/e)^{1/2}$
$n_3=(G/e)^{1/2}(m)$	7.53235E-39	5.98377E-16	29.34309647
n_4	$(hc/e)^{1/2}(c^2/e)$	$(hc/e)^{1/2}(hc^5/G)^{1/2}$	$(hc/e)^{1/2}$
$n_4=(c^2/(Ge)^{1/2})*\lambda$	1.14309E+41	1.43892E+18	29.34309647

TABLE 1

mc^2	1.25880E-23	1.00000E+00	4.90378E+16
n_3n_4e/λ	1.25880E-23	1.00E+00	4.90378E+16
hf	1.25880E-23	1.00000E+00	4.90378E+16
n_3/n_4	6.58945E-80	4.15851E-34	1