The Derivation of the Fine Structure Constant

By

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The electromagnetic force is defined:

$$e_f = 8.94*10^{18}*(Q_1Q_2/r^2)gcm^3/sec^2coulomb^2$$

This can be interpreted in the following way. Let $Q_1=n_1q$ and $Q_2=n_2q$, where q = $1.6*10^{-19}$coulomb/ecu. Then:

$$e_f = 2.30*10^{-19}(n_1*n_2/r^2)(gcm^3/sec^2ecu^2)$$

Under this interpretation, the electromagnetic force constant e has the dimensions $gcm^3/sec^2ecu^2$. I am primarily interested in the dimensions, call it:

$$e = 2.30*10^{-19}gcm^3/sec^2ecu^2$$

The gravitational force is defined:

$$G_f= 6.67*10^{-8} (m_1*m_2/r^2) (cm^3/gsec^2)$$

Again, I am primarily interested in the dimensions, call it:

$$G = 6.67*10^{-8}cm^3/gsec^2$$

Divide G by e $\quad G/e = G/e *(cm^3/gsec^2)/(gcm^3/sec^2ecu^2)$

$\qquad = G/e *(ecu^2/g^2)$

Using dimensional methods, I will create a function whose dimension is the ecu.

Take the square root $\quad = (G/e)^{1/2}*(ecu/g)$

Multiply by m(g) $\quad = (G/e)^{1/2}*m(ecu)$

Call it $n_3$ $\quad n_3 = (G/e)^{1/2}*m(ecu)$

Now multiply G*e $\quad = (G*e)*(cm^3/gsec^2)*(gcm^3/sec^2ecu^2)$

$\qquad = (G*e)*(cm^6/sec^4ecu^2)$

Invert it $\quad = 1/(G*e )*(sec^4ecu^2/cm^6)$

Multiply by $c^4$ $\quad = c^4/( G*e)*(ecu^2)/(cm^2)$

Take the square root $= c^2/(G*e)^{1/2}*(ecu\ /cm)$

Multiply by λ (cm) $= c^2/(G*e)^{1/2}*\lambda\ (ecu)$

Call it $n_4$ $n_4 = c^2/(G*e)^{1/2}*\lambda\ (ecu)$

We have created another function whose dimension is the ecu.

If we now multiply $n_3*n_4$ we arrive at:

$$n_3*n_4 = mc^2*\lambda/e$$

This equation is an extension of Einsteins's famous mass energy relationship. It is important because it connects our concepts of mass, energy, electromagnetism, and radiation, all together into one equation. Every student of physics should be aware of it.

However, Noting that $m\lambda = h/c$, if Einstein was right, we arrive at:

$n_3*n_4 = hc/e$ where $n_3*n_4 = 1/\alpha$ and α is the fine structure constant, not adjusted by 2π.

In this derivation I have used the same kind of dimensional analysis that was used in order to find Planck's system of natural units. The derivation vindicates Eddington's view that the fine structure constant could be derived by pure deduction.

Just a couple of notes to highlight what I think this derivation means.

First, once the speed of light, the force of gravity and the force of electromagnetism were discovered and defined as they are, $n_3$ and $n_4$ also existed as mathematical entities. They were just not discovered until now, so physicists need to deal with these relationships.

Second, $n_3 = n_4 = (hc/e)^{1/2} = 29.34$ at the Planck scale, so that $n_3$ and $n_4$ are an additional two variables that physicists need to consider when they want to attach some significance to Plancks system of natural units. Additionally, at a mass of $1.39*10^{-44}$g, we have $n_4= 1.14*10^{41}$ and $n_3= 7.56*10^{-39}$. This is the mass scale at which $n_4/n_3 = c^4/e^2$ = approximately $10^{80}$. These numbers and relationships give additional credence to Eddington's idea that the fine structure could be derived by deduction and that the numbers $10^{40}$ and $10^{80}$ have significance for physics. Physicists have developed many theories using the Planck conditions as a starting point, thinking of them as the smallest and hottest conditions possible. They might do better by looking for a smallest and coldest mass scale for their starting point.

Finally, there are many physicists who do not accept the Bohr model of the atom. I think that a major problem with our failure to find a realistic model of the atom revolves around the idea that no one knows what an electromagnetic charge is, or what one looks like. It is interesting to note that charge, "q", does not appear, in the fundamental equation:

$$mc^2 = n_3 * n_4 * e/\lambda$$

The ongoing effort to discover how the interaction of matter and radiation results in the creation of charge might get a boost with the discovery of $n_3$ and $n_4$.