Entropy and 'The Arrow of Time': *A Love Story*

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Abstract: In this short note we give a new definition to entropy and derive an interesting relationship between entropy and time. In light of this relationship, we show that The Second Law of Thermodynamics acquires a new meaning as stating that every physical process requires a lapse of time. In simple language, the Second Law says that 'everything happens in time'. This defines 'the arrow of time'. Furthermore, using Planck's Law we derive Boltzmann's Equation for Thermodynamic Entropy and obtain an equation for the number of microstates of a system.

Introduction: While traditionally entropy has been thought of as a 'measure of disorder' and The Second Law of Thermodynamics as declaring the ultimate death of the Universe, our results here show such understanding of entropy and the Second Law to be unnecessary and misleading. The relationship most relevant is that between entropy and time, not entropy and disorder. The Second Law then establishes in a formal way what is clear and obvious: that any physical process takes time.

Notation:

$$\Delta E = E(t) - E(s)$$

$$\Delta t = \tau = t - s$$

$$E_{av} = \bar{E} = \frac{1}{t-s}\int_s^t E(u)du$$

$$\eta = P = \int_s^t E(u)du$$

In a previous paper ('Planck-like' Characterization of Exponential Functions) we have proven the following mathematical characterizations. In the same referenced paper we showed that if *E(t)* is any integrable function the same characterizations hold, but as limits. By assuming exponential functions we get exact equations and avoid limit approximations. We have from this referenced paper the following mathematical results,

$$E(t) = E_0 e^{\nu t} \text{ if and only if } E(s) = \frac{\eta\nu}{e^{\eta\nu/E_{av}} - 1} \tag{1}$$

$$\text{For any integrable function } E(t),\ \lim_{t \to s} \frac{\eta\nu}{e^{\eta\nu/E_{av}} - 1} = E(s) \tag{2}$$

$$E(t) = E_0 e^{\nu t} \text{ if and only if } \Delta E = \eta\nu \tag{3}$$

$$E(t) = E_0 e^{rt} \textit{ if and only if } \frac{\Delta E}{E_{av}} = r\Delta t \tag{4}$$

In the context of Physics, *E(t)* can be thought of as the energy of a system at time *t*.

Thermodynamic Entropy and Planck's Law: In *The Interaction of Measurement* we have shown that

$$E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = \frac{\eta\, \nu}{e^{\eta\, \nu/\kappa \mathcal{T}} - 1} \qquad \text{(if } E(t) \text{ is any exponential function)} \tag{5}$$

or,

$$E_0 \approx \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} \approx \frac{\eta\, \nu}{e^{\eta\, \nu/\kappa \mathcal{T}} - 1} \qquad \text{(if } E(t) \text{ is any integrable function)} \tag{6}$$

where,

$$\eta = \int_0^{\tau} E(t)dt \ \ \textit{is the ' accumulation of E ' over a time interval } \tau$$

$$E_{av} = \frac{\eta}{\tau} \ \ \textit{is the ' average value of E ' over a time interval } \tau$$

$$\Delta E = E(\tau) - E(0) \ \textit{is the incremental ' change of E ' over a time interval } \tau$$

and we define $\mathcal{T} = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ where κ is a scalar constant. Then, $E_{av} = \kappa \mathcal{T}$

These are purely mathematical results independent of Physics. *Planck's Law* in Physics, written as $E_0 = \frac{h\nu}{e^{h\nu/kT} - 1}$, has the same exact form as the above. Note that $E_{av} = kT$ (in degrees Kelvin). Starting with *Planck's Law*, written as $E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1}$ we have that

$$e^{\Delta E/E_{av}} = 1 + \frac{\Delta E}{E_0} = \frac{E}{E_0} \ \text{ and so, } \ \frac{\Delta E}{E_{av}} = \ln\left(\frac{E}{E_0}\right) \tag{7}$$

Thermodynamic Entropy ΔS_θ can be written as $\Delta S_\theta = \frac{\Delta E}{T}$. Thus, from (7) we have that

$$\Delta S_\theta = k \cdot \ln\left(\frac{E}{E_0}\right) \tag{8}$$

We also know from *Statistical Thermodynamics* Boltzmann's Equation that

$$\Delta S_\theta = k \cdot \ln(\Omega) \tag{9}$$

where Ω is the *number of mircostates of the system*

Comparing (8) and (9), we have that.

$$\Omega = \frac{E}{E_0} \tag{10}$$

From (5) and (6) we know that if (8) were an *exact equation* (rather than a limit approximation), then it must mathematically be the case that $E(t) = E_0 e^{\nu(t-t_0)}$. We have then that

$$\Delta S_\theta = k \cdot \ln\left(\frac{E}{E_0}\right) = k \cdot \ln\left(e^{\nu \Delta t}\right) = k\nu \Delta t \qquad (11)$$

and that,

$$\Omega = e^{\nu\Delta t} \qquad (12)$$

Entropy and Time: Thermodynamic entropy ΔS_Θ is typically defined as $\Delta S_\Theta = \Delta E/T$, where ΔE is energy and T is Kelvin temperature of the system. We also have that the average energy of the system is given by $E_{av} = kT$ where k is Boltzmann's constant. In the context of Physics where $E(t)$ is the energy of a system at time t, the quantity $\Delta E/E_{av}$ in (4) above essentially *is entropy* up to the scalar constant k. Following through with this comparison and using (4) and (3) above we have, $\Delta E/E_{av} = \eta r/E_{av} = r\,\Delta t$. This reduces to $\eta/E_{av} = \Delta t$. But this is always true for any function $E(t)$ by definition of E_{av}. This suggests the following definition of entropy:

Definition: The entropy ΔS of a system with energy given by E(t) at any time t is the ratio of 'accumulation of energy' η over 'average energy' E_{av}. I. e. $\Delta S = \eta/E_{av}$.

From the above discussion we have the following interesting relationship between entropy and time.

Basic** 'Entropy vs Time' **Relationship: $\Delta S = \eta/E_{av} = \Delta t.$ (13)

We also have the following property of entropy:

Additive Property of Entropy: If a system goes from state A to state B to state C, then the change in entropy from A to B plus the change in entropy from B to C would equal to the change in entropy from A to C.

Proof: Let Δt_{AB}, Δt_{BC} and Δt_{AC} be the times going from A to B, B to C, and A to C respectively. Clearly $\Delta t_{AB} + \Delta t_{BC} = \Delta t_{AC}$. Thus $\Delta S_{AB} + \Delta S_{BC} = \Delta S_{AC}$, from the above Basic Relationship.

It is noteworthy that once again the quantity 'accumulation of energy' η naturally comes up as more 'primary' and in terms of which entropy and many other physical quantities can be defined. (see: Prime 'physis' and the Mathematical Derivation of Basic Law).

The Second Law of Thermodynamics: The Second Law simply states that in any physical process the change in entropy is positive, i.e. , $\Delta S > 0$. If we were to understand entropy as giving us a measure of randomness or disorder, the Second Law would then be interpreted to mean that the Universe is going from a more orderly state to a more random and chaotic state, and ultimately death.

The above result alters such misleading interpretation of both entropy and the Second Law. Rather, from the above *Basic Relationship* (13) we have that when $\Delta S > 0$ then also $\Delta t > 0$, and visa-versa. The Second Law of Thermodynamics would then be interpreted to mean that there must be a positive lapse of time with every physical process. This makes intuitive sense and is self evident. The Second Law of

Thermodynamics then simply specifies Δt as being positive. According to our *Basic Relationship*, Entropy and Time are intimately embraced to create 'the arrow of time'.

Notes:

- The definition of entropy ΔS given above differs from the thermodynamic entropy ΔS_Θ by a constant determined by the system. More exactly, we have $\Delta S_\Theta = rk\,\Delta S$, where k is Boltzmann's constant and r is a growth factor determined by the system. *Basic Relationship* (13) above then becomes $\Delta S_\Theta = rk\,\Delta t$. The simplicity of the above definition of entropy ΔS, along with the resulting *Relationship* (13), are compelling. This understanding of entropy comes closer to the views of entropy as 'dispersal of energy' or 'spreading of energy' that have been gaining favor with some recently. *(reference)*
- Often entropy can be thought as the measure of 'available energy' to do work. This is perfectly in harmony with our results above. Since entropy ΔS is related to the time Δt to transition from one state to another, the longer that transition takes, the less work is possible. Thus, the higher the entropy ΔS, the less 'available energy' there is to do work.
- The definition of entropy given above and the relationship between entropy and time established clearly shows the reciprocal relationship between entropy and temperature. The higher the entropy, the higher the transition time of the system, the lower the temperature of the system. It all fits harmoniously well together.

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