The New Prime theorems (191) - (240)

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that the new prime theorems (141)- (190) contain infinitely many prime solutions and no prime solutions.

School of mathematics(institute for advanced study)has long been recognized as the leading international center of research and postdoctoral training in pure mathematics.They should support the new prime theorems(1)-(240).
The New Prime theorem (191)

\[ P, jP^{302} + k - j (j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{302} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{302} + k - j (j = 1, \ldots, k - 1). \] (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \] (2)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left[ jq^{302} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \] (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \] (4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \] (5)
We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N,2) = \left| \{ P \leq N : j P^{302} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(302)^{k-1} \phi(\omega) \log^k N} \tag{6}
\]

where \( \phi(\omega) = \prod_p(1-P^{-1}) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \tag{7} \]

we prove that for \( k = 3 \), (1) contain no prime solutions

Example 2. Let \( k > 3 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{8} \]

We prove that for \( k > 3 \) (1) contain infinitely many prime solutions

The New Prime theorem (192)

\[ P, j P^{304} + k - j (j = 1, \ldots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( j P^{304} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[ P, j P^{304} + k - j (j = 1, \ldots, k-1) \tag{1} \]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_p[P - 1 - \chi(P)] \tag{2} \]

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} [j q^{304} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \tag{3} \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{4} \]

We prove that (1) contain infinitely many prime solutions.
If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{306} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi(\omega)\log^k N}
\]

where \( \phi(\omega) = \prod_p (P - 1) \).

Example 1. Let \( k = 3, 5, 17 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

(7)

We prove that for \( k = 3, 5, 17 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 17 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k \neq 3, 5, 17 \), (1) contain infinitely many prime solutions.

The New Prime theorem (193)

\[
P, jP^{306} + k - j (j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{306} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[
P, jP^{306} + k - j (j = 1, \cdots, k - 1)
\]

(1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p [P - 1 - \chi(P)]
\]

(2)

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence
\[
\prod_{j=1}^{k-1} \left[jq^{306} + k - j\right] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]  \hfill (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  \hfill (4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  \hfill (5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ \omega \leq N : jP^{306} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(306)^{k-1} \phi(\omega) \log^k N}
\]  \hfill (6)

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3, 7, 19, 103, 307 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  \hfill (7)

We prove that for \( k = 3, 7, 19, 103, 307 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 7, 19, 103, 307 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  \hfill (8)

We prove that for \( k \neq 3, 7, 19, 103, 307 \) (1) contain infinitely many prime solutions

\[P, jP^{306} + k - j (j = 1, \ldots, k - 1)\]

**The New Prime theorem (194)**

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{306} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.
contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{\omega} [P - 1 - \chi(P)] \] (2)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} [jq^{308} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P-1 \] (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \] (4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \] (5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{308} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(308)^{k-1} \phi(\omega) \log^2 N} \] (6)

where \( \phi(\omega) = \prod_{\omega} (P - 1) \).

Example 1. Let \( k = 3, 5, 23, 29 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \] (7)

We prove that for \( k = 3, 5, 23, 29 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 23, 29 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \] (8)

We prove that for \( k \neq 3, 5, 23, 29 \) (1) contain infinitely many prime solutions

The New Prime theorem (195)

\[ P, jP^{310} + k - j(j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Abstract

Using Jiang function we prove that \( jP^{310} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{310} + k - j(j = 1, \ldots, k - 1).
\]  

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[
J_2(\omega) = \prod_p [P - 1 - \chi(P)]
\]  

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{310} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]  

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N, 2) = \left| \left\{ P \leq N: jP^{310} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(310)^{k-1}\phi^k(\omega)\log^k N} \frac{N}{(310)^{k-1}\phi^k(\omega)\log^k N}
\]  

where \( \phi(\omega) = \prod_p(P - 1). \)

**Example 1.** Let \( k = 3, 11, 311 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  

We prove that for \( k = 3, 11, 311 \), (1) contain no prime solutions.

**Example 2.** Let \( k \neq 3, 11, 311 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  

We prove that for \( k \neq 3, 11, 311 \), (1) contain infinitely many prime solutions
The New Prime theorem (196)

\[ P_j, jP^{312} + k - j(j = 1, \ldots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{312} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P_j, jP^{312} + k - j(j = 1, \ldots, k-1). \tag{1} \]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \tag{2} \]

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left[ jq^{312} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \tag{3} \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{4} \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \tag{5} \]

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[ \pi_k(N, 2) = \left| \{P \leq N : jP^{312} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(312)^{k-1}\phi^k(\omega)\log^k N} N \tag{6} \]

where \( \phi(\omega) = \prod_p (P - 1). \)

Example 1. Let \( k = 3, 5, 7, 13, 53, 79, 157, 313 \). From (2) and(3) we have

\[ J_2(\omega) = 0 \tag{7} \]
We prove that for \( k = 3, 5, 7, 13, 53, 79, 157, 313 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 13, 53, 79, 157, 313 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k \neq 3, 5, 7, 13, 53, 79, 157, 313 \), (1) contain infinitely many prime solutions.

### The New Prime theorem (197)

\[
P, jP^{314} + k - j (j = 1, \ldots, k-1)
\]

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that \( jP^{314} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{314} + k - j (j = 1, \ldots, k-1),
\]

(1) contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{P} [P - 1 - \chi(P)]
\]

(2)

where \( \omega = \prod_{P} \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{314} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P-1
\]

(3)

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]
\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{314} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(314)^{k-1}\phi(\omega) \log^k N} N \] (6)

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have
\[ J_2(\omega) = 0 \] (7)
We prove that for \( k = 3 \), (1) contain no prime solutions.

Example 2. Let \( k > 3 \). From (2) and (3) we have
\[ J_2(\omega) \neq 0 \] (8)
We prove that for \( k > 3 \), (1) contain infinitely many prime solutions.

**The New Prime theorem (198)**

\[ P, jP^{316} + k - j (j = 1, \cdots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{316} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{316} + k - j (j = 1, \cdots, k - 1) \] (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]
\[ J_2(\omega) = \prod_{P} [P - 1 - \chi(P)] \] (2)

where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence
\[ \prod_{j=1}^{k-1} [jq^{316} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \] (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have
\[ J_2(\omega) \neq 0 \] (4)
We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have
\[ J_2(\omega) = 0 \] (5)
We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \{P \leq N : jP^{316} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\phi(k-1)}{(316)^{k-1}\phi(\omega)\log^k N}
\]

(6)

where \( \phi(\omega) = \prod_p (P-1) \).

Example 1. Let \( k = 3, 5, 317 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

(7)

We prove that for \( k = 3, 5, 317 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 317 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k \neq 3, 5, 317 \), (1) contain infinitely many prime solutions.

The New Prime theorem (199)

\[
P, jP^{318} + k - j (j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{318} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[
P, jP^{318} + k - j (j = 1, \cdots, k - 1),
\]

(1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{P} [P - 1 - \chi(P)]
\]

(2)

where \( \omega = \prod_P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} jq^{318} + k - j \equiv 0 \pmod{P}, q = 1, \cdots, P-1
\]

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have
We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\prod_{\omega}(N,2) = \left| \left\{ P \leq N : jP^{318} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(318)^{k-1} \phi^k(\omega) \log^k N}$$

where $\phi(\omega) = \prod_{P}(P - 1)$.

Example 1. Let $k = 3, 7, 107$. From (2) and (3) we have

$$J_2(\omega) = 0$$

We prove that for $k = 3, 7, 107$, (1) contain no prime solutions.

Example 2. Let $k \neq 3, 7, 107$. From (2) and (3) we have

$$J_2(\omega) \neq 0$$

We prove that for $k \neq 3, 7, 107$, (1) contain infinitely many prime solutions

**The New Prime theorem (200)**

$$P, jP^{320} + k - j(j = 1, \cdots, k - 1)$$

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{320} + k - j$ contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let $k$ be a given odd prime.

$$P, jP^{320} + k - j(j = 1, \cdots, k - 1),$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]
\[ J_2(\omega) = \prod_{P} [P - 1 - \chi(P)] \]  

(2)

where \( \omega = \prod_{P} P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [j q^{320} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left\lfloor \frac{N}{(320)^{k-1} \phi(\omega) \log^k N} \right\rfloor \sim \frac{J_2(\omega) \omega^{k-1} \phi(\omega) \log^k N}{(320)^{k-1} \phi^k(\omega) \log^{k+1} N}
\]

(6)

where \( \phi(\omega) = \prod_{P} (P - 1) \).

Example 1. Let \( k = 3, 5, 11, 17, 41 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(7)

We prove that for \( k = 3, 5, 11, 17, 41 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 11, 17, 41 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(8)

We prove that for \( k \neq 3, 5, 11, 17, 41 \), (1) contain infinitely many prime solutions

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**The New Prime theorem (201)**

\( P, j P^{322} + k - j (j = 1, \ldots, k - 1) \)

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract
Using Jiang function we prove that $jP^{322} + k - j$ contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let $k$ be a given odd prime.

$$P, jP^{322} + k - j (j = 1, \ldots, k - 1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)] \quad (2)$$

where $\omega = \prod P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} \left[ jq^{322} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{322} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(322)^{k-1}\phi(\omega)} \frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega) = \prod_p (P - 1)$.

**Example 1.** Let $k = 3, 47$. From (2) and (3) we have

$$J_2(\omega) = 0 \quad (7)$$

we prove that for $k = 3, 47$, (1) contain no prime solutions

**Example 2.** Let $k \neq 3, 47$. From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for $k \neq 3, 47$ (1) contain infinitely many prime solutions
The New Prime theorem (202)

\[ P, jP^{324} + k - j(j = 1, \cdots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{324} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{324} + k - j(j = 1, \cdots, k-1). \]  

(1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \]  

(2)

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left[ jP^{324} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \]  

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(5)

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[ \pi_k(N, 2) = \left[ \left\{ P \leq N : jP^{324} + k - j \text{ prime} \right\} \right] \sim \frac{J_2(\omega)\omega^{k-1}}{(324)^{k-1} \phi(\omega) \log^k N} \]  

(6)

where \( \phi(\omega) = \prod_p (P - 1). \)

Example 1. Let \( k = 3, 5, 7, 13, 19, 37, 109, 163 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(7)

We prove that for \( k = 3, 5, 7, 13, 19, 37, 109, 163 \) (1) contain no prime solutions.
Example 2. Let $k \neq 3,5,7,13,19,37,109,163$. From (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (8)$$

We prove that for $k \neq 3,5,7,13,19,37,109,163$, (1) contain infinitely many prime solutions

The New Prime theorem (203)

$$P, jP^{326} + k - j (j = 1, \cdots, k - 1)$$

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{326} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let $k$ be a given odd prime.

$$P, jP^{326} + k - j (j = 1, \cdots, k - 1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{\omega \neq P} [P - 1 - \chi(P)] \quad (2)$$

where $\omega = \prod_{p} P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jP^{326} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{ P \leq N : jP^{326} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(326)^{k-1} \phi(\omega) \log^k N} N \quad (6)$$
where \(\phi(\omega) = \Pi(P-1)\).

Example 1. Let \(k = 3\). From (2) and (3) we have

\[J_2(\omega) = 0\] (7)

We prove that for \(k = 3\), (1) contain no prime solutions.

Example 2. Let \(k > 3\). From (2) and (3) we have

\[J_2(\omega) \neq 0\] (8)

We prove that for \(k > 3\) (1) contain infinitely many prime solutions.

### The New Prime theorem (204)

\[P, jP^{328} + k - j(j = 1, \ldots, k - 1)\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that \(jP^{328} + k - j\) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \(k\) be a given odd prime.

\[P, jP^{328} + k - j(j = 1, \ldots, k - 1).\] (1)

contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

\[J_2(\omega) = \Pi[P-1-\chi(P)]\] (2)

where \(\omega = \Pi\ P\), \(\chi(P)\) is the number of solutions of congruence

\[\prod_{j=1}^{k-1} [jP^{328} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P-1\] (3)

If \(\chi(P) \leq P - 2\) then from (2) and (3) we have

\[J_2(\omega) \neq 0\] (4)

We prove that (1) contain infinitely many prime solutions.

If \(\chi(P) = P - 1\) then from (2) and (3) we have

\[J_2(\omega) = 0\] (5)

We prove that (1) contain no prime solutions [1,2]

If \(J_2(\omega) \neq 0\) then we have asymptotic formula [1,2]
\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{328} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\phi^{k-1}}{(328)^{k-1}\phi(\omega) \log^k N}
\] (6)

where \( \phi(\omega) = \prod_{P}(P - 1) \).

Example 1. Let \( k = 3, 5, 83 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\] (7)

We prove that for \( k = 3, 5, 83 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 83 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\] (8)

We prove that for \( k \neq 3, 5, 83 \) (1) contain infinitely many prime solutions.

**The New Prime theorem (205)**

\[ P, jP^{330} + k - j(j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{330} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{330} + k - j(j = 1, \ldots, k - 1) \] (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{P}[P - 1 - \chi(P)]
\] (2)

where \( \omega = \prod_{P} P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1}[jq^{330} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\] (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\] (4)

We prove that (1) contain infinitely many prime solutions.
If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{\overline{330}} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(330)^{k-1}\phi(\omega)\log^k N}
\]

(6)

where \( \phi(\omega) = \prod_{p}(P-1) \).

Example 1. Let \( k = 3, 7, 11, 23, 31, 67, 331 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

(7)

We prove that for \( k = 3, 7, 11, 23, 31, 67, 331 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 7, 11, 23, 31, 67, 331 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k \neq 3, 7, 11, 23, 31, 67, 331 \), (1) contain infinitely many prime solutions

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**The New Prime theorem (206)**

\[
P, jP^{\overline{332}} + k - j(j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{\overline{332}} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{\overline{332}} + k - j(j = 1, \cdots, k - 1)
\]

(1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_P [P - 1 - \chi(P)]
\]

(2)

where \( \omega = \prod_P \), \( \chi(P) \) is the number of solutions of congruence
\[
\prod_{j=1}^{k-1} \left[ jq^{332} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P-1 \quad (3)
\]

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (4)\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \quad (5) \]

We prove that (1) contain no prime solutions \([1,2]\).

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{332} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1} N}{(332)^{k-1} \phi(\omega) \log^k N} \quad (6)
\]

where \( \phi(\omega) = \prod_{p} (P-1) \).

Example 1. Let \( k = 3,5,167 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \quad (7) \]

We prove that for \( k = 3,5,167 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3,5,167 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (8) \]

We prove that for \( k \neq 3,5,167 \), (1) contain infinitely many prime solutions.

The New Prime theorem (207)

\[ P, jP^{334} + k - j (j = 1, \ldots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{334} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.
contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \]  

(2)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} (jq^{334} + k - j) \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \]  

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{334} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(334)^{k-1}\phi(\omega) \log^k N} \]  

(6)

where \( \phi(\omega) = \prod_p (P - 1) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(7)

We prove that for \( k = 3 \), (1) contain no prime solutions.

Example 2. Let \( k > 3 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(8)

We prove that for \( k > 3 \), (1) contain infinitely many prime solutions

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**The New Prime theorem (208)**

\[ P, jP^{336} + k - j (j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract
Using Jiang function we prove that \( jP^{336} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{336} + k - j (j = 1, \cdots, k - 1),
\]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p \left[ P - 1 - \chi(P) \right]
\]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1}jq^{336} + k - j \equiv 0 \pmod{P}, q = 1, \cdots, P - 1
\]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left\lfloor \frac{N}{jP^{336} + k - j} \right\rfloor \sim \frac{J_2(\omega)\phi^{k-1}(\omega) N}{(336)^{k-1}\phi^k(\omega) \log^k N}
\]

where \( \phi(\omega) = \prod_P (P - 1) \).

Example 1. Let \( k = 3, 5, 7, 13, 17, 29, 43, 113, 337 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that for \( k = 3, 5, 7, 13, 17, 29, 43, 113, 337 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 13, 17, 29, 43, 113, 337 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3, 5, 7, 13, 17, 29, 43, 113, 337 \), (1) contain infinitely many prime solutions.
The New Prime theorem (209)

\[ P, jP^{338} + k - j (j = 1, \cdots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{338} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{338} + k - j (j = 1, \cdots, k - 1), \]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[ J_2(\omega) = \prod[P - 1 - \chi(P)] \]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} [jq^{338} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \]

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_2(N, 2) = \left| \left\{ P \leq N : jP^{338} + k - j \text{ prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(338)^{k-1}\phi(\omega)\log^k N} \]

(6)

where \( \phi(\omega) = \prod P(P - 1) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]

(7)

We prove that for \( k = 3 \), (1) contain no prime solutions.

Example 2. Let \( k > 3 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]

(8)
We prove that for $k > 3$, (1) contain infinitely many prime solutions

**The New Prime theorem (210)**

\[ P, jP^{340} + k - j (j = 1, \ldots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{340} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{340} + k - j (j = 1, \ldots, k-1), \]

(1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[ J_2(\omega) = \prod_{P} [P - 1 - \chi(P)] \]

(2)

where \( \omega = \prod_{P} \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} [jq^{340} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P-1 \]

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]

(5)

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\]

\[ \pi_k(N,2) = \left| \{ P \leq N : jP^{340} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(340)^{k-1}} \frac{N}{\phi(\omega) \log^k N} \]

(6)

where \( \phi(\omega) = \prod_{P} (P - 1) \).

Example 1. Let \( k = 3, 5, 11 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]

(7)
We prove that for \( k = 3, 5, 11 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 11 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3, 5, 11 \), (1) contain infinitely many prime solutions.

---

**The New Prime theorem (211)**

\[
P, jP^{342} + k - j(j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that \( jP^{342} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{342} + k - j(j = 1, \cdots, k - 1).
\]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{p} [P - 1 - \chi(P)]
\]

where \( \omega = \prod P, \; \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} \left[jq^{342} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1
\]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]
\[ \pi_k(N,2) = \left| \left\{ P \leq N : jP^{344} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(342)^{k-1}\phi(\omega) \log^k N} \] 

(6)

where \( \phi(\omega) = \prod_{P}(P-1) \).

Example 1. Let \( k = 3,7,19 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \] 

(7)

we prove that for \( k = 3,7,19 \), (1) contain no prime solutions

Example 2. Let \( k \neq 3,7,19 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \] 

(8)

We prove that for \( k \neq 3,7,19 \) (1) contain infinitely many prime solutions

The New Prime theorem (212)

\[ P, jP^{344} + k - j (j = 1, \cdots, k-1) \]

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{344} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[ P, jP^{344} + k - j (j = 1, \cdots, k-1). \]

(1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{P} [P - 1 - \chi(P)] \] 

(2)

where \( \omega = \prod_{P} P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} jq^{344} + k - j \equiv 0 \mod P, q = 1, \cdots, P-1 \] 

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \] 

(4)

We prove that (1) contain infinitely many prime solutions.
If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  
(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{344} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(344)^{k-1} \phi(\omega) \log^k N}
\]  
(6)

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3, 5, 173 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  
(7)

We prove that for \( k = 3, 5, 173 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 173 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  
(8)

We prove that for \( k \neq 3, 5, 173 \), (1) contain infinitely many prime solutions

The New Prime theorem (213)

\[
P, jP^{346} + k - j (j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{346} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{346} + k - j (j = 1, \cdots, k - 1).
\]  
(1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{p} [P - 1 - \chi(P)]
\]  
(2)

where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence
\[
\prod_{j=1}^{k-1} \left[j \cdot q^{346} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P-1 \quad (3)
\]

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (4) \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \quad (5) \]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{346} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(346)^{k-1} \phi(\omega) \log^k N} \quad (6) \]

where \( \phi(\omega) = \prod_P (P-1) \).

Example 1. Let \( k = 3,347 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \quad (7) \]

We prove that for \( k = 3,347 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3,347 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (8) \]

We prove that for \( k \neq 3,347 \) (1) contain infinitely many prime solutions

The New Prime theorem (214)

\( P, jP^{348} + k - j (j = 1, \cdots, k-1) \)

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{348} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.
contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \tag{2} \]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left( jq^{348} + k - j \right) \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \tag{3} \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{4} \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \tag{5} \]

We prove that (1) contain no prime solutions [1,2].

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{348} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(348)^{k-1} \phi(\omega) \log^k N} \tag{6} \]

where \( \phi(\omega) = \prod_p (P - 1) \).

Example 1. Let \( k = 3, 5, 7, 13, 59, 349 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \tag{7} \]

We prove that for \( k = 3, 5, 7, 13, 59, 349 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 13, 59, 349 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{8} \]

We prove that for \( k \neq 3, 5, 7, 13, 59, 349 \) (1) contain infinitely many prime solutions.

\[
\text{The New Prime theorem (215)}
\]

\[
P, jP^{350} + k - j(j = 1, \ldots, k - 1)
\]

Chun-Xuan Jiang
Abstract

Using Jiang function we prove that \( jP^{350} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{350} + k - j (j = 1, \ldots, k-1).
\]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[
J_2(\omega) = \prod_{p \neq k} [P - 1 - \chi(P)]
\]

where \( \omega = \prod P \) , \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{350} + k - j] \equiv 0 \pmod{P}, q = 1, \ldots, P-1.
\]

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N,2) = \left\lfloor \frac{N}{(350)^{k-1} \phi(\omega) \log^k N} \right\rfloor
\]

where \( \phi(\omega) = \prod_{p \neq k} (P-1) \).

**Example 1.** Let \( k = 3,11,71 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that for \( k = 3,11,71 \), (1) contain no prime solutions.

**Example 2.** Let \( k \neq 3,11,71 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3,11,71 \), (1) contain infinitely many prime solutions
The New Prime theorem \((216)\)

\[ P, jP^{352} + k - j(j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \(jP^{352} + k - j\) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \(k\) be a given odd prime.

\[ P, jP^{352} + k - j(j = 1, \ldots, k - 1). \tag{1} \]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[ J_2(\omega) = \prod_p \left[ P - 1 - \chi(P) \right] \tag{2} \]

where \(\omega = \prod_p P\), \(\chi(P)\) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left( jq^{352} + k - j \right) \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \tag{3} \]

If \(\chi(P) \leq P - 2\) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{4} \]

We prove that (1) contain infinitely many prime solutions.

If \(\chi(P) = P - 1\) then from (2) and (3) we have

\[ J_2(\omega) = 0 \tag{5} \]

We prove that (1) contain no prime solutions \([1,2]\)

If \(J_2(\omega) \neq 0\) then we have asymptotic formula \([1,2]\)

\[ \pi_k(N, 2) = \left| \{ P \leq N : jP^{352} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega) \phi^{k-1}(\omega)}{(352)^{k-1} \phi^{k}(\omega) \log^k N} \frac{N}{\phi(\omega)} \tag{6} \]

where \(\phi(\omega) = \prod_p (P - 1)\).

Example 1. Let \(k = 3, 5, 17, 23, 89, 353\). From (2) and (3) we have

\[ J_2(\omega) = 0 \tag{7} \]
We prove that for \( k = 3, 5, 17, 23, 89, 353 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 17, 23, 89, 353 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3, 5, 17, 23, 89, 353 \), (1) contain infinitely many prime solutions.

---

**The New Prime theorem (217)**

\[
P, jP^{354} + k - j(j = 1, \ldots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{354} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{354} + k - j(j = 1, \ldots, k - 1),
\]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p \left[ P - 1 - \chi(P) \right]
\]

where \( \omega = \prod_p \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} jq^{354} + k - j \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]
\[
\pi_k(N,2) = \left\lfloor P \leq N : jP^{354} + k - j = \text{prime} \right\rfloor \sim \frac{J_2(\omega)\omega^{k-1}}{(354)^{k-1}\phi(\omega) \log^+ N} N
\]  

(6)

where \( \phi(\omega) = \prod_p (P-1) \).

Example 1. Let \( k = 3, 7 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

(7)

We prove that for \( k = 3, 7 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 7 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k \neq 3, 7 \), (1) contain infinitely many prime solutions

---

**The New Prime Theorem (218)**

\( P, jP^{356} + k - j (j = 1, \ldots, k-1) \)

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{356} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{356} + k - j (j = 1, \ldots, k-1),
\]

(1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p [P-1 - \chi(P)]
\]

(2)

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{356} + k - j] \equiv 0 \pmod P, q = 1, \ldots, P-1
\]

(3)

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(4)

We prove that (1) contain infinitely many prime solutions.
If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  \( (5) \)

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N, 2) = \left\lfloor \frac{J_2(\omega)\omega^{k-1}N}{(356)^{k-1}\phi(\omega)\log^k N} \right\rfloor \]  \( (6) \)

where \( \phi(\omega) = \prod (P - 1) \).

Example 1. Let \( k = 3, 5, 179 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  \( (7) \)

We prove that for \( k = 3, 5, 179 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 179 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  \( (8) \)

We prove that for \( k \neq 3, 5, 179 \), (1) contain infinitely many prime solutions

**The New Prime theorem (219)**

\[
P, jP^{358} + k - j(j = 1, \ldots, k - 1)
\]

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{358} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{358} + k - j(j = 1, \ldots, k - 1),
\]  \( (1) \)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function \([1,2]\)

\[
J_2(\omega) = \prod \left( P - 1 - \chi(P) \right)
\]  \( (2) \)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence
\[
\prod_{j=1}^{k-1} \left[j q^{358} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1
\]  
(3)

If \(\chi(P) \leq P - 2\) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  
(4)

We prove that (1) contain infinitely many prime solutions.

If \(\chi(P) = P - 1\) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  
(5)

We prove that (1) contain no prime solutions [1,2]

If \(J_2(\omega) \neq 0\) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : j P^{358} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(358)^{k-1} \phi(\omega) \log^2 N} \frac{N}{\phi(P)}
\]  
(6)

where \(\phi(\omega) = \prod_{p} (P - 1)\).

Example 1. Let \(k = 3,359\). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  
(7)

We prove that for \(k = 3,359\), (1) contain no prime solutions.

Example 2. Let \(k \neq 3,359\). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  
(8)

We prove that for \(k \neq 3,359\), (1) contain infinitely many prime solutions

**The New Prime theorem (220)**

\[P, j P^{360} + k - j (j = 1, \cdots, k - 1)\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \(j P^{360} + k - j\) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \(k\) be a given odd prime.
contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]
\[ J_2(\omega) = \prod_{p} [P - 1 - \chi(P)] \]  
(2)
where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence
\[ \prod_{j=1}^{k} \left[jq^{360} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P-1 \]  
(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have
\[ J_2(\omega) \neq 0 \]  
(4)
We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have
\[ J_2(\omega) = 0 \]  
(5)
We prove that (1) contain no prime solutions [1,2]
If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]
\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{360} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(360)^{k-1} \phi(\omega) \log^k N} \]  
(6)
where \( \phi(\omega) = \prod_{p} (P-1) \).

Example 1. Let \( k = 3, 5, 7, 11, 13, 19, 31, 37, 41, 61, 73, 181 \). From (2) and (3) we have
\[ J_2(\omega) = 0 \]  
(7)
We prove that for \( k = 3, 5, 7, 11, 13, 19, 31, 37, 41, 61, 73, 181 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 11, 13, 19, 31, 37, 41, 61, 73, 181 \). From (2) and (3) we have
\[ J_2(\omega) \neq 0 \]  
(8)
We prove that for \( k \neq 3, 5, 7, 11, 13, 19, 31, 37, 41, 61, 73, 181 \), (1) contain infinitely many prime solutions

The New Prime theorem (221)
Using Jiang function we prove that \( jP^{362} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{362} + k - j (j = 1, \ldots, k-1)
\]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{p \mid P} [P - 1 - \chi(P)]
\]

(2)

where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1}jq^{362} + k - j \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_1(N, 2) = \left| \{P \leq N : jP^{362} + k - j = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(362)^{k-1} \phi(\omega) \log^k N}
\]

(6)

where \( \phi(\omega) = \prod_{p} (P - 1) \).

**Example 1.** Let \( k = 3 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

(7)

we prove that for \( k = 3 \), (1) contain no prime solutions

**Example 2.** Let \( k > 3 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

(8)

We prove that for \( k > 3 \) (1) contain infinitely many prime solutions
The New Prime theorem (222)

\[ P, jP^{364} + k - j(j = 1, \ldots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract
Using Jiang function we prove that \( jP^{364} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[ P, jP^{364} + k - j(j = 1, \ldots, k-1), \tag{1} \]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function \[1,2]\n
\[ J_2(\omega) = \prod_{p}[P - 1 - \chi(P)] \tag{2} \]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} jq^{364} + k - j \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \tag{3} \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \tag{4} \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \tag{5} \]

We prove that (1) contain no prime solutions \[1,2\]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \[1,2\]

\[ \pi_k(N,2) = \left| \left\{ P \leq N : jP^{364} + k - j = prime \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(364)^{k-1}\phi(\omega) \log^+ N} \frac{N}{N} \tag{6} \]

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3, 5, 29, 53 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \tag{7} \]

We prove that for \( k = 3, 5, 29, 53 \) (1) contain no prime solutions.
Example 2. Let \( k \neq 3, 5, 29, 53 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  \( (8) \)

We prove that for \( k \neq 3, 5, 29, 53 \), (1) contain infinitely many prime solutions

**The New Prime theorem (223)**

\[
P, jP^{366} + k - j(j = 1, \cdots, k-1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{366} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{366} + k - j(j = 1, \cdots, k-1).
\]  \( (1) \)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p [P - 1 - \chi(P)]
\]  \( (2) \)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{366} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P-1
\]  \( (3) \)

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  \( (4) \)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  \( (5) \)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left\lfloor P \leq N : jP^{366} + k - j = \text{prime} \right\rfloor \sim \frac{J_2(\omega) \omega^{k-1}}{(366)^{k-1} \phi(\omega) \log^k N}
\]  \( (6) \)
where $\phi(\omega) = \prod_{p}(P-1)$.

Example 1. Let $k = 3, 7, 367$. From (2) and (3) we have

$$J_2(\omega) = 0$$

(7)

We prove that for $k = 3, 7, 367$, (1) contain no prime solutions.

Example 2. Let $k \neq 3, 7, 367$. From (2) and (3) we have

$$J_2(\omega) \neq 0$$

(8)

We prove that for $k \neq 3, 7, 367$ (1) contain infinitely many prime solutions.

**The New Prime theorem (224)**

$$P, jP^{368} + k - j(j = 1, \cdots, k-1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{368} + k - j$ contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let $k$ be a given odd prime.

$$P, jP^{368} + k - j(j = 1, \cdots, k-1).$$

contain infinitely many prime solutions or no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P}(P-1-\chi(P))$$

(2)

where $\omega = \prod_{p} P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} \left[ jq^{368} + k - j \right] \equiv 0 \mod P, q = 1, \cdots, P-1$$

(3)

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0$$

(4)

We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have
\[ J_2(\omega) = 0 \] (5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_1(N, 2) = \left| \left\{ P \leq N : jP^{368} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(368)^{k-1}\phi(\omega) \log^k N} \] (6)

where \( \phi(\omega) = \prod (P - 1) \).

Example 1. Let \( k = 3, 5, 17, 47 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \] (7)

We prove that for \( k = 3, 5, 17, 47 \) (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 17, 47 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \] (8)

We prove that for \( k \neq 3, 5, 17, 47 \) (1) contain infinitely many prime solutions

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**The New Prime theorem (225)**

\[ P, jP^{370} + k - j (j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{370} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{370} + k - j (j = 1, \ldots, k - 1) \]. (1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{P}[P - 1 - \chi(P)] \] (2)

where \( \omega = \prod P, \ \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} j q^{370} + k - j \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \] (3)
If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0$$  \hspace{1cm} (4) $$

We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0$$  \hspace{1cm} (5) $$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{370} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{\delta-1}}{(370)^{\delta-1} \phi(\omega) \log^4 N}$$  \hspace{1cm} (6) $$

where $\phi(\omega) = \prod_{\rho}(P - 1)$.

Example 1. Let $k = 3, 11$. From (2) and (3) we have

$$J_2(\omega) = 0$$  \hspace{1cm} (7) $$

We prove that for $k = 3, 11$, (1) contain no prime solutions.

Example 2. Let $k \neq 3, 11$. From (2) and (3) we have

$$J_2(\omega) \neq 0$$  \hspace{1cm} (8) $$

We prove that for $k \neq 3, 11$, (1) contain infinitely many prime solutions.

The New Prime theorem (226)

$$P, jP^{372} + k - j (j = 1, \ldots, k - 1)$$

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{372} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let $k$ be a given odd prime.

$$P, jP^{372} + k - j (j = 1, \ldots, k - 1).$$  \hspace{1cm} (1) $$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function \[1,2\]
\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \]  
(2)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence
\[
\prod_{j=1}^{k-1} \left(jq^{372} + k - j \right) \equiv 0 \pmod{P}, q = 1, \ldots, P - 1
\]  
(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have
\[ J_2(\omega) \neq 0 \]  
(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have
\[ J_2(\omega) = 0 \]  
(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]
\[
\pi_k(N, 2) = \left\lfloor \frac{N}{jP^{372} + k - j \text{ prime}} \right\rfloor \sim \frac{J_2(\omega) \omega^{k-1}}{(372)^{k-1} \phi(\omega) \log^k N}
\]  
(6)

where \( \phi(\omega) = \prod_p (P - 1) \).

Example 1. Let \( k = 3, 5, 7, 13, 373 \). From (2) and (3) we have
\[ J_2(\omega) = 0 \]  
(7)

We prove that for \( k = 3, 5, 7, 13, 373 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 13, 373 \). From (2) and (3) we have
\[ J_2(\omega) \neq 0 \]  
(8)

We prove that for \( k \neq 3, 5, 7, 13, 373 \), (1) contain infinitely many prime solutions

**The New Prime theorem (227)**

\( P, jP^{374} + k - j (j = 1, \ldots, k - 1) \)

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract
Using Jiang function we prove that \( jP^{374} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{374} + k - j(j = 1, \cdots, k - 1),
\]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function \([1,2]\)

\[
J_2(\omega) = \prod_p \left[ P - 1 - \chi(P) \right]
\]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{374} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1
\]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N, 2) = \left\lfloor \frac{N}{(374)^{k-1} \phi(\omega) \log^k N} \right\rfloor 
\]

where \( \phi(\omega) = \prod_p (P - 1) \).

Example 1. Let \( k = 3, 23 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that for \( k = 3, 23 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 23 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3, 23 \), (1) contain infinitely many prime solutions.
The New Prime theorem (228)

\[ P, jP^{376} + k - j (j = 1, \cdots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{376} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{376} + k - j (j = 1, \cdots, k-1), \quad (1) \]

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{P} \left[ P - 1 - \chi(P) \right] \quad (2) \]

where \( \omega = \prod_{P} P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left[ jP^{376} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P-1 \quad (3) \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (4) \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \quad (5) \]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left\lfloor N \right\rfloor \frac{J_2(\omega) \omega^{k-1}}{(376)^{k-1} \phi(\omega) \log^k N} N \quad (6) \]

where \( \phi(\omega) = \prod_{P} (P - 1) \).

Example 1. Let \( k = 3, 5 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \quad (7) \]

We prove that for \( k = 3, 5 \), (1) contain no prime solutions.

Example 2. Let \( k > 5 \). From (2) and (3) we have
We prove that for \( k > 5 \), (1) contain infinitely many prime solutions.

**The New Prime theorem (229)**

\[ P, jP^{378} + k - j (j = 1, \cdots, k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that \( jP^{378} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{378} + k - j (j = 1, \cdots, k-1), \]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2] \)

\[ J_2(\omega) = \prod_{p} [P - 1 - \chi(P)] \]  

where \( \omega = \prod_{p} \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k} [jq^{378} + k - j] \equiv 0 \mod P, q = 1, \cdots, P-1 \]  

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(5)

We prove that (1) contain no prime solutions \([1,2] \)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2] \)

\[ \pi_k(N,2) = \left| \left\{ P \leq N : jP^{378} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(378)^{k-1}\phi(\omega) \log^{k} N} \]  

(6)

where \( \phi(\omega) = \prod_{p} (P-1) \).
Example 1. Let \( k = 3, 7, 19, 23, 127, 379 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  \hspace{1cm} (7)

We prove that for \( k = 3, 7, 19, 23, 127, 379 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 7, 19, 23, 127, 379 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (8)

We prove that for \( k \neq 3, 7, 19, 23, 127, 379 \), (1) contain infinitely many prime solutions.

**The New Prime theorem (230)**

\[ p, j p^{380} + k - j (j = 1, \cdots, k - 1) \]

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

**Abstract**

Using Jiang function we prove that \( j p^{380} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ p, j p^{380} + k - j (j = 1, \cdots, k - 1), \]  \hspace{1cm} (1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[ J_2(\omega) = \prod \frac{p - 1 - \chi(P)}{p} \]  \hspace{1cm} (2)

where \( \omega = \prod p \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} (jq^{380} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, P - 1 \]  \hspace{1cm} (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  \hspace{1cm} (5)

We prove that (1) contain no prime solutions [1,2].
If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left\lfloor \frac{N}{jP^{380} + k - j = \text{prime}} \right\rfloor \sim \frac{J_2(\omega)\omega^{k-1}}{(380)^{k-1}\phi(\omega) \log^k N} \tag{6}
\]

where \( \phi(\omega) = \prod_p (P-1) \).

Example 1. Let \( k = 3, 5, 11, 191 \). From (2) and(3) we have

\[
J_2(\omega) = 0 \tag{7}
\]

We prove that for \( k = 3, 5, 11, 191 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 11, 191 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0 \tag{8}
\]

We prove that for \( k \neq 3, 5, 11, 191 \), (1) contain infinitely many prime solutions.

The New Prime theorem (231)

\[
P, jP^{382} + k - j(j = 1, \ldots, k-1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{382} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{382} + k - j(j = 1, \ldots, k-1). \tag{1}
\]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_p [P-1 - \chi(P)] \tag{2}
\]

where \( \omega = \prod_p P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} \left[jq^{382} + k - j \right] \equiv 0 \pmod{P}, q = 1, \ldots, P-1 \tag{3}
\]
If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]  

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{382} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(382)^{k-1}\phi(\omega)\log^k N}
\]  

(6)

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3,383 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]  

(7)

we prove that for \( k = 3,383 \), (1) contain no prime solutions

Example 2. Let \( k \neq 3,383 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]  

(8)

We prove that for \( k \neq 3,383 \) (1) contain infinitely many prime solutions

The New Prime theorem (232)

\[
P, jP^{384} + k - j(j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{384} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{384} + k - j(j = 1, \cdots, k - 1).
\]  

(1)

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function \([1,2]\)
\[ J_2(\omega) = \prod_P [P - 1 - \chi(P)] \] \hspace{1cm} (2)
where \(\omega = \prod_P P\), \(\chi(P)\) is the number of solutions of congruence
\[ \prod_{j=1}^{k-1} \left[ j P^{384} + k - j \right] \equiv 0 \pmod{P}, q = 1, \cdots, P-1 \] \hspace{1cm} (3)
If \(\chi(P) \leq P - 2\) then from (2) and (3) we have
\[ J_2(\omega) \neq 0 \] \hspace{1cm} (4)
We prove that (1) contain infinitely many prime solutions.
If \(\chi(P) = P - 1\) then from (2) and (3) we have
\[ J_2(\omega) = 0 \] \hspace{1cm} (5)
We prove that (1) contain no prime solutions \([1,2]\)
If \(J_2(\omega) \neq 0\) then we have asymptotic formula \([1,2]\)
\[ \pi_k(N,2) = \left| \left\{ P \leq N : j P^{384} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{(384)^{k-2} \phi(\omega) \log^k N} N \] \hspace{1cm} (6)
where \(\phi(\omega) = \prod_{P} (P - 1)\).

Example 1. Let \(k = 3, 5, 7, 13, 17, 97, 193\). From (2) and (3) we have
\[ J_2(\omega) = 0 \] \hspace{1cm} (7)
We prove that for \(k = 3, 5, 7, 13, 17, 97, 193\) (1) contain no prime solutions.

Example 2. Let \(k \neq 3, 5, 7, 13, 17, 97, 193\). From (2) and (3) we have
\[ J_2(\omega) \neq 0 \] \hspace{1cm} (8)
We prove that for \(k \neq 3, 5, 7, 13, 17, 97, 193\), (1) contain infinitely many prime solutions

\textbf{The New Prime theorem (233)}

\[ P, j P^{386} + k - j (j = 1, \cdots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract
Using Jiang function we prove that \( jP^{386} + k - j \) contain infinitely many prime solutions and
no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{386} + k - j(j = 1, \cdots, k - 1).
\]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function \([1,2]\)

\[
J_2(\omega) = \prod_{p} [P - 1 - \chi(P)]
\]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} \left[jq^{386} + k - j\right] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1
\]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions \([1,2]\)

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula \([1,2]\)

\[
\pi_k(N, 2) = \left\lfloor \frac{J_2(\omega) \omega^{k-1} N}{(386)^{k-1} \phi^{k}(\omega) \log^{k} N} \right\rfloor
\]

where \( \phi(\omega) = \prod_{p} (P - 1) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that for \( k = 3 \), (1) contain no prime solutions.

Example 2. Let \( k > 3 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k > 3 \) (1) contain infinitely many prime solutions

**The New Prime theorem (234)**

\[
P, jP^{388} + k - j(j = 1, \cdots, k - 1)
\]

Chun-Xuan Jiang
Abstract

Using Jiang function we prove that \( jP^{388} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[
P, jP^{388} + k - j (j = 1, \cdots, k-1).
\]

contain infinitely many prime solutions or no prime solutions.

**Proof.** We have Jiang function [1,2]

\[
J_2(\omega) = \prod_{P}[P-1 - \chi(P)]
\]

where \( \omega = \prod_{P} P, \ \chi(P) \) is the number of solutions of congruence

\[
\prod_{j=1}^{k-1} [jq^{388} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P-1
\]

If \( \chi(P) \leq P-2 \) then from (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P-1 \) then from (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N,2) \sim \frac{N}{\phi(\omega) \log^k N}
\]

where \( \phi(\omega) = \prod_{P} (P-1) \).

**Example 1.** Let \( k = 3, 5, 389 \). From (2) and (3) we have

\[
J_2(\omega) = 0
\]

We prove that for \( k = 3, 5, 389 \) (1) contain no prime solutions.

**Example 2.** Let \( k \neq 3, 5, 389 \). From (2) and (3) we have

\[
J_2(\omega) \neq 0
\]

We prove that for \( k \neq 3, 5, 389 \) (1) contain infinitely many prime solutions
The New Prime theorem (235)

\[ P, jP^{390} + k - j(j = 1, \cdots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{390} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{390} + k - j(j = 1, \cdots, k - 1). \quad (1) \]

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{p}[P - 1 - \chi(P)] \quad (2) \]

where \( \omega = \prod P, \quad \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} j q^{390} + k - j \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \quad (3) \]

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \quad (4) \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \quad (5) \]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{390} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \phi^{k-1}(\omega)}{(390)^{k-1} \phi^k(\omega) \log^k N} \frac{N}{N} \quad (6) \]

where \( \phi(\omega) = \prod_{P}[P - 1]. \)

Example 1. Let \( k = 3, 7, 11, 31, 79, 131. \) From (2) and (3) we have

\[ J_2(\omega) = 0 \quad (7) \]
We prove that for $k = 3, 7, 11, 31, 79, 131$, (1) contain no prime solutions.

Example 2. Let $k \neq 3, 7, 11, 31, 79, 131$. From (2) and (3) we have

$$J_2(\omega) \neq 0$$

We prove that for $k \neq 3, 7, 11, 31, 79, 131$, (1) contain infinitely many prime solutions

**The New Prime theorem (236)**

Let $k$ be a given odd prime.

$$P, jP^{3j^2} + k - j(j = 1, \ldots, k - 1)$$

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{3j^2} + k - j$ contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let $k$ be a given odd prime.

$$P, jP^{3j^2} + k - j(j = 1, \ldots, k - 1).$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_p [P - 1 - \chi(P)]$$

where $\omega = \prod_p P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} j q^{3j^2} + k - j \equiv 0 \quad \text{(mod P)}, q = 1, \ldots, P - 1$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0$$

We prove that (1) contain infinitely many prime solutions.

If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]
\[ \pi_k(N,2) = \left| \{ P \leq N : jP^{392} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(392)^{k-1}\phi(\omega) \log^k N} \]  \hspace{1cm} (6)

where \( \phi(\omega) = \prod (P - 1) \).

Example 1. Let \( k = 3, 5, 29, 197 \). From (2) and (3) we have
\[ J_2(\omega) = 0 \]  \hspace{1cm} (7)

We prove that for \( k = 3, 5, 29, 197 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 29, 197 \). From (2) and (3) we have
\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (8)

We prove that for \( k \neq 3, 5, 29, 197 \), (1) contain infinitely many prime solutions.

The New Prime theorem (237)

Using Jiang function we prove that \( jP^{394} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.

\[ P, jP^{394} + k - j (j=1,\ldots,k-1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{394} + k - j \) contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_{P} \left[ P - 1 - \chi(P) \right] \]  \hspace{1cm} (2)

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} \left[ jq^{394} + k - j \right] \equiv 0 \mod (P), q = 1,\ldots, P - 1 \]  \hspace{1cm} (3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (4)

We prove that (1) contain infinitely many prime solutions.
If $\chi(P) = P - 1$ then from (2) and (3) we have

$$J_2(\omega) = 0$$

(5)

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left\{ P \leq N : jP^{396} + k - j = \text{prime} \right\} \sim \frac{J_2(\omega)\omega^{k-1} N}{(394)^{k-1}\phi(\omega) \log^k N}$$

(6)

where $\phi(\omega) = \prod_p (p-1)$.

Example 1. Let $k = 3$. From (2) and (3) we have

$$J_2(\omega) = 0$$

(7)

We prove that for $k = 3$, (1) contain no prime solutions.

Example 2. Let $k > 3$. From (2) and (3) we have

$$J_2(\omega) \neq 0$$

(8)

We prove that for $k > 3$, (1) contain infinitely many prime solutions

---

**The New Prime theorem (238)**

$$P, jP^{396} + k - j (j = 1, \cdots, k - 1)$$

Chun-Xuan Jiang

Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $jP^{396} + k - j$ contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let $k$ be a given odd prime.

$$P, jP^{396} + k - j (j = 1, \cdots, k - 1)$$

(1)

contain infinitely many prime solutions and no prime solutions.

**Proof.** We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)]$$

(2)

where $\omega = \prod_p P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [jP^{396} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1$$

(3)

If $\chi(P) \leq P - 2$ then from (2) and (3) we have
We prove that (1) contain infinitely many prime solutions.

\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (4)

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  \hspace{1cm} (5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[
\pi_k(N, 2) = \left| \left\{ P \leq N : jP^{396} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(396)^{k-1}\phi(\omega)\log^k N} \]  \hspace{1cm} (6)

where \( \phi(\omega) = \prod_p (P-1) \).

Example 1. Let \( k = 3, 5, 7, 13, 19, 23, 37, 67, 199, 397 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  \hspace{1cm} (7)

We prove that for \( k = 3, 5, 7, 13, 19, 23, 37, 67, 199, 397 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 7, 13, 19, 23, 37, 67, 199, 397 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  \hspace{1cm} (8)

We prove that for \( k \neq 3, 5, 7, 13, 19, 23, 37, 67, 199, 397 \), (1) contain infinitely many prime solutions.

The New Prime theorem (239)

\[ P, jP^{398} + k - j (j = 1, \ldots, k - 1) \]

Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( jP^{398} + k - j \) contain infinitely many prime solutions and no prime solutions.

Theorem. Let \( k \) be a given odd prime.

\[ P, jP^{398} + k - j (j = 1, \ldots, k - 1), \]  \hspace{1cm} (1)

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]
\[ J_2(\omega) = \prod_{p}(P - 1 - \chi(P)) \]  

(2)

where \( \omega = \prod_{p}P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} j^{q^{398}} + k - j \equiv 0 \pmod{P}, q = 1, \ldots, P - 1 \]  

(3)

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(4)

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(5)

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N,2)\equiv\left\{P \leq N: j^{P^{398}} + k - j = \text{prime}\right\} \sim \frac{J_2(\omega)\phi^{k-1}(\omega)}{(398)^{k-1}\phi(\omega)\log^k N} \]  

(6)

where \( \phi(\omega) = \prod_{p}(P - 1) \).

Example 1. Let \( k = 3 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]  

(7)

We prove that for \( k = 3 \), (1) contain no prime solutions.

Example 2. Let \( k > 3 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]  

(8)

We prove that for \( k > 3 \), (1) contain infinitely many prime solutions

The New Prime theorem (240)

\( P, j^{P^{400}} + k - j \)  

Chun-Xuan Jiang  
Jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that \( j^{P^{400}} + k - j \) contain infinitely many prime solutions and no prime solutions.

**Theorem.** Let \( k \) be a given odd prime.
contain infinitely many prime solutions and no prime solutions. 

Proof. We have Jiang function [1,2]

\[ J_2(\omega) = \prod_p [P - 1 - \chi(P)] \]  

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ \prod_{j=1}^{k-1} [jq^{400} + k - j] \equiv 0 \pmod{P}, q = 1, \cdots, P - 1 \]  

If \( \chi(P) \leq P - 2 \) then from (2) and (3) we have

\[ J_2(\omega) \neq 0 \]

We prove that (1) contain infinitely many prime solutions.

If \( \chi(P) = P - 1 \) then from (2) and (3) we have

\[ J_2(\omega) = 0 \]

We prove that (1) contain no prime solutions [1,2]

If \( J_2(\omega) \neq 0 \) then we have asymptotic formula [1,2]

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : jP^{400} + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(400)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \]

where \( \phi(\omega) = \prod P^{-1} \).

Example 1. Let \( k = 3, 5, 11, 17, 41, 101, 401 \). From (2) and (3) we have

\[ J_2(\omega) = 0 \]

We prove that for \( k = 3, 5, 11, 17, 41, 101, 401 \), (1) contain no prime solutions.

Example 2. Let \( k \neq 3, 5, 11, 17, 41, 101, 401 \). From (2) and (3) we have

\[ J_2(\omega) \neq 0 \]

We prove that for \( k \neq 3, 5, 11, 17, 41, 101, 401 \), (1) contain infinitely many prime solutions.

Remark. The prime number theory is basically to count the Jiang function \( J_{n+1}(\omega) \) and Jiang prime \( k \)-tuple singular series \( \sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod \left( 1 - \frac{1 + \chi(P)}{P} \right) \left( 1 - \frac{1}{P} \right)^{-\epsilon} [1,2] \), which can count the number of prime numbers. The prime distribution is not random. But Hardy-Littlewood prime \( k \)-tuple
singular series \( \sigma(H) = \prod_p \left( 1 - \frac{\nu(p)}{p} \right) \left( 1 - \frac{1}{p} \right)^{-1} \) is false [3-17], which cannot count the number of prime numbers\[3\].

**References**


[3] Chun-Xuan Jiang, The Hardy-Littlewood prime \( k \)-tuple conjecture is false.(http://wbabin.net/xuan.htm#chun-xuan)


Szemerédi’s theorem does not directly to the primes, because it cannot count the number of primes.

Cramér’s random model cannot prove any prime problems. The probability of \( 1/\log N \) of being prime is false. Assuming that the events “\( P \) is prime”, “\( P + 2 \) is prime” and “\( P + 4 \) is prime” are independent, we conclude that \( P, P + 2, P + 4 \) are simultaneously prime with probability about
There are about \( \frac{N}{\log^3 N} \) primes less than \( N \). Letting \( N \to \infty \) we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler (1707-1783)

It will be another million years, at least, before we understand the primes.

Paul Erdos (1913-1996)

Of course, the primes are a deterministic set of integers, not a random one, so the predictions given by random models are not rigorous (Terence Tao, Structure and randomness in the prime numbers, preprint). Erdos and Turán (1936) contributed to probabilistic number theory, where the primes are treated as if they were random, which generates Szemerédi’s theorem (1975) and Green-Tao theorem (2004). But they cannot actually prove and count any simplest prime examples: twin primes and Goldbach’s conjecture. They don’t know what prime theory means, only conjectures.