The New Prime theorem（25）

Hardy-Littlewood conjecture M:

$$x^3 + y^3 + k$$

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Abstract

Using Jiang function we prove Hardy-Littlewood conjecture M: $x^3 + y^3 + k$ [4].

Theorem 1. Let k be an odd number. We define prime equation

$$P_3 = P_1^3 + P_2^3 + k. \tag{1}$$

For every odd integer k there are infinitely many primes P_1 and P_2 such that P_3 is a prime.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [(P-1)^2 - \chi(P)], \tag{2}$$

where $\chi(P)$ is the number of solutions of congruence

$$q_1^3 + q_2^3 + k \equiv 0 \pmod P \tag{3}$$

where $q_i = 1, \cdots, P-1, i = 1, 2$.

From (3) we have

$$J_3(\omega) \neq 0. \tag{4}$$

We prove that there are infinitely many prime solutions in (1).

We have the best asymptotic formula [1,2]

$$\pi_2(N,3) = \left|\{P_1, P_2 \le N : P_3 = prime\}\right| \sim \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N}. \tag{5}$$

Example 1.

$$P_3 = P_1^3 + P_2^3 + 1. \tag{6}$$

From (2) we have

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)] \neq 0. \tag{7}$$

The table below gives the values of $\chi(P)$.

P	3	5	7	11	13	17	19	23	29	31
$\chi(P)$	1	3	0	9	0	15	18	21	27	27

Theorem 2. Let k be an even number. Suppose prime equaiton

$$P_3 = (P_1 + 1)^3 + P_2^3 + k\,. \qquad (8)$$

We have Jiang function [1,2]

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)]\,, \qquad (9)$$

where $\chi(P)$ is the number of solutions of congruence.

$$(q_1 + 1)^3 + q_2^3 + k \equiv 0 \pmod P\,. \qquad (10)$$

where $q_i = 1, \cdots, P-1, i = 1, 2.$

From (10) we have

$$J_3(\omega) \neq 0\,. \qquad (11)$$

We prove that there are has infinitely many prime solutions in (8).
We have asymptotic formula [1,2]

$$\pi_2(N,3) = \left|\{P_1, P_2 \le N : P_3 = prime\}\right| \sim \frac{J_3(\omega)\omega}{6\phi^3(\omega)} \frac{N^2}{\log^3 N}\,. \qquad (12)$$

Remark. The prime number theory is basically to count the Jiang function $J_{n+1}(\omega)$ and Jiang prime k-tuple singular series $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1,2], which can count the number of prime number. The prime distribution is not random. But Hardy prime k-tuple singular series $\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [3-8], which can not count the number of prime numbers.

References

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Szemerédi's theorem does not directly to the primes, because it can not count the number of primes. It is unusable. Cramér's random model can not prove prime problems. It is incorrect. The probability of $1/\log N$ of being prime is false.

Assuming that the events "P is prime", "$P+2$ is prime" and "$P+4$ is prime" are independent, we conclude that P, $P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N. Letting $N \to \infty$ we obtain the prime conjecture, which is false.

The tool of additive prime number theory is basically the Hardy-Littlewood prime tuple conjecture, but can not prove and count any prime problems[6].