Relativistic Spirals

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Abstract

A closed form analytic model has been presented and matched to the morphology of spiral galaxies[9]. This model consists of an Archemedean spiral which matches the physical attributes of spiral galaxies as a result of relativistic effects due to great distances on a rotating rigid body. This effect was first posed by Minkowski[5]. We present a derivation of this analytical model though the application of Lortentz transformations.

A closed form analytic model has been previously presented and matched to the morphology of spiral galaxies.[9] This model consists of an Archemedean spiral which is the result of relativistic effects due to great distances which affect rotating rigid bodies. This effect was first posed by Minkowski[5] and can now be verified through direct observation.

It has been shown that spiral galaxies have a flat velocity rotation profile[1][2][3] and this can only be the result of a linear distribution of matter having a constant linear density[9]. The contradiction between the behaviour of matter composing a spiral galaxy, namely as a straight line, and its morphology, an Archemedean spiral, is resolved through Lorentz transformations [5].

Consider a very large circular platform. Rotate the platform at some angular velocity ω. Place a clock at one of the outer edges and an identical clock near the centre. Consider an observer, carrying a third identical clock, next to the inner rotating clock. The inner clock and observer's clock both measure the passage of time at the same rate. However, if the observer looks to the outer clock, the outer clock will keep time at a slower rate than the inner clocks. Suppose the observer moves to the outer portion of the rotating platform. The outer clock and observer's clock would then both keep time at the same rate while looking to the inner clock from the outer rim will reveal the inner clock keeping time at a faster rate than the outer two clocks. This is the result of the fact that the outer clock is moving faster than the inner clock and the difference in the rate of keeping time between the two clocks is resolved by Lorentz transformations due to the different tangential speeds of the inner and outer locations.

The velocity of each clock is determined by:

$$v_i = \omega R_i \tag{1}$$

and

$$\gamma_i = \sqrt{\frac{1}{1 - \frac{v_i^2}{c^2}}} \tag{2}$$

where where c is the speed of light, v_i is the tangential velocity of each clock considered and R_i is the distance of each respective clock to the centre of rotation.

At this particular juncture we note that the acceleration of, say, the outer clock is:

$$a = \frac{v^2}{R} \tag{3}$$

and the acceleration due to the presence of a massive body is:

$$g = G\frac{M}{R^2} \tag{4}$$

and from principle of equivalence we have:

$$a = g. \tag{5}$$

We can therefore conclude:

$$\gamma = \sqrt{\frac{1}{1 - G\frac{M}{Rc^2}}} \tag{6}$$

and thereby derive the effects on the measurements of space and time due to a gravitational field from mass M at distance R from the centre of mass of the massive body [5].

Consider a straight line drawn from one side of a platform, a few hundred thousand light years in diameter, to the opposite side through the centre. Say we have massive bodies, such as stars and the like, distributed along the straight line. As the platform rotates, all the galactic material so placed on the platform will behave and perceive other galactic material as a rigid linear body.

Say there is an observer A on the platform at distance R, many thousands of light years, from the centre who wishes to determine his or her tangential velocity. Using some distant reference point very far from, and off of, the platform, the observer would measure a tangential velocity of v. An observer, say observer C, at the distant reference point also measures the speed of this particular galactic location as v. Consider also another observer, observer B, on the platform's straight line at radial distance $R/2$. This observer conducts the same measurements as the outer observer and, surprisingly, also obtains a measurement of v as the inner location's tangential velocity. The distant observer C would also obtain the same value of tangential velocity of observer B. This surprising result is the effect of Lorentz transformations on the clocks and measuring rods of the rotating observers as compared to the measuring instruments of the distant observer in an inertial reference frame.

To demonstrate, should the observer at location R from the centre of the rotating platform use a standard measuring rod to measure the circumference of the platform at his or her location, the measuring rod would be shortened by a factor of $1/\gamma$ in the tangential direction and a larger than expected measure of the circumference be obtained. Since the measuring rod would not be affected if directed towards the centre of the platform, an unaffected measure would be made of the location's radial distance. Therefore, such an observer would measure the circumference divided by twice the radial distance as greater than π. This effect becomes more pronounced the further from the centre the measure is made. Furthermore, should the observer be in a non-rotating reference frame near a massive body, the same experiment would yield an answer less than π and the effect would become more pronounced the closer to the centre the measures are made.

From this line of reasoning we state that it is not the stellar and other galactic material that has been altered by being in a rotating reference frame, but it is the platform itself that has deviated from the properties of a rigid body. We therefore propose that a massive body alters spacial and temporal measurements in such a way as to evoke "positive curvature" and a rotation alters spacial and temporal measurements in such a way as to evoke "negative curvature".

Let us now examine the effects of negative curvature on measurements of space and time and apply results we achieve to the effects of a straight line drawn on a large rotating platform.

Examining once again the Lorentz factor, γ.

$$\gamma = \sqrt{\frac{1}{1 - \frac{\omega^2 R^2}{c^2}}}$$

From the properties of negative curvature, if a measuring rod of length R, the length matching the radial distance to the centre of the galaxy is used to measure an arc length to determine an angle of one radian, a smaller angular measure would be made than in a non-rotating reference frame. Furthermore the period of revolution to circumambulate the galaxy would also dilate with increased radial distance. The measure of θ as an angle subtending an arc length would deviate from the same measure in a non-rotating reference frame by a factor of $1/\gamma$. Considering a constant value of ω_0 for the angular velocity of the platform without relativistic effects, an observer in an inertial reference frame looking at the rotating platform would see:

$$\frac{d\theta}{dt} = \frac{\omega_0}{\gamma}$$

where

$$\gamma = \sqrt{\frac{1}{1-\frac{\omega_0^2 R^2}{\gamma^2 c^2}}}$$

$$\gamma^2 = \frac{1}{1-\frac{\omega_0^2 R^2}{\gamma^2 c^2}}$$

$$\gamma^2 (1 - \frac{\omega_0^2 R^2}{\gamma^2 c^2}) = 1$$

$$\gamma^2 - \frac{\omega_0^2 R^2}{c^2} = 1$$

$$\gamma^2 = 1 + \frac{\omega_0^2 R^2}{c^2}.$$

If time is in years and distance in light years, $c = 1$ then:

$$\gamma = \sqrt{1 + \omega_0^2 R^2}. \tag{7}$$

If $\omega_0 R << 1$ then $\gamma \approx 1$ and $\omega = \omega_0$ and the platform would appear to behave as a rigid body and the line drawn through the centre would appear straight to an observer in an inertial reference frame. If, however, $\omega_0 R >> 1$ then $\gamma \approx \omega_0 R$ and since $\omega_0/\gamma = \omega$ we would have $\omega \longrightarrow \frac{1}{R}$ and the line drawn on the platform would appear as an Archemedean spiral. Furthermore, the points on the line would all have a constant tangential velocity with $v = \omega_0$ in light years per year and radians per year.

In terms of the dimensions of a spiral galaxy, if such a galaxy has a constant rotational velocity profile of 300 kps, $(10^{-3}c)$, we see that ωR begins to exceed the speed of light at a radial distance of one thousand light years. Obviously, at greater distances from the centre of the galaxy, say thousands of light years and more, relativistic effects play a major rôle.

Using parametric equations, we have:

$$T = \frac{2\pi\theta}{v}$$

as a measure of angular displacement and along the radial axis we have:

$$T = \frac{R}{c}.$$

Combining yields:

$$R = \frac{2\pi\theta}{v/c}$$

which is the equation presented previously [9]. This is the equation of an Archimedean spiral which is the projection onto a flat sheet of paper of a rotating straight line in a four dimensional Minkowski space through considerations of Lorentz transformations. From this galactic spiral relation we have a closed form analytical model of a spiral galaxy from which can be derived Roxy's Ruler[10] and various physical measures of spiral galaxies can be made, including their distances.[11]

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