On the Relation between Mathematics, Natural Sciences, And Scientific Inquiry

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In this article, we will shortly review a few old thoughts and recent thoughts on the relation between Mathematics and the Natural Sciences. Of course, the classic references to this open problem will include Wigner’s paper (1964); a more recent review article is Darvas (2008). But it appears that this issue is partly on the domain of natural philosophy and also philosophy of inquiry. Therefore we will begin with a review on some known thoughts of Kant, Bacon, Popper, etc.

Our hope here is to find out clues to reveal the hidden structure of Nature, just as what Planck did a century ago. (An early note to our scientific colleagues: In writing this article we choose to switch off our role as ‘practical scientist’ and switch on the ‘free thinker’ mode, therefore you can sit back and relax, because chance is what we write here is not related to what you’re doing; this is more on science as a whole. But of course if you’re interested in this kind of article, you can read on.)

In the meantime, we’ve written a rather serious article on this issue, but after midnight our thought becomes twisted, and now we are going to rewrite it again in the style of Scott Adams’ Dilbert comics. This belongs to our favorite comic strips. If at certain point you feel like we’re going too far (probably saying to yourself: Heck, what kind of tablets these guys have swallowed?), perhaps you should stop reading or send this file to recycle bin. Otherwise, you can continue reading and make up your mind later on.

The Hidden Structure of Nature: What it is, what it was

It appears as a fair guess to say that the greatest Natural philosopher was Kant. One of his most cited remark is perhaps the distinction between ‘phenomena’ and ‘noumena’ (from ‘nous’). To put this idea a bit simpler, we can say that phenomena refer to processes or symptoms that appear to the eyes, while noumena refer to the hidden configuration or inner structures which are beyond what meet the eyes.

But that notion of ‘noumena’ is quite problematic, because it does not clarify how ‘hidden’ or which deeper level that we’re looking for. If for instance, we discuss here the elementary particles, then does it mean that present hadron physics theories or strong forces already reflect the noumena, or shall we find out hidden structure beyond the hadrons, perhaps something like sub-quark or Planck scale models?
In this regards, some physicists already mention that there is a scale invariance character of elementary particles, which suggests that we can always reveal new structure at deeper and deeper scale. Perhaps it is quite safe to say that the restriction here is not on theoretical side, but more on the precision of measurement apparatus.

If in accordance with Kant the phenomena are qualitatively distinct from the ‘noumena’ (the hidden structure of Nature), then problem of finding <noumena> will be more adverse if we ask not only what Nature is today, but also what Nature was in the past. In this regards, it is quite apparent that the uncertainties of the problem become twofold, one concerns the deep structure of Nature itself, and the next concerns the premise of the smooth continuation of time.

Most evolution theories apparently are based on this premise of a smooth progression of things (some modern models are based on dynamical equation like Lotka-Volterra equation, but how to define time itself remains an open issue). On the other side, there are new theories based on possibility of ‘sudden changes’ happening on large scales, for instance the concept self-organized criticality introduced by Per Bak et al., emergence theory, spontaneous symmetry breaking, etc. (Darvas 2008).

In such a model based on the self-organized criticality, sudden changes can happen after a long period of stasis. For example, consider a pile of sand: initially it can pile up almost vertically, until sometime it will change such that a slope will form what is known as ‘critical angle’ (see Figure 1).

![Figure 1. Sudden change to form critical angle.](image)

Hopefully the above example can give illustration how the sudden changes can happen during such critical phenomena. Various other critical phenomena can be related to this self-organized criticality, so that it is quite problematic to conceive how smooth
continuation of changes can take place ‘naturally’. Another well-known example is the geographical layering formation near Yukatan area, Mexico. As reported by Alvarez et al., they indicate some kind of periodic changes in the past at the order of thousand years.

Nowadays, the self-organized criticality phenomena have been studied extensively in various context; for a quick look see for instance Boldyrev (arXiv:hep-th/9610080), and Ambjorn, Jurkiewics & Loll (arXiv: hep-th/0712.2485, gr-qc/0711.0273). One can also find that introducing the discontinuous progression of ‘time’ will lead to a quite different Galilean law of motion, and so forth.

One can also note here that in some ancient thinking, large natural changes can take place in the same time with large social upheavals. From the viewpoint of modern dynamics theory, whether such a large climate or environmental change really can affect social upheaval remains mystery, although there has been study on the relation between human/population evolution processes and their environment (by T. Barnosky from Berkeley Univ.). From a viewpoint, this may explain why some people feel that they can predict anything except to predict when the sky would fall upon them (remember the Asterix comics, for instance).

Nonetheless, we should limit our discussions here on self-organized criticality only in the domain of Natural phenomena. Meanwhile, other people may find that those sudden changes may also be related to Kuhn’s idea of ‘paradigm changes’ in the history of science (Gholson & Barker, 1985). For example, one can notice from history of modern science that the long-stagnant period between Planck’s blackbody radiation (1901) until 1921 was a precursor to the rapid development in short period (1922-1928), where the modern Physics began. As Weinberg once remarked such a rapid change is so remarkable in history such that scientists nowadays refer to ‘Classical Physics’ for all things happened before this era. Nonetheless, in this article we don’t discuss such a possible parallelism any further.

On the Methods of Inquiry: From Bacon, Popper, to Habermas

As the night goes very late, now we will continue this rambling note on how scientists may possibly discover something new in their fields. In this section, it is safe to say that you can forget all what you already learned on scientific methods, because this section is not about that classic teaching on science. This section is more about where to begin the scientific process itself. As we all know, how to invent and how to discover are perhaps one of the most fundamental questions for all living scientists (including physicists). In writing this section we would rely solely on a few irrelevant experiences with pets and also to a crystal ball which tells nothing.

It seems worth to mention here Darvas’ (2008) note on the distinctive standpoints between Bacon, Polanyi and Popper. Bacon emphasized methodological processes which should be given attention in science; somewhat a more philosophical part of what Galileo did experimentally.
In the mean time, Polanyi gave emphasize on the ‘personal knowledge.’ By personal here he meant human mind which consists of things he/she learnt (objective knowledge), things he/she thought (tacit knowledge), and also things he/she perceived (subjective knowledge). In other words, according to Polanyi, one’s personal knowledge does not necessarily mean to be always subjective, though it may include subjective knowledge.

Karl Popper who wrote his seminal book ‘Objective Knowledge’ apparently as a response to Polanyi’s book ‘Tacit Knowledge’ disagreed strongly with this idea of personal knowledge. Popper himself apparently emphasized the role of knowing ‘episteme’, via continuing criticism. In his model, validation of theory is not possible to achieve via experiments, they can only support or reject a hypothesis. For further discussion on this issue, see Darvas’ review (2008).

We can make further remark here that Polanyi’s assertion of personal knowledge and tacit knowledge today has begun to be implemented in the so-called Knowledge Management. This is a modern method to organize the unstructured parts of human knowledge, for instance see the OneNote feature in recent version of MS Office.

In the context of Knowledge Management, one can predict that in the future our present methods of file management will be improved to enable people organize better their tacit knowledge. For instance, scientists perhaps would prefer to organize their files according to their specific ‘mind-mapping’ diagram, instead of standard ‘vertical’ folder systems. The distinction is shown in Figure 2 and Figure 3 below.

Figure 2. The common File/Information structure.
It is clear that mind-map diagram enables the users to track his/her files according to his/her interests, because most people think visually. Of course, the present method to organize files (folders etc), which is based on cabinet system around 1950s, can be retained, provided they can be integrated with the visual/mindmap approach.

Now we discuss some recent thoughts on scientific programs. Meanwhile, Jurgen Habermas, a leading philosopher from the Frankfurt School, opened a whole new can of worms outside of this traditional debate on the ‘objectivity of science.’ He suggests a quite different argument compared to Popper’s objective science. First, we can refer to Lakatos’ idea (see Gholson & Barker, 1985) of ‘scientific program,’ i.e. the progress of scientific development was actually determined by a group of respective scientists in each area, who also would write recommendations to the governments. While this standard practice is quite common in the most developed countries, especially after the WWII, it has been pointed out by Habermas (1968) that in this respect the scientific development programs themselves are not free of interests, ranging from industrial interests, a country’s economics preservation, energy interests, and so on. This is not to say that this practice is wrong by itself, but it is to indicate that it becomes quite difficult to describe these programs as ‘objective knowledge’, at least in the sense of those ancient Greek scholars who seek knowledge as part of their effort to understand the Logos.

In Habermas’ view, it is impossible to perceive that modern sciences follow the same path of these ancient Greek scholars, because in today’s modern world, the Logos disappears and it is replaced with ‘scientific programs.’ To put in other words, what we
study in modern days are not BioLogos, but perhaps BioPrograms, not ZooLogos but ZooPrograms and so on. For example, in Bacon’s worldview one can sense that the ultimate ‘program’ of science is to conquer the world surrounding human. As shown by Fritjof Capra (The Turning Point) this kind of philosophy of science led to environmental degradation, etc. See Figure 4 & Figure 5.

Figure 4. Scheme of method of Inquiry by Ancient Greek scholars

Figure 5. Scheme of Scientific Programs
To summarize, the methods of Inquiry in our modern times have been influenced by the so-called scientific programs. Of course, at this point one can ask whether is it possible to do research which meet the scientific programs but at the same time meet the ideals of those ancient Greek scholars? And also which is the best possible methods of Inquiry, which can lead one into a new invention or scientific discovery? As we pointed out in the beginning of this section, this question apparently belongs to the most fundamental questions for a scientist.

There are actually a few well-known methods of Inquiry, depending on one’s preference:

(a) **Einfühlung**: this may be a favorite method for Einstein, because he wrote that a physicist should sense something subtle in Nature before he works on the formalism itself.

(b) **Generalizing Math**: this method may be called as Dirac’s trick, i.e. consider one equation and try to generalize its math. Thereafter you can look for its plausible implications: Does the new equation imply new physics? At least this method works for Dirac equation, and yield prediction of positron. Another example here is that one can recognize that possible breakthroughs in mathematics come from relaxing Euclid’s axioms one by one, for instance by relaxing the fifth axiom (there is only one parallel through a given point to a given line) one can find geometries which go beyond flat surface: i.e. hyperbolic geometry (Lobachevsky-Bolyai-Gauss) – there are many parallels through a given point to a given line, and elliptic geometry (Riemann) – there is no parallel through a given point to a given line. Then go further and combine these geometries, since our universe is not homogeneous but heterogeneous, and consider the Smarandache’s multispace, which is formed by a space which can be Euclidean and another space non-Euclidean, or even many spaces put together such that an axiom is valid in one space and invalid in various ways in other spaces (Smarandache geometries).

(c) **Antithesis-Synthesis Dialectic**: this method apparently is more favored by Popper, who suggests that scientific efforts move step by step nearing the hidden truth. Dialectic method was introduced by Hegel, who says that things make progress via creating antithesis and synthesis of what already exist. In other words, one should find out what others have done in a field, and then move on with something that others have not done before.

(d) **Smarandache’s Neutrosophic Method**: this method is a generalization of Hegel’s dialectic, and suggests that scientific research will progress via studying the opposite ideas and the neutral ideas related to them in order to have a bigger picture.
(e) **Music** (sense of art): you may pick up violin or play flute, guitar or piano, and voilà! You discover another great thing like the next SuperDuper-General Relativity theory. Sounds a bit like exaggeration? Perhaps, but according to a study, students with musical ability tend to perceive mathematical principles better. This effect can be explained from the viewpoint that in traditional school, emphasis is given on the left side of the brain, while actually to maximize human brain’s potential, one should use both sides in equal way (see also Darvas 2008).

(f) **Irrelevant Fiction Stories/Books.** You may have heard that Bohr likes Dickens, Einstein and others also liked fiction stories such as Sherlock Holmes. If those books are not available near you, perhaps you can begin doing permutation on random words taken from a dictionary, and find out possible meaning of their combination.

(g) **Climbing or Going to Mountain**: at least this method worked for Heisenberg and plenty of other physicists who sense better grasp on their problems while they were going to mountain. Some people say that going to mountain will give you a sense of unity with the entire Universe, see for example J. Redfield’s book (*The Celestine Prophecy*). Not a bad thing to try, at least.

(h) **Lateral Thinking**: if you think that the traditional scientific method is a bit too methodical for you, then perhaps you can try DeBono’s lateral thinking. Another way may be called as ‘diagonal thinking’, i.e. start with a known premise from one field of science, and then derive conclusions in other field. For example, you start with quantum principles and then derive conclusions for cosmology (i.e. quantum cosmology). Or start with antimatter/antihydrogen and find conclusions in Newtonian mechanics (e.g. is there classical antimatter?) or in Smarandache’s unmatter. And so on. See Figure 6.
If this method doesn’t sound good to you, perhaps you can try to extend it a bit further, i.e. do diagonal thinking twice and you may call it ‘zigzag thinking’ (See Figure 7). For instance, to put quantum principles to cosmology is one thing, but you can also find relation from cosmology and particle physics, which is a very active field nowadays, called ‘cosmo-particle physics’. And so on, you can also invent your own thinking way which enables you to adapt your specific abilities to your fields of interest.
After citing some of those possible methods of Inquiry, now we’re going to discuss on the relation between mathematics and symmetries behind the Nature itself.

Planck and the Symmetries of Nature

In his note on Planck, his favorite figure, Einstein wrote in 1932 (P.M. Robitaille, “Max Planck,” *Progress in Physics* vol. 4, Oct. 2007):

> Many kinds of men devote themselves to science, and not all for the sake of science herself. There are some who come into her temple because it offers them the opportunity to display their particular talents. To this kind of men, science is a kind of sport in the practice of which they exult, just as an athlete exults in the exercise of his muscular prowess. There is another class of men who came into the temple to make an offering of their brain pulp in the hope of securing a profitable return. These men are scientists only by the chance of some circumstance which offered itself when making a choice of career. Should an angle of God descends and drive from the temple of science all those who belong to the categories I have mentioned, I fear the temple would be nearly emptied. But a few worshippers would still remain – some from former times and some from ours. To this latter belongs our Planck. And that is why we love him...”

According to the above very interesting remark on Planck, Einstein pointed out 3 distinctive attitudes on science which someone (or some groups of scientists) may display: sport, expected return, and true believers. If we wish to find out some parallels between this note and Habermas’ viewpoint as discussed above, then perhaps it is quite appropriate to compare those interests and ‘expected return’ motives; and also between ‘true believers’ and what Habermas called as ‘liberative knowledge.’

In this regards, it is also worth to mention here that Max Planck’s greatest achievement, i.e. the discovery of the true statistical description of blackbody radiation is a good example on how mathematics derivation (with a fair number of premises) can lead scientists to a new and unexpected kind of knowledge.

As discussed by Darvas (2008), mathematics role in science is unavoidable, but how actually mathematics correspond to the Nature itself remains unexplainable, or in Wigner’s word “unreasonable effective”. In other words, we can accept the role of mathematics to describe Nature because of its effectiveness, although it is unreasonable. By doing so, of course we don’t refer here to the ancient belief that Nature itself is inherently mathematical (as Pythagoras would say: “The whole thing is a number.”). What we refer here is just another saying of Pythagoras: “Mathematics is the way to
understand the Universe.” (Darvas 2008) In other words, it is because simply mathematics is the only consistent and effective tool that humankind can use to analyze the world surrounding us.

By mathematics here we do not only refer to the symmetries, invariance, and transformation principles that scientists ought to use in order to find the pattern of Nature, but we can also use Wigner’s definition:

“Mathematics is the science of skillful operations with concepts and rules invented just for this purpose. The principal emphasis is on the invention of concepts.” (Darvas 2008, p. 10)

Or in other words we can find a somewhat simplistic description of mathematics: “A symbolic and formal formulation to express concepts.” For instance, there are plenty of formulations to describe logic without introducing ‘the principle of excluded middle’, from Lukasiewicz, until Zadeh’s fuzzy logic. Recently, a new kind of logic is developed by F. Smarandache, called ‘Neutrosophic Logic’, in his effort to unify the mathematical logic, statistics, and philosophy in one theoretical footing. Further implication of this new model of triple-infinite valued logic can be found in mathematical domain known as ‘Information Fusion Theory’ (see for example Dezert-Smarandache Theory on paradoxist information).

Therefore, apparently we can say that for the practical (pure) mathematicians, to conceive new mathematics, one does not have to care of its implications. It is task for physical sciences to think of these implications in real world. In other words, we can write the following scheme to describe how the set of mathematical theories can intersect with the set of physical theories, and also intersect with the set of observables. See Figure 8.
Figure 8. The set of mathematical, physical theories, and observables

One last remark on this section is that some physicists may not agree with what we discuss above, especially those who belong or call themselves ‘positivist.’ For example, Hawking (arXiv:hep-th/9409195, p.1, 1994) once noted that:

“a physical theory is just a mathematical model, and it is meaningless to ask whether it corresponds to reality.”

Bohr himself was also widely reputable as one of the most positivists among others, which led to his famous debate with Einstein on the interpretation of Quantum Mechanics. Einstein of course belongs to ‘traditional’ physicists who somehow believe that one should find out the deep physics behind Nature, that is why he sought for some kind of ‘field’ structure to explain the quantum effects, and therefore he considered that Quantum Theory is incomplete. (This open problem has been discussed at length in the Solvay Conference on Physics XXII held in Brussels, 2001).

However, from a viewpoint the positivists may be useful, because it should be apparent that in practice we can only speak of the physical observables (measured by some kind of apparatus setting), therefore we don’t know what reality is. In particular, if we put Kant’s word ‘noumena’ instead of ‘reality’, then Hawking’s quote above becomes more make-sense:

“a physical theory is just a mathematical model, and it is meaningless to ask whether it corresponds to noumena.”
Therefore a better scheme to represent the classic dichotomy between positivists and the so-called ‘realists’ is as follows (Figure 9):

![Diagram showing mathematical, physical theories, observables, measurement process, and noumena.]

Figure 9. The set of mathematical, physical theories, and observables

From the scheme shown in Figure 6 it should be clearer why the positivists assert that one can only know (speaking of physical theories) the physical observables via measurement process, but not what Nature really is.

We can also conclude from Figure 9 that it is possible that the Noumena does not necessarily fit into our Mathematical knowledge, which seems quite a contradiction with Pythagorean’s belief. (Of course, it is also possible to suppose that the set of Noumena inherently correspond to the Mathematical theories.)

**Extracting Knowledge from Geometry: Some possible routes**

Now if you feel some relief from reading in preceding section that at least in physical theories one can expect the ‘gluing’ part between mathematical ideas and physical observables, you will never know how physical theories can become so weird, depending on the mathematical notions where they have started from.

For example, one common problem in physical sciences is how to ‘extract’ knowledge in geometry, for instance a planet’s motion or trajectory of satellites. And a fundamental
The mathematical concept behind this geometry is the definition of ‘distance.’ For the beginners, traditionally we use the Cartesian coordinates as follows:

$$ds^2 = dx^2 + dy^2 + dz^2.$$  \hspace{1cm} (1)

But then physicists began to include time as the fourth component of the metric, to become Minkowski metric:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2,$$  \hspace{1cm} (2)

which is known as the basis of the Special Relativity theory (1905). In the meantime, General Relativity theory uses the non-flat metric with constant curvature, which was introduced by Gauss, Riemann etc.:

$$g = g_{ab} \, dx_a \otimes dx^b.$$  \hspace{1cm} (3)

By virtue of the equivalence principle, this pseudo-Riemann metric with constant curvature corresponds to the gravitation phenomena.

While it’s instructive to study this pseudo-Riemann metric in order to understand the General Relativity theory, one can consider another dimension(s) to become 5-dimensional or 6-dimensional metrics and so on. There are also some new theories with extra-dimensions, including higher-dimensional gravities, multidimensional gravity theories, and also Smarandache’s multispace theories.

This kind of metrics with extra-dimension(s) has become so advanced in the so-called superstring theories, where the most recent theory is so-called 26-dimensional Bosonic string.

It is less mentioned in literature that Riemann himself in one of his talk did make a deliberate remark, mentioning that even the concept of distance and metric are merely construction of human mind. He also suggested possibility to study metric where the metric interval is expressed as the fourth exponent of the distance, i.e. for Minkowski metric it can be written:

$$ds = \sqrt{dx^4 + dy^4 + dz^4 + (ic \cdot dt)^4}.$$  \hspace{1cm} (4)

While this metric looks quite awkward, it may be useful for studying gravitation theories, in particular in the context of generalization of pseudo-Riemann metric, for example using the Finsler geometries (e.g. the so-called Berwald-Moor metric), or Smarandache geometries endowed with semi-metrics. (Rabounski, 2010 [9])

It is worth to note here, that one can also consider an extra dimension to Minkowski metric in terms of velocity component, so the metric becomes:
\[
    ds^2 = dx^2 + dy^2 + dz^2 - c^2 \cdot dt^2 - \tau^2 \cdot dv^2,
\]

Which is the so-called Carmeli metric, and the velocity component corresponds to the galaxy’s velocity.

Now, it is possible to find out the symmetries, invariance, and transformation laws corresponding to the above (1)-(5) metrics, and also to figure out their implications to the physical world. Symmetry itself can be defined as “invariance with respect to a transformation group” (Esposito & Marmo, 2005).

Another way is to introduce the Hausdorff dimension into the metric, which one allows to consider non-integer dimension. This seems to correspond to the fact that this Earth and other planetary surfaces are far from smooth; therefore the metric of smooth surface is only an approximation. The problem then is how to express the differential geometry principles for this non-smooth metric. Now we face quite a paradox because a surface with Hausdorff non-integer dimension can be non-differentiable or non-integrable, just like Weirstrass function. Then how can one define differential geometry for non-differentiable surfaces? A particularly noteworthy example in this regards is perhaps Nottale’s Scale Relativity Theory which defines differentiation on such non-differentiable geometry. Another model of universe based on the non-integrable geometry has been presented by Maciejewski et al. (2002), while Ronchetti & de Sabbata (2002) discussed a quantum gravity model based on the notion of Hausdorff dimension. These novel approaches are mentioned here as mere examples on how different theories can emerge from different assumption of the non-smooth geometry.

At this point, perhaps it is not appropriate to speak of mathematics as the inherent properties of Nature anymore (as Pythagoreans would say), we can only guess what is the most consistent geometry corresponding to a given set of Natural phenomena (known to these days). We can only guess it and hopefully will find the true geometrical structure of Nature, possibly via studying the most generalized type of the metric.

The same principles apparently also apply to the physics of elementary particles or bioinformatics. Without reiterating here what Darvas (2008) has described, especially concerning the role of complex numbers in describing codon, or C. M.Yang’s method using quasi-28-gon, one can note that in elementary particles or bioinformatics, the role of metric in standard physics has been replaced with the ‘symmetry principles’ of certain groups.

And with respect to the group theories, then it appears that these symmetry principles can be used to extract new knowledge, just as the role that symmetry consideration may have played during formulation of Newton’s equations or Maxwell’s equations (Darvas 2008).

Nonetheless, there are other types of governing dynamics, for instance the spontaneous symmetry breaking, which can lead to another type or new symmetry principle. How exactly this approach will affect our perception of bioinformatics or the structure of life
itself, remains an open question. For example, does life come from some phenomena related to the spontaneous symmetry breaking of some chemical compounds?

Concluding Note

We have shortly discuss in this article, how scientists including physicists, mathematicians and bioinformatics specialists etc., rely on some special properties in mathematics (via symmetries, transformation and invariance principles) to reveal new kinds of knowledge. These properties are supposed to be able to give some clues of the dynamics of the Nature (or better perhaps, of the dynamics of some given observable phenomena).

Nonetheless, as with the choice of the groups or the metric to be used, it remains an open question to the scientists themselves. In this regard, one should not force his/her own conception to the Nature. Instead, one can begin to learn and respect the Nature.

Concerning how far the contradiction between these approaches can be, one can rephrase an old saying reflecting the (quite antagonistic) Baconian world view: “If you torture the data long enough, Nature will confess.” The modern version of the same ‘attitude’ toward Nature perhaps can be written: “If you torture geometry long enough, Nature will confess.”

Returning to the Einstein’s note on Plank as cited above, the somewhat protagonist view of scientists would learn from Nature and seek to understand it, instead of just forcing Nature to “behave” just as what he/she commend. In other words, apparently it would be better if the physical explanation can be extracted directly from the metric itself plus some new concepts, instead of retaining the same concept but having to “torture” the geometry. In this sense, perhaps one can understand why the General Relativity theory is so fascinating, because it just reinterprets the pseudo-Riemann metric and gives it new physical meaning.

Kaluza-Klein theory also remains beautiful because it only introduces minimal modification to GTR, by including a fifth-component into the metric. But at this point, we don’t want to make early remark on other modern theories including supersymmetry, string theories, etc.

Last but not least, by making this quite strong wording on ‘torture’ of geometry, of course we do not mean that only a handful of approaches are plausible, and other theories shall be forbidden. With regards to mathematical theories, one is free to conceive any kind of idea he/she had, nonetheless at the same time when one develops physical theories, it should be better if they can explain or predict some phenomena where the theories can be put compared with observation. Or if we are allowed to quote what Prof. Gell-Mann once remarked: physicists should find a balance between abstraction and phenomena, just like in Odyssey story one should sail between Scylla and Charibdis.
As for this end of this article, allow me to repeat here a great wisdom saying: May the force be with you.

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References