The New Prime theorem (19)

\[ P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2 \]

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Abstract
Using Jiang function we prove that such that \( P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2 \) has infinitely many prime solutions.

**Theorem.** The prime equation

\[ P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2 \]

has infinitely many prime solutions.

**Proof.** We have Jiang function[1]

\[ J_n(\omega) = \prod_{p} [(P-1)^{n-1} - \chi(P)] \] \hspace{1cm} (2)

where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence

\[ (q_1 q_2 \cdots q_n)^2 - 2 \equiv 0 \pmod{P}, \quad q_i = 1, \cdots, P-1, i = 1, \cdots, n-1 \] \hspace{1cm} (3)

From (3) we have

\[ \left( \frac{2}{P} \right) = (-1)^{\frac{P-1}{8}}, \text{ if } \left( \frac{2}{P} \right) = 1 \text{ then } \chi(P) = 2(P-1)^{n-2}, \text{ if } \left( \frac{2}{P} \right) = -1 \text{ then } \chi(P) = 0 \]

Substituting it into (2) we have.

\[ J_n(\omega) = \prod_{p \leq 3} [(P-1)^{n-2}(P-2^{n-1})^{\frac{P-1}{8}}] \neq 0 \] \hspace{1cm} (4)

We prove that (1) has infinitely many prime solutions. \( J_n(\omega) \subset \phi^{n-1}(\omega) \)

We have the best asymptotic formula

\[ \pi_2(N, n) = \left| \left\{ P_1, \cdots, P_{n-1} \leq N : P_n = \text{prime} \right\} \right| = \frac{J_n(\omega)\omega}{2 \times (n-1)! \phi^{n-1}(\omega) \log^n N} \] \hspace{1cm} (5)

**Example 1.** Let \( n = 2 \). From (1) we have

\[ P_2 = P_1^2 - 2 \] \hspace{1cm} (6)

From (4) we have

\[ J_2(\omega) = \prod_{3 \leq p} [(P-2^{n-1})^{\frac{P-1}{8}}] \neq 0 \] \hspace{1cm} (7)

**Example 2.** Let \( n = 3 \). From (1) we have
\[ P_3 = (P_1 P_2)^2 - 2. \] (8)

From (4) we have
\[ J_3(\omega) = \prod_{3 \leq p} \left[ (P-1)(P-2-(-1)^{\frac{p^2-1}{8}} \right] \neq 0. \] (9)

Note. The prime numbers theory is to count the Jiang function \( J_{n+1}(\omega) \) and Jiang singular series
\[
\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_p \left( 1 - \frac{1+\chi(P)}{P} \right) (1 - \frac{1}{P})^{-k} \] [1], which can count the number of prime number. The prime number is not random. But Hardy singular series \( \sigma(H) = \prod_p \left( 1 - \frac{\nu(P)}{P} \right) (1 - \frac{1}{P})^{-k} \) is false. [2], which can not count the number of prime numbers.

References