The New Prime theorem (17)

\[ P_n = 2P_1P_2 \cdots P_{n-1} \pm 1 \]

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Abstract

Using Jiang function we prove that such that \( P_n = 2P_1P_2 \cdots P_{n-1} \pm 1 \) has infinitely many prime solutions.

**Theorem.** The prime equation

\[ P_n = 2P_1P_2 \cdots P_{n-1} + 1 \]

has infinitely many prime solutions.

**Proof.** We have Jiang function [1]

\[ J_n(\omega) = \prod_{p} [(P-1)^{n-1} - \chi(P)] , \]

where \( \omega = \prod_{p} P \), \( \chi(P) \) is the number of solutions of congruence

\[ 2q_1q_2 \cdots q_{n-1} + 1 \equiv 0 \pmod{P}, \quad q_i = 1, \cdots, P - 1, \quad i = 1, \cdots, n - 1 , \]

From (3) we have

\[ \chi(P) = (P-1)^{n-2} \]

Substituting (4) into (2) we have

\[ J_n(\omega) = \prod_{3 \leq p} [(P-1)^{n-2}(P-2)] \neq 0 . \]

We prove that (1) has infinitely many prime soutions. \( J_n(\omega) \subseteq \phi^{n-1}(\omega) \).

We have the best asymptotic formula [1]

\[ \pi_2(N, n) = \left| \left\{ P_1, \cdots, P_{n-1} \leq N : P_n = \text{prime} \right\} \right| \sim \frac{J_n(\omega)\omega}{(n-1)!\phi^{n}(\omega)} \frac{N^{n-1}}{\log N} . \]

Example 1. Let \( n = 2 \). From (1) we have

\[ P_2 = 2P_1 + 1 \]

From (5) we have

\[ J_2(\omega) = \prod_{3 \leq p} (P-2) \neq 0 \]

Example 2. Let \( n = 3 \). From (1) we have

\[ P_3 = 2P_1P_2 + 1 . \]

From (5) we have

\[ J_3(\omega) = \prod_{3 \leq p} [(P-1)(P-2)] \neq 0 . \]
In the same way we are able to prove that
\[ P_m = 2P_1P_2 \cdots P_{n-1} - 1 \quad (11) \]
has infinitely many prime solutions.

Note. The prime numbers theory is to count the Jiang function \( J_{a+1}(\omega) \) and Jiang singular series
\[ \sigma(J) = \frac{J_2(\omega)(\omega)^{\omega-1}}{\varphi(\omega)} = \prod_p \left( 1 - \frac{\chi(P)}{P} \right) \left( 1 - \frac{1}{P} \right)^{-k} \quad [1], \]
which can count the number of prime number. The prime number is not random. But Hardy singular series
\[ \sigma(H) = \prod_p \left( 1 - \frac{\nu(P)}{P} \right) \left( 1 - \frac{1}{P} \right)^{-k} \quad \text{is false} \quad [2-5], \]
which can not count the number of prime numbers.

References