The New Prime theorem (16)

\[ P_j = (j^n P + (k - j)^n, j = 1, \ldots, k - 1 \]

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Abstract

Using Jiang function we prove that there exist infinitely many primes \( P \) such that each of \( (j^n P + (k - j)^n \) is a prime.

**Theorem.** Let \( k \) be a given prime.

\[ P_j = (j^n P + (k - j)^n (j = 1, \ldots, k - 1, n = 1, 2, \ldots) \]  

There exist infinitely many prime \( P \) such that each of \( (j^n P + (k - j)^n \) is a prime.

**Proof.** We have Jiang function[1]

\[ J_2(\omega) = \prod_{p}(P - 1 - \chi(P)), \]  

where \( \omega = \Pi_P \), \( \chi(P) \) is the number of solutions of congruence

\[ \Pi_{j=1}^{k-1}[(j)^n q + (k - j)^n] \equiv 0 \pmod{P}, q = 1, \ldots, P - 1. \]  

From (3) we have \( \chi(2) = 0 \), if \( P < k \) then \( \chi(P) \leq P - 2, \chi(k) = 1 \), if \( k < P \) then \( \chi(P) \leq k - 1 \). From (3) we have

\[ J_2(\omega) \neq 0. \]  

We prove that there exist infinitely many primes \( P \) such that each of \( (j^n P + (k - j)^n \) is a prime.

Jiang function is a subset of Euler function: \( J_2(\omega) \subset \phi(\omega) \).

We have asymptotic formula

\[ \pi_k(N, 2) = \left| \left\{ P \leq N : (j)^n P + (k - j)^n \text{ prime} \right\} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi(k)} \frac{N}{\log^k N}. \]  

where \( \phi(\omega) = \Pi_P (P - 1) \).

Example 1. Let \( k = 3 \). From (1) we have

\[ P_j = P + 2^n, \quad P_2 = 2^n P + 1 \]  

We have Jiang function

\[ J_2(\omega) = \Pi_{\leq P} (P - 3) \neq 0 \]  

We prove that there exist infinitely many primes \( P \) such that \( P_1 \) and \( P_2 \) are all prime.
Reference