The New Prime theorem (13)

\[ n \times a^n \pm 1 \text{ and } n \times 2^n \pm 1 \]

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Abstract
Using Jiang function we prove that \( n \times a^n \pm 1 \) have infinitely many prime solutions and \( n \times 2^n \pm 1 \) have finite prime solutions.

**Theorem.** We define the irreducible prime equation

\[ P_1 = n \times (P - 1)^n + 1 \quad (1) \]

For every positive integer \( n \) there exist infinitely many primes \( P \) such that \( P_1 \) is a prime.

**Proof.** We have Jiang function[1]

\[ J_2(\omega) = \prod_{P} [P - 1 - \chi(P)], \quad (2) \]

where \( \omega = \prod P \), \( \chi(P) \) is the number of solutions of congruence

\[ n \times (q - 1)^n + 1 \equiv 0 \pmod{P}, \quad q = 1, \ldots, P - 1. \quad (3) \]

From (3) we have that if \( n = 3b + 2 \) then \( \chi(3) = 1, \chi(3) = 0 \) otherwise, \( \chi(P) < P - 1 \). We have

\[ J_2(\omega) \neq 0. \quad (4) \]

We prove that there exist infinitesimally many primes \( P \) such that \( P_2 \) is a prime.

We have asymptotic formula [1]

\[ \pi_2(N, 2) = \left| \left\{ P \leq N : n \times (P - 1)^n + 1 = \text{prime} \right\} \right| \sim \frac{J_2(\omega) N}{n(\omega) \log^2 N} \quad (5) \]

where \( \phi(\omega) = \prod (P - 1) \).

Let \( P = 3 \). From (1) we have Cullen equation

\[ P_1 = n \times 2^n + 1 \quad (6) \]

From (5) we have
We prove the finite Cullen primes.

In the same way we are able to prove that $n \times a^n - 1$ has infinitely many prime solutions, $n \times 2^n - 1$ has definite prime solutions and $h \times 2^n \pm 1$ have finite prime solutions.

Reference