The New Prime theorem（9）
There are finite Fermat primes

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Abstract

Using Jiang function we prove the finite fermat primes.

Theorem. Suppose the prime equation

$$P_1 = (P-1)^{2^n} + 1. \qquad (1)$$

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(q-1)^{2^n} + 1 \equiv 0 \pmod P, \qquad q = 1, \cdots, P-1. \qquad (3)$$

From (3) we have $\chi(P) = 2^n$ if $P \equiv 1 \pmod{2^{n+1}}$, $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N,2) = \left|\left\{P \le N : (P-1)^{2^n} + 1 = prime\right\}\right| \sim \frac{J_2(\omega)}{2^n \phi^2(\omega)} \frac{N}{\log^2 N}. \qquad (4)$$

When $P = 3$. From (1) we have the equation of Fermat number [2]

$$P_1 = 2^{2^n} + 1 \qquad (5)$$

From (4) we have

$$\pi_2(3,2) = \left|\left\{3 \le N : 2^{2^n} + 1 = prime\right\}\right| \sim \frac{J_2(\omega)}{2^n \phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \quad \text{as } n \to \infty \qquad (4)$$

From (4) we prove the finite Fermat primes.

In the same way we are able to prove that $4^{2^n} + 1$ and $6^{2^n} + 1$ have finite prime solutions [2]

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.

[2] P. Ribenboim, The new book of prime number records, 3rd edition, spring-Verlag, New York, NY, 1995.