

The New Prime theorem (6)

$$P, jP+9-j (j=1,2,4,5,7,8)$$

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Abstrat

Using Jiang function we prove that there exist infinitely many primes P such that each $jP+9-j$ is a prime.

Theorem

$$P, jP+9-j (j=1,2,4,5,7,8) \quad (1)$$

There exist infinitely many primes P such that each of $jP+9-j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P-1-\chi(P)], \quad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod (jq+9-j)(j=1,2,4,5,7,8) \equiv 0 \pmod{P} \quad (3)$$

$q=1, \dots, P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=3$, $\chi(7)=3$, $\chi(P)=6$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = 3 \prod_{11 \leq P} (P-7) \neq 0. \quad (4)$$

We prove that there exist infinitely many primes P such that $jP+9-j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_7(N, 2) = |\{P \leq N : jP+9-j = \text{prime}\}| \sim \frac{J_2(\omega)\omega^6}{\phi^7(\omega)} \frac{N}{\log^7 N}, \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Reference

- [1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. <http://www.wbabin.net/math/xuan2.pdf>.