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Title
QUANTUM SIMILARITIES OR RADAR SCATTERING AS A GAUGE THEORY

ABSTRACT

As a continuation of a preceding paper “The trapped light” a preliminary attempt of how to construct a spinor theory of radar scattering or radar signal-target interaction with the gauge theories of quantum mechanics is presented. This way radar signals and radar targets may become visible macroscopic objects to be put in analogy with Standard Model particles and interactions. The basic idea is that particles and forces are all of electromagnetic nature, light, and appear different due to the size and shape of interacting objects.
For the purpose that we propose here, you must first deal with a generic radar signal in spinor form (note: it is TEM, TE, TM or evanescent. I mean “radar signal” in a wide sense, ie for ex. also into a waveguide).
This is done by deriving a spinor representation of the TE TM through the Dirac equation for plane wave, starting rigorously from Maxwell's equations without any use of equivalent V, I as in previous papers.
As a byproduct the representation is extended to the TEM.
Then I introduce a tentative procedure to express the deflection of the field in a different direction, and its variation in frequency, and rest mass. This is accomplished through the interaction with $SU(2)\otimes U(1)$ gauge fields ie electroweak interactions.
Some simple but illustrative examples are given.
The ideas set out here need of course further research.

INTRODUCTION

For the purpose that we propose here, you must first deal with a generic radar signal in spinor form (note: it is TEM, TE, TM or evanescent).
As it is known in the theory of waveguide, the propagation for the TE and TM modes can be represented by an appropriate equivalent transmission line [1] and voltage and current V, I.
In previous papers ([2], [3]) has examined this representation with the equivalent V, I compared with the Dirac equation.
She then demonstrated equivalence between the two representations.
We now want to derive a spinor representation of TE TM in waveguide through the Dirac equation for plane wave, starting from Maxwell's equations rigorously, without any use of equivalent V, I.

To show how and why the Maxwell and the Dirac equations are connected between them, in the following Clifford algebra is used in a formulation which is essentially that of the STA (Hestenes [4]).
When you try to do the same thing with the equations in the classical formalism, the important reasons behind this correspondence is in fact masked by the different character of Maxwell's equations than the Dirac equation, the latter being a matrix equation in terms of Dirac matrices γ_μ.
The STA algebra can treat γ_μ as algebraic elements (the basics of 4D space-time) and thus enables a much more direct correspondence between the Dirac equation and Maxwell's equations.
The Clifford algebra used here is slightly different (see Appendix 1) but is substantially the Hestenes STA, unless the use of "signature" (+++-) with $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = +1$ and $\hat{T}^2 = -1$ instead of " STA signature" (---+) with $\gamma_k^2 = -1, \gamma_0^2 = +1$, $\gamma_k (k = 1,2,3)$.

MAXWELL EQUATIONS

Maxwell's equations are obtained by introducing a Clifford number or "even number" (Appendix 1):

(1) $$F = (E_t + jE_l) + Tji(H_t + jH_l)$$

(in MKSA units should be written with $\sqrt{\varepsilon}E$ and $\sqrt{\mu}H$).
The condition of analyticity for F

(2) $$\partial^* F = 0$$

(3) $$\partial^* = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y} + j\frac{\partial}{\partial z} + \mathrm{T}\frac{\partial}{\partial \tau}$$

provides equaling components:

(4)

$$\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)iH_l+\frac{\partial}{\partial z}iH_t+\frac{\partial}{\partial \tau}E_t=0$$

$$\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)iH_t-\frac{\partial}{\partial z}iH_l+\frac{\partial}{\partial \tau}E_l=0$$

$$\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)E_l+\frac{\partial}{\partial z}E_t+\frac{\partial}{\partial \tau}iH_t=0$$

$$\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)E_t-\frac{\partial}{\partial z}E_l+\frac{\partial}{\partial \tau}iH_l=0$$

where they are placed E_t etc. equal to:

(5)

$$E_t=E_x+iE_y$$

$$E_l=E_z+iE_\tau$$

$$H_t=H_x+iH_y$$

$$H_l=H_z+iH_\tau$$

The (4) are the Cauchy Riemann conditions or the conditions of analyticity $\partial^* F=0$ for F. They coincide with the Maxwell equations for the conjugate F^* (basically changing the sign of the y, z components). Note that this property corresponds exactly to the known properties of analytic functions on the plane, to which it is reduced in two-dimensional case: the conditions of analyticity $\partial^* f=0$ coincide with the field equations for the field which has as components those of f^*.
From another point of view, and are equal regardless of the result, we can instead say "analyticity for F also leads to the analyticity for $F\hat{i}$" that so contains the physical components of $\vec{E}$ and $\vec{H}$ along the axes x, y, z.
So:

(6) $$\partial^* F\hat{i}=0$$

where:

(7) $$F\hat{i}=\vec{E}+\mathrm{T}ji\vec{H}=E_x\hat{i}+E_y\hat{j}+E_z\hat{k}+Tji(H_x\hat{i}+H_y\hat{j}+H_z\hat{k})$$

In (7) then the components y and z are not the same of (5) but the same with a change of sign.
In (7) I also put $E_\tau = 0$ and $H_\tau = 0$ in order to have Maxwell equations in empty space; with $E_\tau \neq 0$ and $H_\tau \neq 0$ terms would appear formally related to electric and magnetic charge and currents.
(Note: strictly speaking the true quality of are those of "time-like bivectors" (Hestenes, [3]), so $F\hat{i}\hat{T}$ should be considered and not $F\hat{i}$. But to avoid too heavy notations use (7) which is sufficient for present purposes).
Is immediate and very smart from (7) to derive the Maxwell's equations with div and rot. It starts with the 3D property in Clifford algebra (Appendix 1):

(8) $$\vec{\partial}_V a = \vec{\partial}_V \bullet a + \vec{\partial}_V \wedge a = \vec{\partial}_V \bullet a + \left(\hat{i}\hat{j}\hat{k}\right)\left(\vec{\partial}_V \times a\right)$$

(9) $$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} = \vec{\partial}_V$$

and therefore the operators div and rot are "embedded" in the 'Clifford algebra, through the formula:

(10) $$\vec{\partial}_V a = diva + \left(\hat{i}\hat{j}\hat{k}\right)\left(rota\right)$$

Then immediately derive Maxwell's equations with div and rot.
In fact we can rewrite (6) as follows:

(11) $$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} + \frac{\partial}{\partial \tau}\hat{T}\right)(\vec{E} + Tji\vec{H}) = (\vec{\partial}_V + \frac{\partial}{\partial \tau}\hat{T})\left(\vec{E} + \hat{i}\hat{j}\hat{k}\hat{T}\vec{H}\right) = 0$$

Developing with (10) and separating the indices comes quickly:

(12) $$rot\vec{E} = -\frac{\partial \vec{H}}{\partial \tau}, rot\vec{H} = \frac{\partial \vec{E}}{\partial \tau}, div\vec{E} = 0, div\vec{H} = 0$$

which are Maxwell equations with div and rot.

DESCRIPTION OF TE TM FIELDS WITH MAXWELL FIELDS IN WAVEGUIDE, COMPONENTS OF A TENSOR OR SPACE TIME BIVECTOR F

Let us express the Maxwell equations for TE or TM.
I take in (4) $E_l = E_z + iE_\tau = 0$ (TE):

$$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) iH_l + \frac{\partial}{\partial z} iH_t + \frac{\partial}{\partial \tau} E_t = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) iH_t - \frac{\partial}{\partial z} iH_l + \frac{\partial}{\partial \tau} E_l = 0$$

(13)

$$\frac{\partial}{\partial z} E_t + \frac{\partial}{\partial \tau} iH_t = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) E_t + \frac{\partial}{\partial \tau} iH_l = 0$$

Now proceed on the assumption of exponential dependence $e^{+i\omega t}$ as is the custom in the IEEE conventions.
Take in the fourth (13) $\frac{\partial}{\partial \tau} = i\omega$:

(14) $$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) E_t = -i\omega iH_l$$

from which iH_l to be replaced in the first:

(15) $$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) E_t \frac{i}{\omega} + \frac{\partial}{\partial z} iH_t + i\omega E_t = -\omega_0^2 E_t \frac{i}{\omega} + \frac{\partial}{\partial z} iH_t + i\omega E_t = 0$$

The first and third equations thus become a TE equations and become precisely:

$$\frac{\partial}{\partial z} iE_t = i\omega H_t$$

(16)

$$\frac{\partial}{\partial z} H_t = i\omega(1 - \frac{\omega_0^2}{\omega^2}) iE_t$$

(Note that to obtain the field components along the x, y axes the (7) holds, ie there is an operation () * conjugation in between).
Do now in (4) $H_l = H_z + iH_\tau = 0$ (TM):

$$\frac{\partial}{\partial z} iH_t + \frac{\partial}{\partial \tau} E_t = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) iH_t + \frac{\partial}{\partial \tau} E_l = 0$$

(17)

$$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) E_l + \frac{\partial}{\partial z} E_t + \frac{\partial}{\partial \tau} iH_t = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) E_t - \frac{\partial}{\partial z} E_l = 0$$

I take in the the second $\frac{\partial}{\partial \tau} = i\omega$

(18) $$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) iH_t + i\omega E_l = 0$$

from which iE_l replacing in the third

(19) $$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial_x} + i\frac{\partial}{\partial_y}\right)\frac{-1}{\omega} H_t + \frac{\partial}{\partial z} E_t - \omega H_t = \frac{1}{\omega}\omega_0^2 H_t + \frac{\partial}{\partial z} E_t - \omega H_t = 0$$

and so arrive to the equations for a TM:

$$\frac{\partial}{\partial z} iH_t = -i\omega E_t$$

(20)

$$\frac{\partial}{\partial z} E_t = \omega(1 - \frac{\omega_0^2}{\omega^2}) H_t$$

In summary I have the complete TE and TM equations derived in the hypothesis $\frac{\partial}{\partial \tau} = i\omega$.

Using the subscript TE or TM to recall how I learned.

$$
(21)\qquad
\begin{aligned}
&\frac{\partial}{\partial z} iE_{TE} - i\omega H_{TE} = 0\\
&\frac{\partial}{\partial z} iH_{TE} + i\omega(1-\frac{\omega_0^2}{\omega^2})E_{TE} = 0\\
\\
&\frac{\partial}{\partial z} iH_{TM} + i\omega E_{TM} = 0\\
&\frac{\partial}{\partial z} iE_{TM} - i\omega(1-\frac{\omega_0^2}{\omega^2})H_{TM} = 0
\end{aligned}
$$

Emphasized again that is passed to the components of field along the x, y axes with:

$$(7\mathrm{bis})\qquad \vec{E} + \mathrm{T}ji\vec{H} = F\hat{\imath} = \left(E_{TE} + \mathrm{T}jiH_{TE} + E_{TM} + \mathrm{T}jiH_{TM}\right)\hat{\imath}\ .$$

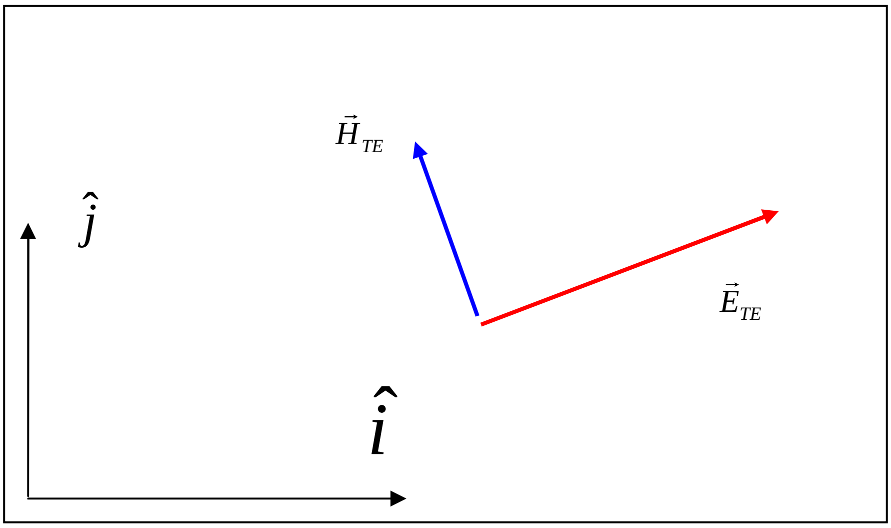
Let me give some physical interpretation.
Take the case TE.
For propagation along z is $\frac{\partial}{\partial z} = -ik_z$ and from the first of the two you have:

$$(22)\qquad k_z E_{TE} - i\omega H_{TE} = 0$$

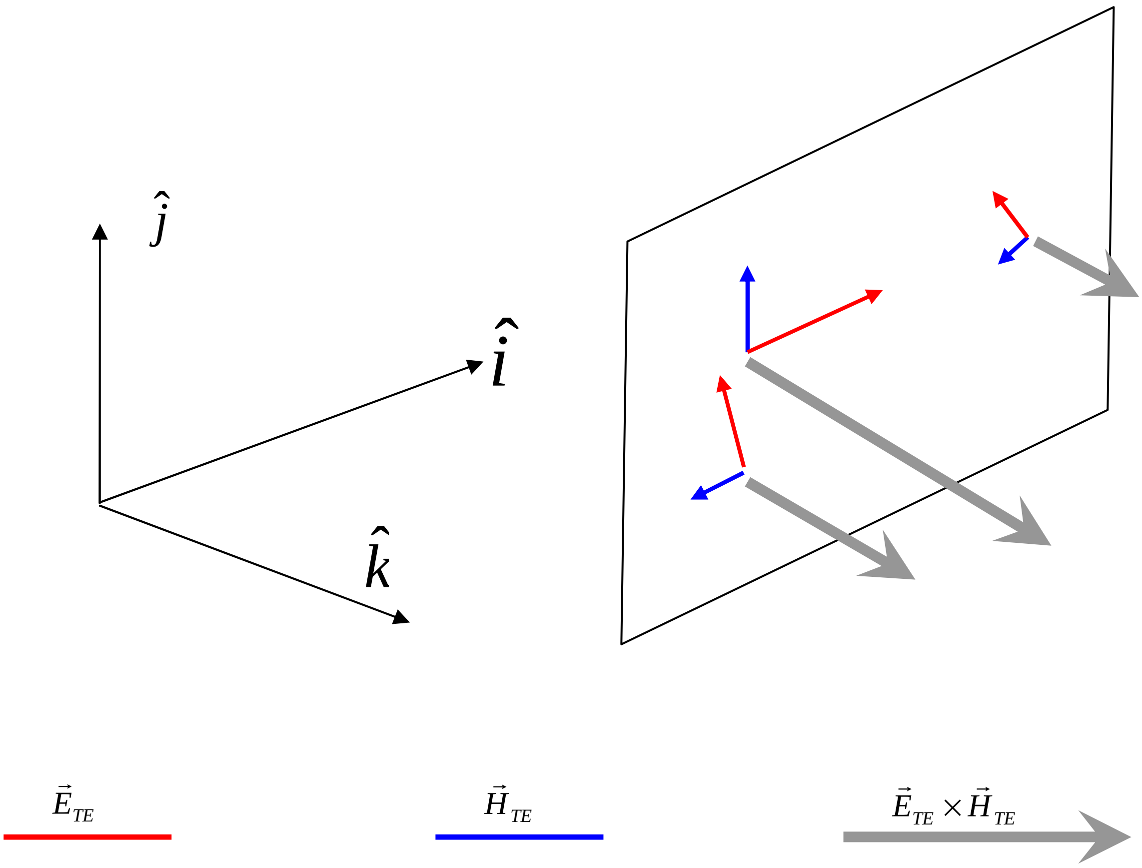
To grasp the physical meaning multiply by $\hat{\imath}$ from right

$$(23)\qquad k_z \vec{E}_{TE} - i\omega\vec{H}_{TE} = 0$$

This shows that $\vec{E}_{TE}, i\vec{H}_{TE}$ are parallel. The meaning is this: at every point, whatever be $\vec{E}_{TE}$, $\vec{H}_{TE}$ is 90° with respect to $\vec{E}_{TE}$, and precisely rotated by $\hat{\imath}$ towards $\hat{\jmath}$.



On the x, y plane at each point $\vec{E}_{TE}$ is 90 ° with respect to $\vec{H}_{TE}$ so as to give rise to a Poynting vector $\vec{E}_{TE} \times \vec{H}_{TE}$ different in amplitude, but always directed toward the positive z:



At this point I do a hypothesis of alternative representation of the TE and TM fields, namely a spinor representation relative to the total energy that propagates in the waveguide (and, as we shall, rest energy or mass, and polarization).

ELIMINATION OF (X, Y) DEPENDENCE AND FIELD DESCRIPTION WITH (FICTITIOUS) FIELDS E, H, COMPONENTS OF A SPINOR ψ.

Let us now turn to an alternative representation of the TE and TM fields, namely a spinor representation, but subject to the condition of properly express the value of the energy that is propagated (and also, as we shall see, the polarization).
We go into more detail.

The electromagnetic fields in waveguide, respectively TE and TM, we have so far dealt with the Maxwell equations.

This means that they are described as follows:
- As regards the fields, through the components E_t, E_Z H_t and H_Z;
- As regards energy, the energy and momentum tensor T_{ik};
- As regards the momentum or energy that propagates in the waveguide in the z direction, at any point in the plane (x, y) from the Poynting vector $\vec{E}_t \times \vec{H}_t$.
I intend to show that this description can be replaced with another equivalent, in which the energy that propagates in the waveguide in the z direction is given by a energy momentum vector $\vec{P} = \psi\hat{T}\psi^*$ through the spinor ψ associated to $\vec{P}$ (see Appendix 2).
In this second mode so the TE and TM modes are described overall by a energy momentum vector $\psi\hat{T}\psi^*$. In other words, the TE and TM modes are physically described only by the total energy that propagates in the waveguide in the z direction that is by $\psi\hat{T}\psi^*$ while the Maxwell field components E_t, H_t (and E_Z and H_Z), are effectively ignored.
I intend to show that with the only condition for the spinor ψ to satisfy the Dirac equation (see Appendix 3) follows for the electromagnetic field a double opportunity of state TE / TM and a double state of circular polarization.
Let us now eliminate the dependence on x, y in order to obtain an overall description of the field as a whole.
Consider the case TE.
Equations for the TE

(21)
$$\frac{\partial}{\partial z} iE_{TE} - i\omega H_{TE} = 0$$
$$\frac{\partial}{\partial z} iH_{TE} + i\omega(1-\frac{\omega_0^2}{\omega^2})E_{TE} = 0$$

ie

(24)
$$k_z E_{TE} - i\omega H_{TE} = 0$$
$$k_z H_{TE} + i\omega(1-\frac{\omega_0^2}{\omega^2})E_{TE} = 0$$

Rewrite multiplying $\hat{i}$ from right:

(25)
$$k_z \vec{E}_{TE} - i\omega \vec{H}_{TE} = 0$$
$$k_z \vec{H}_{TE} + i\omega(1-\frac{\omega_0^2}{\omega^2})\vec{E}_{TE} = 0$$

The first of these two shows that $\vec{E}_{TE}$ and $i\vec{H}_{TE}$ are parallel.

Separate the dependence on x, y in the form

(26)
$$\vec{E}_{TE} = \mathrm{E}_{TE}(z,t)\vec{e}(x,y) = \mathrm{E}_{TE}(z,t)e(x,y)\hat{i} = \mathrm{E}\vec{e}$$
$$\vec{H}_{TE} = \mathrm{H}_{TE}(z,t)\vec{h}(x,y) = \mathrm{H}_{TE}(z,t)h(x,y)\hat{i} = \mathrm{H}\vec{h}$$

Note: you can alter at will the scale of amplitudes between $\mathrm{E} \leftrightarrow \vec{e}$ and $\mathrm{H} \leftrightarrow \vec{h}$ while maintaining the values of $\vec{E}_{TE}$ and $\vec{H}_{TE}$.
By the TE equations (21)

$$\frac{\partial}{\partial z} i\vec{E}_{TE} - i\omega\vec{H}_{TE} = 0$$
$$\frac{\partial}{\partial z} i\vec{H}_{TE} + i\omega(1-\frac{\omega_0^2}{\omega^2})\vec{E}_{TE} = 0$$

we obtain

(27)
$$\frac{\partial}{\partial z} i\mathrm{E}\vec{e} - i\omega\mathrm{H}\vec{h} = 0 \qquad \rightarrow \qquad k_z\mathrm{E}\vec{e} - i\omega\mathrm{H}\vec{h} = 0$$
$$\frac{\partial}{\partial z} i\mathrm{H}\vec{h} + i\omega(1-\frac{\omega_0^2}{\omega^2})\mathrm{E}\vec{e} = 0$$

The first of two $k_z\mathrm{E}\vec{e} - i\omega\mathrm{H}\vec{h} = 0$ shows that $\mathrm{E}\vec{e}$ and $i\mathrm{H}\vec{h}$ are parallel.
In the hypothesis, but it will be true, $\mathrm{E} \propto i\mathrm{H}$ ie $\mathrm{E}\hat{i} \propto i\mathrm{H}\hat{i}$ ie still $\vec{\mathrm{E}} \propto i\vec{\mathrm{H}}$, I can write:

(28)
$$\vec{e} = \mathrm{A}\vec{h}$$

which:

(29)
$$\frac{\partial}{\partial z}\mathrm{EA} + i\omega\mathrm{H} = 0$$
$$\frac{\partial}{\partial z}\mathrm{H} + i\omega(1-\frac{\omega_0^2}{\omega^2})\mathrm{EA} = 0$$

Choose A is equivalent to implicitly define E,H (See Appendix 4).
Choosing:

(30)
$$\mathrm{A} = \frac{\omega}{\omega + \omega_0}$$

obtained:

(31)
$$\frac{\partial}{\partial z}\mathrm{E}_{TE} + (i\omega + i\omega_0)i\mathrm{H}_{TE} = 0$$
$$\frac{\partial}{\partial z}i\mathrm{H}_{TE} + (i\omega - i\omega_0)\mathrm{E}_{TE} = 0$$

and then the equations directly in the required "Dirac form".
With appropriate variants can repeat the procedure for TM and you have the complete equations

(32)
$$\frac{\partial}{\partial z}\mathrm{E}_{TE} + (i\omega + i\omega_0)i\mathrm{H}_{TE} = 0$$
$$\frac{\partial}{\partial z}i\mathrm{H}_{TE} + (i\omega - i\omega_0)\mathrm{E}_{TE} = 0$$
$$\frac{\partial}{\partial z}i\mathrm{H}_{TM} + (i\omega + i\omega_0)\mathrm{E}_{TM} = 0$$
$$\frac{\partial}{\partial z}\mathrm{E}_{TM} + (i\omega - i\omega_0)i\mathrm{H}_{TM} = 0$$

Recalling for $i\omega$ its meaning $\frac{\partial}{\partial \tau} = i\omega$ we arrive at

(33)
$$\frac{\partial}{\partial z}\mathrm{E}_{TE} + (\frac{\partial}{\partial \tau} + i\omega_0)i\mathrm{H}_{TE} = 0$$
$$\frac{\partial}{\partial z}i\mathrm{H}_{TE} + (\frac{\partial}{\partial \tau} - i\omega_0)\mathrm{E}_{TE} = 0$$
$$\frac{\partial}{\partial z}i\mathrm{H}_{TM} + (\frac{\partial}{\partial \tau} + i\omega_0)\mathrm{E}_{TM} = 0$$
$$\frac{\partial}{\partial z}\mathrm{E}_{TM} + (\frac{\partial}{\partial \tau} - i\omega_0)i\mathrm{H}_{TM} = 0$$

Observe separately the equations for the TE and the TM.
The equations for the TE now appear in a particularly symmetrical shape.
They can be seen as providing, at rest, the proposed solution "electric" $e^{+i\omega_0 t}$.
(That is, consistent with the initial assumptions, an exponential dependence "electric” with positive ω).
However, the same equations also provide, at rest, a solution "magnetic" $e^{-i\omega_0 t}$, exponential dependence with negative ω.
Symmetric equations for the TM shall provide, at rest, the proposed solution "magnetic" $e^{+i\omega_0 t}$.

(That is consistent with the initial assumptions, a solution "magnetic" with exponential dependence with positive ω).
However, the same equations also provide, at rest, a solution "electric" $e^{-i\omega_0 t}$, with exponential dependence with negative ω. In summary, the equations in this symmetrical form should no longer have to submit the subscript TE or TM, but rather provide a complete set of solutions "electric" and "magnetic" in the double possibility of exponential dependence $e^{\pm i\omega t}$. This double possibility of exponential dependence means, given the significance of the 'imaginary' i which is nothing but $\hat{i}\hat{j} = i$, the bivector operator of rotations in the x, y plane, a dual state of polarization.
That said, we can now compare these equations with those of Dirac for a plane wave.
The Dirac equation for plane wave in z are (Appendix 3):

(34)
$$\begin{aligned}
\frac{\partial}{\partial z}\psi_3 + \left(\frac{\partial}{\partial \tau} + im\right)\psi_1 &= 0 \\
-\frac{\partial}{\partial z}\psi_4 + \left(\frac{\partial}{\partial \tau} + im\right)\psi_2 &= 0 \\
\frac{\partial}{\partial z}\psi_1 + \left(\frac{\partial}{\partial \tau} - im\right)\psi_3 &= 0 \\
-\frac{\partial}{\partial z}\psi_2 + \left(\frac{\partial}{\partial \tau} - im\right)\psi_4 &= 0
\end{aligned}$$

Rewrite the (33), rearranged as follows

(35)
$$\begin{aligned}
\frac{\partial}{\partial z} i\mathrm{H}_{TM} + \left(\frac{\partial}{\partial \tau} + i\omega_0\right)\mathrm{E}_{TM} &= 0 \\
\frac{\partial}{\partial z} i\mathrm{H}_{TE} + \left(\frac{\partial}{\partial \tau} - i\omega_0\right)\mathrm{E}_{TE} &= 0 \\
\frac{\partial}{\partial z}\mathrm{E}_{TM} + \left(\frac{\partial}{\partial \tau} - i\omega_0\right) i\mathrm{H}_{TM} &= 0 \\
\frac{\partial}{\partial z}\mathrm{E}_{TE} + \left(\frac{\partial}{\partial \tau} + i\omega_0\right) i\mathrm{H}_{TE} &= 0
\end{aligned}$$

With a few more boring, but simply steps it is noted that they are also the Dirac equation for plane wave (26), where it is simply done the following name change in ψ components:

(36)
$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = \begin{bmatrix} \mathrm{E}_{TM} \\ j\mathrm{E}_{TE}\, j \\ i\mathrm{H}_{TM} \\ ij\mathrm{H}_{TE}\, j \end{bmatrix}$$

Here, however, the term TE or TM becomes misleading because for example ψ_1, ψ_3 can be TM but also TE.
Indeed ....
As we have seen, the equations in this symmetrical form should no longer have to submit the subscript TE or TM, but rather provide a complete set of solutions "electric" and "magnetic" in the double possibility of exponential dependence. This double possibility of exponential dependence means , given the significance of the 'imaginary' $\hat{i}\hat{j} = i$, a dual state of polarization.
Therefore this may be more significant further change of name

$$(37) \qquad \begin{bmatrix} \mathrm{E}_{TM} \\ \mathrm{E}_{TE} \\ \mathrm{H}_{TM} \\ \mathrm{H}_{TE} \end{bmatrix} = \begin{bmatrix} \mathrm{E}_{+} \\ \mathrm{E}_{-} \\ \mathrm{H}_{+} \\ \mathrm{H}_{-} \end{bmatrix}$$

which I get the equations of the form:

$$(38) \qquad \begin{aligned} &\frac{\partial}{\partial z} i\mathrm{H}_{+} + \left(\frac{\partial}{\partial \tau} + i\omega_0\right)\mathrm{E}_{+} = 0 \\ &\frac{\partial}{\partial z} i\mathrm{H}_{-} + \left(\frac{\partial}{\partial \tau} - i\omega_0\right)\mathrm{E}_{-} = 0 \\ &\frac{\partial}{\partial z} \mathrm{E}_{+} + \left(\frac{\partial}{\partial \tau} - i\omega_0\right) i\mathrm{H}_{+} = 0 \\ &\frac{\partial}{\partial z} \mathrm{E}_{-} + \left(\frac{\partial}{\partial \tau} + i\omega_0\right) i\mathrm{H}_{-} = 0 \end{aligned}$$

These then are the Dirac equation (34) with the following name change in the ψ components :

$$(39) \qquad \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = \begin{bmatrix} \mathrm{E}_{+} \\ j\mathrm{E}_{-}j \\ i\mathrm{H}_{+} \\ ij\mathrm{H}_{-}j \end{bmatrix}$$

We'll see after the reason of the use of the subscript (+) or (-).
For a further discussion see Appendix 5.

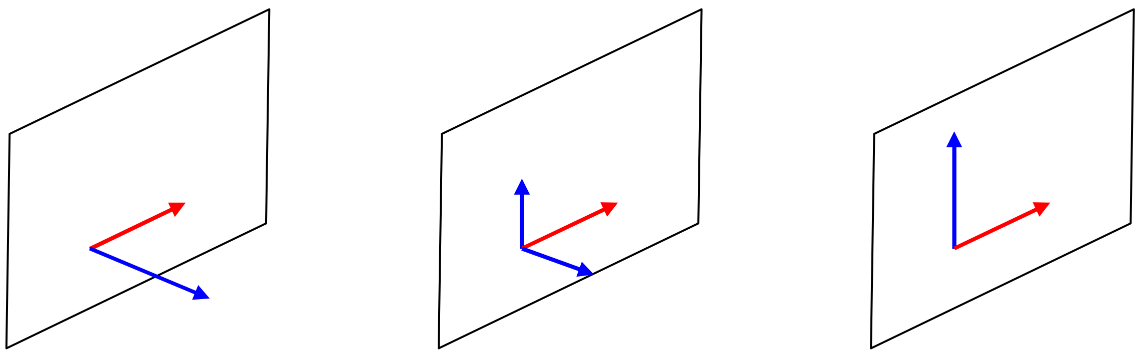
INTUITIVE INTERPRETATION OF E, H

Notwithstanding that E, H are components of a spinor ψ, can give a physical interpretation "intuitive" for E, H.
(Note: not to be confused symbols as E, H, field components, with symbols E, H, components of a spinor ψ)
Since the transverse field components are given by (7a) we can read E, H as "fictitious" field components using the same formula. A fictitious field follows:

(40) $$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\mathrm{E}_{TE} + \mathrm{T}ji\mathrm{H}_{TE} + \mathrm{E}_{TM} + \mathrm{T}ji\mathrm{H}_{TM}\right)\hat{\imath}$$

From a physical point of view we are thus confronted with a possible dual description of the fields in waveguide.
Consider (for example) a TE. I summarize with the aid of a drawing.

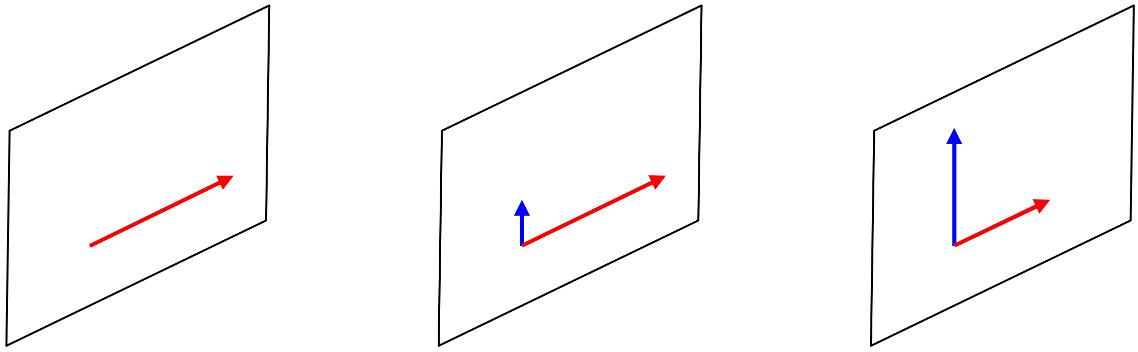
Description of the TE field with Maxwell fields (real fields), components of a space-time bivector F.
Transverse fields E_t, H_t and longitudinal field H_z.



$\omega = \omega_0$ $\qquad$ $\omega \to \infty$

electric fields ——— $\qquad$ magnetic fields ———

Description of the TE field with a spinor ψ: fields (fictitious fields) E,H.
Only transverse fields E,H.



Note: we will see that with a choice of scale factors may be invoked for the fictitious fields E,H the condition on the Poynting vector

(41) $$\vec{\mathrm{E}} \times \vec{\mathrm{H}} = \oiiint \vec{E}_t \times \vec{H}_t dV$$

Now all this can be useful if we want to create us a visual image of E,H.
But we have seen that E,H are components of a Dirac spinor ψ.
We can then also create us directly a visual image of the components of ψ.
We can see how ψ directly generates the fictitious field $\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}}$.
For brevity only summarize the results, for a complete discussion refer to Appendix 5.
Let ψ a spinor solution of Dirac equation

(42) $$\psi = \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4$$

Let:

(43) $$\psi_+ = (\psi_1 + \mathrm{T}j\psi_3)$$
$$\psi_- = (j\psi_2 + \mathrm{T}\psi_4)$$

(it may be interesting to note that these are the parts which respectively commute or not with i, and whose meaning in quantum mechanics is the separation of the solutions at opposite spin).

Fictitious field $\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}}$ is obtained directly from

(44) $$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \psi_+ \hat{i} + \psi_- (-j)\hat{i}$$

This also justifies the reason for the name with the subscripts (+) and (-).
It is apparent

(45) $$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\mathrm{E}_+ + \mathrm{T}ji\mathrm{H}_+ + \mathrm{E}_- + \mathrm{T}ji\mathrm{H}_-\right)\hat{i}$$

fully equivalent to the previous (44).

CHOICE OF SCALE FACTORS

We start from the Dirac equation (34) for free particle with the components ψ_1 and ψ_3.

(46) $$\begin{aligned} &\frac{\partial}{\partial z}\psi_3 + \left(\frac{\partial}{\partial \tau} + im\right)\psi_1 = 0 \\ &\frac{\partial}{\partial z}\psi_1 + \left(\frac{\partial}{\partial \tau} - im\right)\psi_3 = 0 \end{aligned}$$

These may be a TE or a TM depending on if resolved at rest with $\psi_1 \neq 0$ or with $\psi_3 \neq 0$.
TE case.
These solved for $\psi_1 \neq 0$ at rest provide (placing $m \to \omega_0$)

(47) $$\psi_1 = e^{-i\omega t + ik_z z} \qquad \psi_3 = Be^{-i\omega t + ik_z z} \qquad B = \frac{\sqrt{\omega - \omega_0}}{\sqrt{\omega + \omega_0}}$$

(48) $$k_z^2 = \omega^2 - \omega_0^2$$

We use $v_g = \dfrac{d\omega}{dk_z}$, formula for the group velocity in the waveguide.

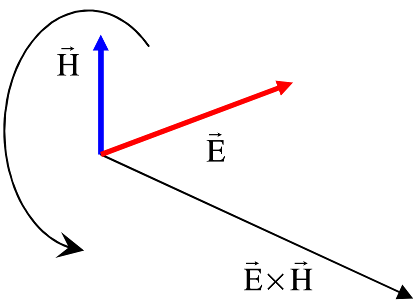
From (48) is obtained

(49) $k_z = \sqrt{\omega^2 - \omega_0^2}$ so

(50) $v_g = \dfrac{d\omega}{dk_z} = \sqrt{1 - \dfrac{\omega_0^2}{\omega^2}}$

In summary,

$$\psi = \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4$$
$$\psi_+ = (\psi_1 + \mathrm{T}j\psi_3)$$
$$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \psi_+ \hat{i}$$
$$\vec{\mathrm{E}} + Tji\vec{\mathrm{H}} = (1 + Tj\frac{\sqrt{\omega - \omega_0}}{\sqrt{\omega + \omega_0}})e^{-i\omega t + ik_z z}\hat{i} \text{ or}$$
$$\vec{\mathrm{E}} + Tji\vec{\mathrm{H}} = e^{-i\omega t + ik_z z}\hat{i} + Tji\frac{\sqrt{\omega - \omega_0}}{\sqrt{\omega + \omega_0}}e^{-i\omega t + ik_z z}\hat{j}$$



Equations of TM.
We can (for instance) start from (46) for the components ψ_1 and ψ_3, but this time looking for a solution with ψ_3 different from zero at rest. Will get a TM solution as opposed to the previous polarization.
The (46) in fact have solutions

$$\psi_3 = e^{+i\omega t - ik_z z} \qquad \psi_1 = Be^{+i\omega t - ik_z z} \qquad B = \frac{\sqrt{\omega - \omega_0}}{\sqrt{\omega + \omega_0}}$$

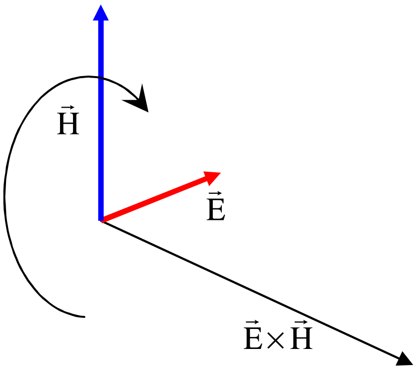
(51)
$k_z^2 = \omega^2 - \omega_0^2$ da cui:

In summary, we have:

$$\vec{\mathrm{E}}+Tji\vec{\mathrm{H}}=(\frac{\sqrt{\omega-\omega_0}}{\sqrt{\omega+\omega_0}}+Tj)e^{+i\omega t-ik_z z}\hat{i}$$

namely:

$$\vec{\mathrm{E}}+Tji\vec{\mathrm{H}}=Tjie^{+i\omega t-ik_z z}\hat{j}+\frac{\sqrt{\omega-\omega_0}}{\sqrt{\omega+\omega_0}}e^{+i\omega t-ik_z z}\hat{i}$$



We note that H in case TE play the role of "small component" [8] of the wave function, while E is the "large component". In contrast to the TM (see Appendix 6). We perform now the explicit calculation of $\psi\hat{T}\psi^*$ for the TE.

$$\psi=\mathrm{E}_{TE}+Tj(i\mathrm{H}_{TE})=(\psi_1+\mathrm{T}j\,\psi_3)$$
$$\psi\hat{T}\psi^*=(\psi_1+Tj\,\psi_3)\hat{T}(\psi_1{}^*-\psi_3{}^*Tj)$$

(52) $$\psi\hat{T}\psi^*=(\psi_1\psi_1{}^*+\psi_3\psi_3{}^*)\hat{T}+Tj(\psi_3\psi_1{}^*+\psi_1\psi_3{}^*)\hat{T}$$

or by $\psi_{TE}=\mathrm{E}_{TE}+Tj(i\mathrm{H}_{TE})$

$$\psi\hat{T}\psi^*=(\mathrm{E}\ +Tji\mathrm{H})\hat{T}(\mathrm{E}^*+Tji\mathrm{H}^*)$$

(53) $$\psi\hat{T}\psi^*=(\mathrm{EE}^*+\mathrm{HH}^*)\hat{T}+Tji(\mathrm{HE}^*-\mathrm{EH}^*)\hat{T}$$

I take ½ and replace $\hat{i}\hat{j}\hat{k}\hat{T}=Tji$ so with some step:

(54) $$\frac{1}{2}\psi\hat{T}\psi^*=\frac{1}{2}(\mathrm{EE}^*+\mathrm{HH}^*)\hat{T}-\frac{1}{2}\hat{i}\hat{j}\hat{k}(\mathrm{HE}^*-\mathrm{EH}^*)$$

For correspondence already established (40) are valid relations:

$$\vec{\mathrm{E}} = \mathrm{E}\hat{i}$$
$$\vec{\mathrm{H}} = \mathrm{H}\vec{i}$$

so

$$\vec{\mathrm{E}}\hat{i} = \mathrm{E} \leftrightarrow \mathrm{E}^* = \hat{i}\,\vec{\mathrm{E}}$$
$$\vec{\mathrm{H}}\hat{i} = \mathrm{H} \leftrightarrow \mathrm{H}^* = \hat{i}\,\vec{\mathrm{H}}$$

Substituting in (54) we have:

(55) $$\frac{1}{2}\psi\hat{T}\psi^* = \frac{1}{2}(\left|\vec{\mathrm{E}}\right|^2 + \left|\vec{\mathrm{H}}\right|^2)\hat{T} - \frac{1}{2}\hat{i}\hat{j}\hat{k}(\vec{\mathrm{H}}\vec{\mathrm{E}} - \vec{\mathrm{E}}\vec{\mathrm{H}})$$

Recalling the formulas (Appendix 1)

$$\frac{1}{2}(ab - ba) = a \wedge b$$
$$a \times b = -(\hat{i}\hat{j}\hat{k})(a \wedge b)$$

finally we arrive at:

(56) $$\frac{1}{2}\psi\hat{T}\psi^* = \frac{1}{2}(\left|\vec{\mathrm{E}}\right|^2 + \left|\vec{\mathrm{H}}\right|^2)\hat{T} - (\vec{\mathrm{E}}\times\vec{\mathrm{H}})$$

This is the expression of the energy momentum four-vector according to the techniques and notations (ie $\psi\hat{T}\psi^*$) relevant to quantum mechanics.
For the electromagnetic field momentum and energy density is calculated from (7) and provides:

(57) $$\frac{1}{2}F\hat{\mathrm{T}}F^* = \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}} - \vec{E}\times\vec{H}$$

(see also Hestenes, [4]).
This is the fourth row of the field energy momentum tensor and provides energy and momentum density. It should be noted that (56) and (57) are formally identical, which justifies the name for E,H as fictitious fields. But remember that E,H transform like components of a spinor ψ. See also Appendix 7 as an exercise.
The volume integral of (57) is instead a four-vector (Pauli, [9]):

(58) $$\vec{P} = \oiiint \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}}dV - \oiiint \vec{E}\times\vec{H}dV$$

In the integration circulating terms of the Poynting vector offset one another and remains the only contribution of the Poynting vector in the z direction, which is provided by transverse fields.
One can thus write

$$\vec{P} = \oiiint \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}}dV - \oiiint \vec{E}_t \times \vec{H}_t dV$$

which explicitly becomes, with the appropriate transverse fields TE

(59) $$\vec{P} = \oiiint \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}}dV - \oiiint \vec{E}_{TE} \times \vec{H}_{TE} dV$$

But it was placed with (26):

$$\vec{E}_{TE} = \mathrm{E}_{TE}(z,t)\vec{e}(x,y) = \mathrm{E}_{TE}(z,t)e(x,y)\hat{i} = \vec{\mathrm{E}}e*$$
$$\vec{H}_{TE} = \mathrm{H}_{TE}(z,t)\vec{h}(x,y) = \mathrm{H}_{TE}(z,t)h(x,y)\hat{i} = \vec{\mathrm{H}}h*$$

and then in (59) can be substituted

$$\vec{E}_{TE} = \vec{\mathrm{E}}e*$$
$$\vec{H}_{TE} = \vec{\mathrm{H}}h*$$

which with some step (Appendix 8):

(60) $$\vec{E}_{TE} \times \vec{H}_{TE} = (\vec{\mathrm{E}} \times \vec{\mathrm{H}})eh*$$

In summary this leads to transform (59) in the form

(61) $$\vec{P} = \oiiint \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}}dV - (\vec{\mathrm{E}} \times \vec{\mathrm{H}})\oiiint eh*dV$$

To match the total momentum to that expressed by (56) through ψ establishes the scale of amplitudes taking:

(62) $$\oiiint eh*dV = 1$$

From this follows

(63) $$\vec{P} = \oiiint \frac{1}{2}\left(\left|\vec{E}\right|^2 + \left|\vec{H}\right|^2\right)\hat{\mathrm{T}}dV - (\vec{\mathrm{E}} \times \vec{\mathrm{H}}) = \frac{1}{2}\psi\hat{T}\psi* = \frac{1}{2}(\left|\vec{\mathrm{E}}\right|^2 + \left|\vec{\mathrm{H}}\right|^2)\hat{T} - (\vec{\mathrm{E}} \times \vec{\mathrm{H}})$$

Summarize.
Describes the propagation as a whole, with total momentum and energy.
For this you choose the level of amplitude for E,H that match the total momentum in the z direction

(64) $$\vec{\mathrm{E}}\times\vec{\mathrm{H}} = \oiiint \vec{E}_t \times \vec{H}_t dV$$

(note: $\vec{\mathrm{E}}\times\vec{\mathrm{H}}$ is the development of $\psi\hat{T}\psi^*$).
In both formulations group velocity in z is the same.
As both descriptions are relativistic, this equality of speed and total momentum in z direction ensures the correctness and coincidence of representations not only of the momentum, but of the whole (momentum, energy, and rest energy or mass). See also Appendix 9.

EXTENSION TO TEM

For a TEM ($E_l = 0$ e $H_l = 0$) equations (4) become

(65) $$\frac{\partial}{\partial z} iH_t + \frac{\partial}{\partial \tau} E_t = 0$$
$$\frac{\partial}{\partial z} E_t + \frac{\partial}{\partial \tau} iH_t = 0$$

and simultaneously with $E_l = 0$ and $H_l = 0$ must also be

(66) $$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) iH_t = 0$$
$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) E_t = 0$$

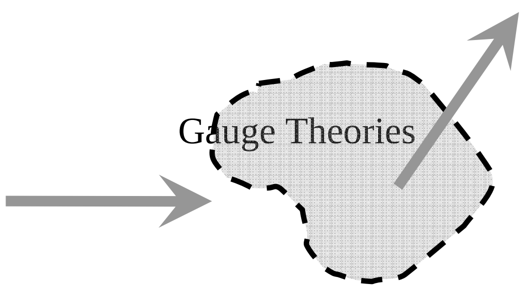
ie the transverse fields E_t and H_t are analytic on x, y plane.
Maxwell's equations are reduced to two, and simply provide two different solutions that differ only in the polarization.
For the transition to the spinor representation is legal to refer again to the same reasoning that led to equations (33). Obviously these are reduced from four to two, being perfectly coincident equations for the TE or TM when $\omega_0 = 0$.
In fact, (33) become:

$$
\begin{aligned}
&\frac{\partial}{\partial z}\mathrm{E}_{TE} + \left(\frac{\partial}{\partial\tau}\right) i\mathrm{H}_{TE} = 0\\
&\frac{\partial}{\partial z} i\mathrm{H}_{TE} + \left(\frac{\partial}{\partial\tau}\right)\mathrm{E}_{TE} = 0\\
&\frac{\partial}{\partial z} i\mathrm{H}_{TM} + \left(\frac{\partial}{\partial\tau}\right)\mathrm{E}_{TM} = 0\\
&\frac{\partial}{\partial z}\mathrm{E}_{TM} + \left(\frac{\partial}{\partial\tau}\right) i\mathrm{H}_{TM} = 0
\end{aligned} \tag{67}
$$

ie two by two equal and are reduced to

$$
\begin{aligned}
&\frac{\partial}{\partial z} i\mathrm{H} + \frac{\partial}{\partial\tau}\mathrm{E} = 0\\
&\frac{\partial}{\partial z}\mathrm{E} + \frac{\partial}{\partial\tau} i\mathrm{H} = 0
\end{aligned} \tag{68}
$$

These are formally identical to (65) but the big difference is that E,H are components of a spinor. Therefore all the previous considerations apply, including (64) for the choice of the scale of magnitude.
We are now able to handle a generic radar signal (note: it is TEM, TE, TM or evanescent. I mean “radar signal” in a wide sense, ie for ex. also interactions into a waveguide).
It raises the possibility of a spinor theory of scattering or signal - target interaction with the gauge theories of quantum mechanics.
Consider, for example, a TEM radar signal incident on a target from a certain direction.



Spinor theory of radar scattering

Propose a procedure that takes place through interaction with a $SU(2)\otimes U(1)$ gauge fields (see also [10]) and that may express the deflection of the field in a different direction, and its (possible) change of frequency.

DEFLECTION AND CHANGE OF FREQUENCY OF A RADAR SIGNAL

To illustrate the guidelines of the procedure introduce it first for a signal in a waveguide.
I start for this from the Dirac equation (Appendix 3) to rewrite it in this form

(69) $\vec{\partial}\psi + m\psi i\hat{T} = 0 \qquad \vec{\partial} = \hat{i}\partial_x + \hat{j}\partial_y + \hat{k}\partial_z + \hat{T}\partial_\tau)$

and introducing a local gauge transformation on ψ

(70) $\psi \to \psi' = \psi e^{+i\varphi}$

where φ is a function of the coordinates

(71) $\varphi = \varphi(x, y, z, t)$

The transformation (70) is the electromagnetic gauge transformation and is part of $SU(2)$.
With it is

$$\vec{\partial}\psi' = \vec{\partial}(\psi e^{+i\varphi}) = (\vec{\partial}\psi)e^{+i\varphi} + (\hat{i}\,\psi\partial_x + \hat{j}\,\psi\partial_y + \hat{k}\,\psi\partial_z + \hat{T}\,\psi\partial_\tau)e^{+i\varphi}$$

or

$$\vec{\partial}\psi' = (\vec{\partial}\psi)e^{+i\varphi} + (\hat{i}\partial_x\varphi + \hat{j}\partial_y\varphi + \hat{k}\partial_z\varphi + \hat{T}\partial_\tau\varphi)\psi i e^{+i\varphi}$$

Now replaced in (69) and is (after simplification of an exponential from right):

(72) $\vec{\partial}\psi' + m\psi' i\hat{T} = \vec{\partial}\psi + (\hat{i}\partial_x\varphi + \hat{j}\partial_y\varphi + \hat{k}\partial_z\varphi + \hat{T}\partial_\tau\varphi)\psi i + m\psi i\hat{T} = 0$

If (69) is true, this is no longer true because the presence of a foreign term. So to force the Dirac equation valid for ψ still valid for ψ' we introduce in (69) instead of $\vec{\partial}\psi$ a covariant derivative $\vec{D}\psi$ that delete the foreign term. Should be in place of $\vec{\partial}\psi$

(73) $\vec{\partial}\psi \to \vec{D}\psi = \vec{\partial}\psi - eA\psi i$

where

(74) $eA = \hat{i}\partial_x\varphi + \hat{j}\partial_y\varphi + \hat{k}\partial_z\varphi + \hat{T}\partial_\tau\varphi = e(\vec{A} + \hat{T}\Phi)$

The equation is then amended

(75) $\bar{\partial}\psi - eA\psi i + m\psi i\hat{T} = 0$

and will have for example in place of TE solution (48) I recall

(48) $\vec{k}_z^2 = \omega^2 - \omega_0^2$

a new and different solution where both the frequency and the direction of propagation will be changed.
In this context (75) is used to express the frequency variation and the variation of the direction of propagation of the TE. But it is formally the Dirac equation for a particle-electron in the presence of electromagnetic potentials $\vec{A},\Phi$, namely the scalar potential Φ and the vector potential $\vec{A}$.
To appreciate the kind of result we can therefore refer to this example.
As you know, instead of the formula for free particle, analogous to (48)

(76) $\vec{p}^2 = E^2 - m^2$

by (75) and with $\vec{A},\Phi \neq 0$ constants (see for example Schiff) we have:

(77) $(\vec{p} - e\vec{A})^2 = (E - e\Phi)^2 - m^2$

The result then is this: the scalar potential Φ changes the energy and the vector potential $\vec{A}$ changes the momentum $\vec{p}$.
Similarly we can therefore conclude that a gauge transformation (70), and with φ (71) function of the coordinates and time, will change ω (see also [2]) and $\vec{k}$ of a TE.
As a hypothesis we can then to conclude that with this we will be able to express at will the desired change in frequency and direction of propagation of a TE in waveguide. (The latter means a change of direction of the waveguide).
Note that to have the same effect in a TM must change the sign to the coupling parameter e.

After this introduction we briefly illustrate the same situation for a TEM. Also mention here the procedure as a preliminary attempt.
Consider the action of a transformation

(78) $\psi \rightarrow \psi' = \psi e^{-TjiZt - iUt}$

Transformation with i is the electromagnetic gauge transformation and is a part of $SU(2)$. Transformation with Tji is $U(1)$.
The (78) involves the introduction of an appropriate covariant derivative that leads to the equation:

(79) $\partial^* \psi + j i \psi Z + T \psi i U = 0$

Developing full gives:

(80)
$$\frac{\partial}{\partial z}\psi_3 + \frac{\partial}{\partial \tau}\psi_1 + i\psi_3 Z + \psi_1 i U = 0$$
$$\frac{\partial}{\partial z}\psi_1 + \frac{\partial}{\partial \tau}\psi_3 + i\psi_1 Z + \psi_3 i U = 0$$

These equations provide, in the absence of gauge fields, a TEM solution that can be both right and left.
Let's see what possible solutions exist in the presence of gauge fields.
Seeking a solution in the form (TEM "right").

(81)
$$\psi_1 = e^{-i\omega t + ik_z z}$$
$$\psi_3 = e^{-i\omega t + ik_z z}$$
$$k_Z = \omega$$

Substituting (81) in (80) with k_Z and ω indeterminate are actually solutions of the form (81) with the condition:

(82) $(k_Z + Z)^2 = (\omega - U)^2$

So from an initial condition in the absence of fields with $k_Z = \omega$ must happen that k_Z and ω undergo a change as to satisfy (82).
From a physical point of view a TEM can increase or decrease the frequency through the interaction with an object (or a "target"). For example a moving target that communicates a Doppler ω_d.
However, a TEM can increase or decrease ω, but must do maintaining the condition of equality between ω and k (which means propagation speed c = 1). It follows from (82) that the action of U and Z is not permissible with the signs that appear there, that is (for positive U and Z) with an increase of k_Z and a decrease of ω.
Therefore the only possible hypothesis is that under the transformation (78):
a) U and Z appear both and not single;
b) U and Z have equal value and opposite sign and then
c) there are " coupling charges" to U and Z opposite.
We appear in (80) the presence of "coupling charges" to U and Z. Quite subtle I use the following arbitrary names:
call $[Y/2]$ the coupling charge to Z;
call $[T3]$ the coupling charge to U.
The (80) thus becomes:

$$\frac{\partial}{\partial z}\psi_3 + \frac{\partial}{\partial \tau}\psi_1 + \left[Y/2\right] i\psi_3 Z + [T3]\psi_1 iU = 0$$

(83)

$$\frac{\partial}{\partial z}\psi_1 + \frac{\partial}{\partial \tau}\psi_3 + \left[Y/2\right] i\psi_1 Z + [T3]\psi_3 iU = 0$$

Solve with:

(84) $$\left[\frac{Y}{2}\right] = +\frac{1}{2}$$

(85) $$[T3] = -\frac{1}{2}$$

Is thus the solution TEM "right":

(86) $$\left(k_Z + \frac{1}{2}Z\right)^2 = \left(\omega + \frac{1}{2}U\right)^2$$

This solution is now physically compatible and is the action a "moving target" which imparts a Doppler ω_d with an increased frequency from ω to $\omega + \omega_d$. The action of this object is so identified with the field produced by the gauge transformation (78). Now consider the solution TEM "left" in the absence of fields:

(87) $$\psi_1 = e^{+i\omega t - ik_z z}$$
$$\psi_3 = e^{+i\omega t - ik_z z}$$
$$k_Z = \omega$$

Interacting with the same target first and then under the action of gauge fields produced by the transformation (78) we found under hypothesis (84) (85) the following solution of (83):

(88) $$\left(k_Z - \frac{1}{2}Z\right)^2 = \left(\omega - \frac{1}{2}U\right)^2$$

This leads to the absurd situation where the same target to communicate a positive doppler to TEM "right" and a negative Doppler to TEM "left", which is not what physically happens.
Are we supposed to "coupling charge" of the TEM "left" to U and Z equal to:

(89) $$\left[\frac{Y}{2}\right] = -\frac{1}{2}$$

(90) $$[T3] = +\frac{1}{2}$$

therefore opposite to those of TEM "right".
This will find the correct solution (86).
It is therefore necessary for the coupling charges to the gauge fields the following situation:

	$\left[Y\!/_{2}\right]$	$[T3]$
TEM “left”	$\frac{1}{2}$	$-\frac{1}{2}$
TEM “right”	$-\frac{1}{2}$	$\frac{1}{2}$

which clearly recalls the classification of neutrinos in the Standard Model (obviously without being able to assign any meaning to the symbols, which I chose to art):

	$\left[Y\!/_{2}\right]$	$[T3]$
ν_L	$\frac{1}{2}$	$-\frac{1}{2}$
$\bar{\nu}_R$	$-\frac{1}{2}$	$\frac{1}{2}$

This leads to a possible interpretation in terms of analogy with the action of Z°.
However this is not what interests me now.
What interests me is to have demonstrated the following:
for the ω of a TEM can increase or decrease through the interaction with a radar target must consider the effect of a gauge transformation with i and Tji simultaneously:

(91) $$\psi \rightarrow \psi' = \psi e^{+b\varphi(t)}$$

where

(92) $$b = \frac{i(1-Tj)}{2}$$

Note: we change signs in the (78) in the right way and introduce a single coupling charge to i and Tji.
This holds for ω.
What about the direction of propagation?
Extend the (91) to the case where φ is a function of the coordinates of space and time.
We can repeat the whole procedure followed with the formulas (71) (77) and in particular come to the conclusion

(93) $\vec{\partial}\psi - QA_{Z^0}\psi b = 0$

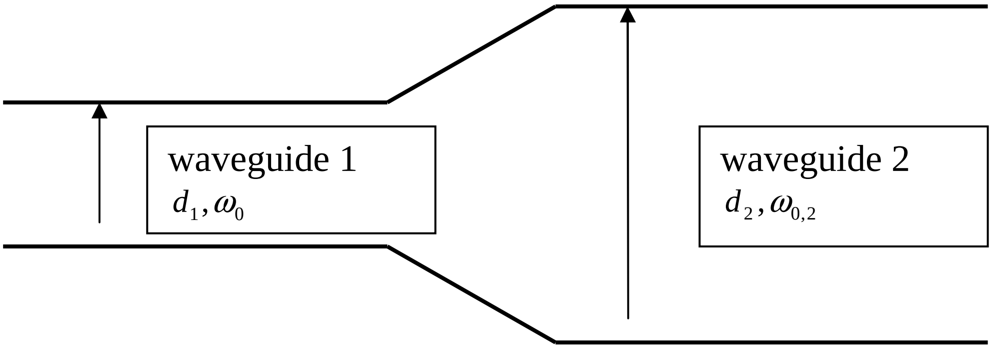
where

(94) $QA_{Z^0} = \hat{i}\partial_x\varphi + \hat{j}\partial_y\varphi + \hat{k}\partial_z\varphi + \hat{T}\partial_\tau\varphi = Q(\vec{A}_{Z^0} + \hat{T}\Phi_{Z^0})$

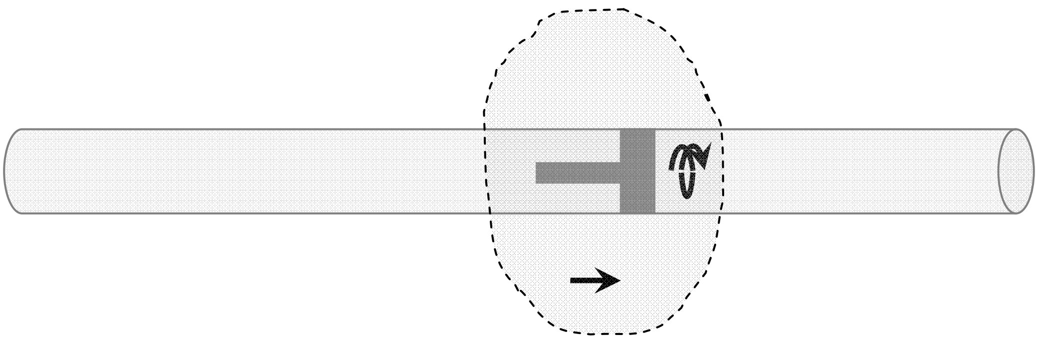
and you will have for example in place of the solution TEM R (81) with $\vec{k}^2 = \omega^2$ a new and different solution where both ω and $\vec{k}$ will be changed.
The (93) is formally the Dirac equation for a particle of zero mass, in the presence of potential $A_{Z^0} = \vec{A}_{Z^0} + \hat{T}\Phi_{Z^0}$ namely a scalar potential Φ_{Z^0} and a vector potential $\vec{A}_{Z^0}$.
(I use this notation for the potential they represent an action that resembles that of the Z° particle).
The effect of the scalar potential has already been examined with (86).
From the action of the vector potential $\vec{A}_{Z^0}$ instead, we must expect effects on the direction of propagation.
So we can conclude for the moment that with a gauge transformation (91) we can properly express at will the change in frequency and direction of propagation of a radar signal in free space.
I repeat that the potential introduced by (91) recall an action reminiscent of the particle Z°, but that was not what I was interested in showing at this time. What interested me was to identify a possible procedure to deflect a TEM.

INTERPRETATION OF THE ACTION OF GAUGE FIELDS

I intend to show that the varying potentials are interpreted as forces exerted by physical objects.
In the case of the electromagnetic potential Φ it is easily interpretable.
I interpreted in [2] the action of the electromagnetic potential Φ as a equivalent "waveguide 2" with a different cutoff $\omega_{0,2}$ (ie size d_2).

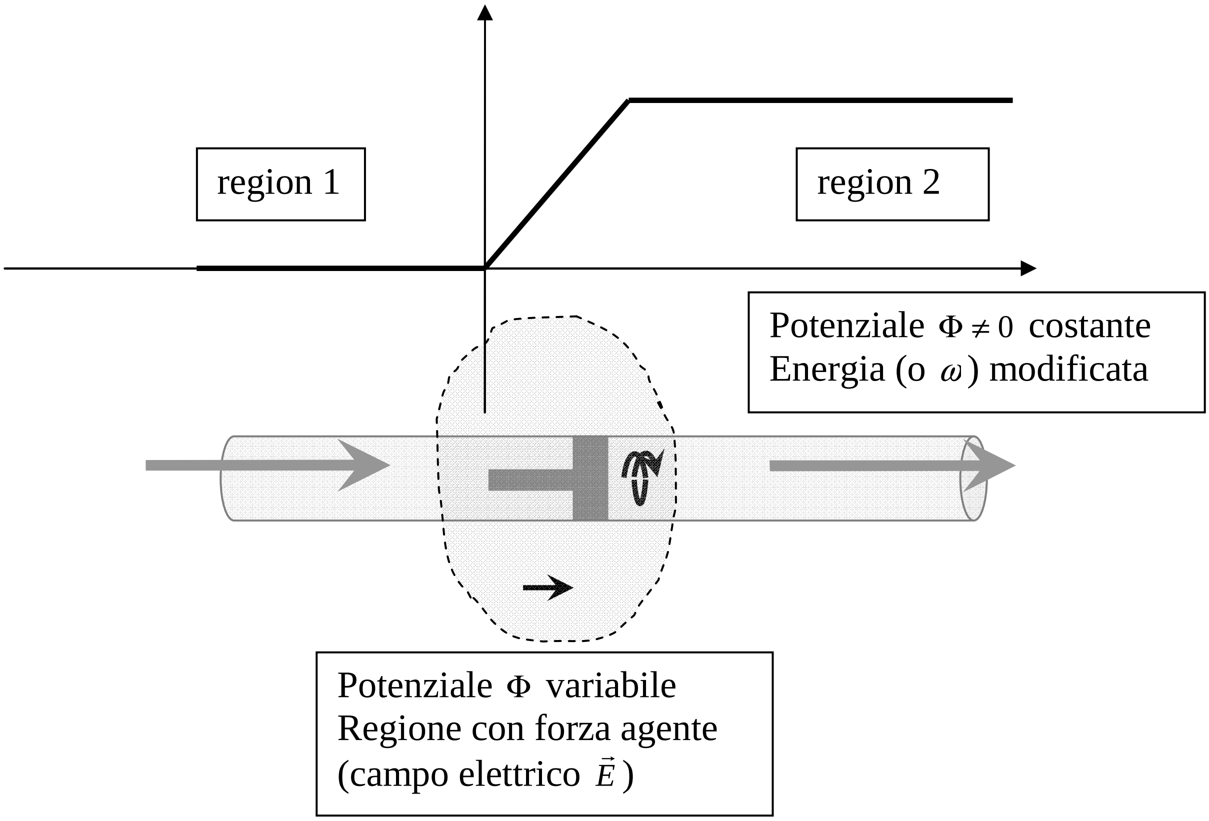


Alternatively, rather than considering a equivalent "waveguide 2" with cutoff $\omega_{0,2}$
I've interpreted in [10] the action of Φ with the action of someone or something that has changed ω in ($\omega - U$). That something is detectable by the mathematical point of view in the operation $\psi \rightarrow \psi' = \psi e^{-iUt}$, who acted on ψ increasing (if $U \leq 0$) the ω.
From the electromagnetic point of view such an action is produced by the interaction with an object in the waveguide that imparts a Doppler frequency (here positive).
A push.

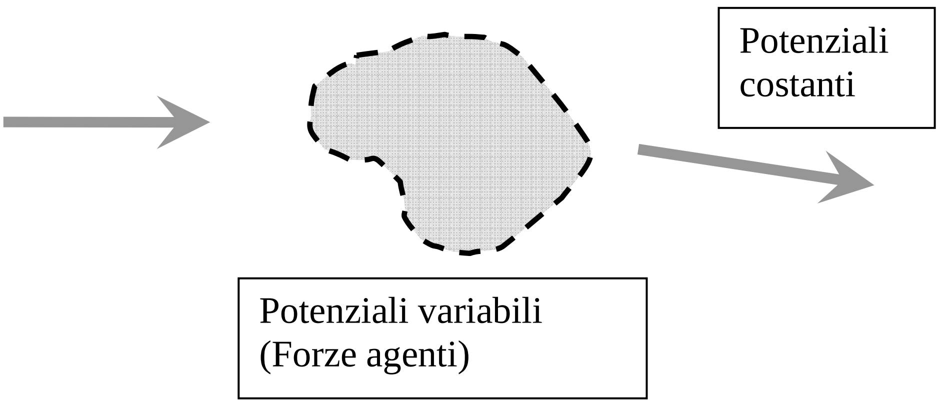


Anyhow, the resulting action is that of "electromagnetic force" exerted by the γ particle or the potential Φ.
However you want to interpret, there are two regions:
-a first region where the potential is manifested changing from the value $\Phi = 0$ to the final value Φ.
In this region Φ is variable and there is a force, an electric field $\vec{E} = -grad\Phi$.

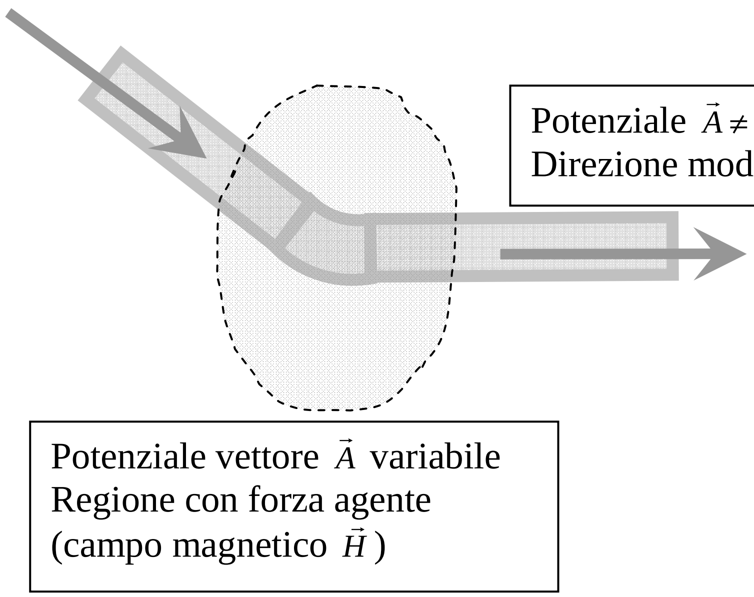
-a region where there is a potential $\Phi \neq 0$ constant. In this region Φ is constant and the energy (the ω) is changed and that remains.



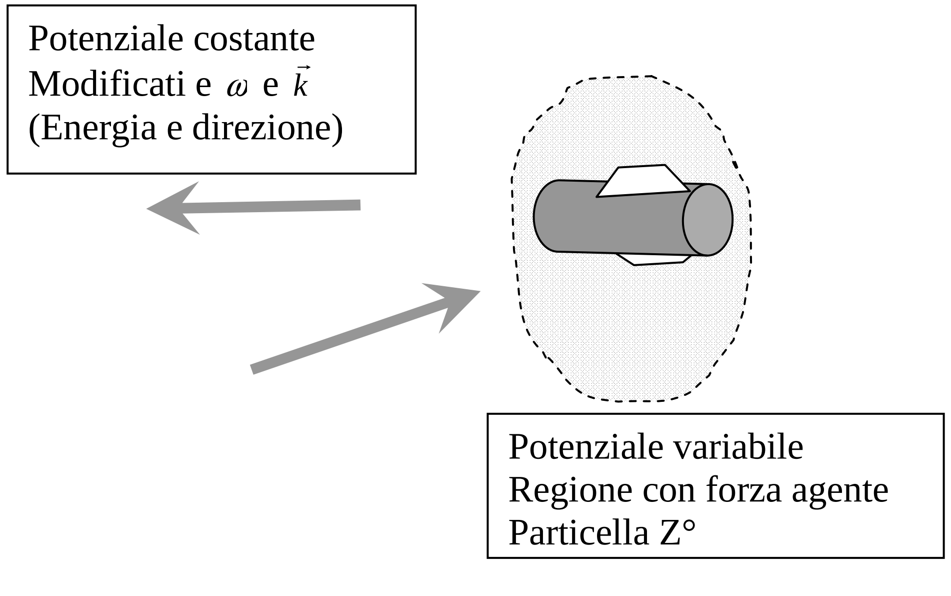
This example illustrates a very general situation in which we can interpret the behavior of gauge fields.
It starts from an initial situation in the absence of potentials.
There is a region of interaction in which potentials occur, until reaching their final value.
In this region, with variable potentials, actions that are produced can be attributed to force field (the derivatives of the potentials).
Reached their final value the potentials remains constant. Any subsequent change would entail a new force.



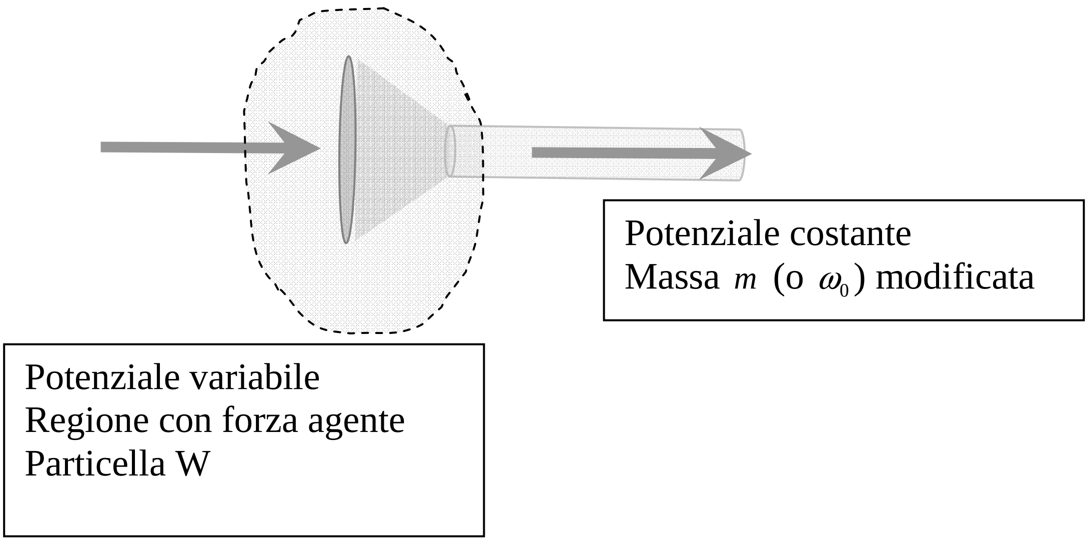
Take the case of the vector potential $\vec{A}$.
In this case $\vec{A}$ variable means magnetic field. In the short interaction region manifests an effect similar to the effect of a magnetic field on an electric charge: a deflection. The situation is succinctly summarized in the figure.



The gauge fields expressed by (93) lead instead to the following interpretation



Finally in [10] have examined a gauge field the effect is similar to the action of the W particle, which is interpreted as shown.



CONCLUSIONS

It was rigorously derived a spinor representation of TE TM through the Dirac equation for plane wave, alternative to Maxwell's equations.
Doing this the TE and TM modes are physically described only by the total energy that propagates in a waveguide in the z direction and this is provided by a four vector $\psi\hat{T}\psi^*$ while the components of the Maxwell field E_t, H_t (and E_Z and H_Z), are effectively ignored.
With the unique mathematical condition for the spinor ψ . satisfy the Dirac equation, is automatically for the electromagnetic field a double possibility of state TE / TM and a double state of circular polarization.
All this also transfers to the special case of TEM. In this case describes the electromagnetic field in free space (specifically: radar signals in free space) and the Dirac equation are those of the neutrino.
Using the spinor representations thus obtained opens up the possibility of a spinor theory of radar scattering or radar-target interaction, briefly outlined here, developed with the gauge theories of quantum mechanics. Some simple but illustrative examples are given.
Further study is of course needed in order to verify if this approach can gain the status of a viable theory.

ACKNOWLEDGEMENTS

I wish to thank Dr. Alberto Bicci for the suggestions and support given on the subject.

APPENDIX 1

I use a Clifford algebra based on 4 elements $\hat{i}\ \ \hat{j}\ \ \hat{k}\ \ \hat{\mathrm{T}}$ (axis unit vectors in spacetime, sometimes referred to the authors with other symbols, such e_1, e_2, e_3, e_0). They have the following properties:

(1) $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = +1 \qquad \hat{\mathrm{T}}^2 = -1$

and all anticommute between them, eg

$\hat{j}\hat{i} = -\hat{i}\hat{j} \qquad \hat{\mathrm{T}}\hat{k} = -\hat{k}\hat{\mathrm{T}}$ etc

Possibly I use the symbols $i\ \ j\ \ \mathrm{T}$ to generalize the usual imaginary unit i of the x, y plane

(2) $i = \hat{i}\hat{j} \qquad j = \hat{i}\hat{k} \qquad \mathrm{T} = \hat{i}\hat{\mathrm{T}}$

All this, combined with the rule concerning the conjugates

(3) $(AB)^* = B^* A^*$

generates all properties of interest.

It's enough to admit that fact that $\hat{i}\ \ \hat{j}\ \ \hat{k}\ \ \hat{\mathrm{T}}$ do not change by conjugation (as it is intuitive that it should be) to derive for example, or rediscover, the usual rule for the conjugate i^*:

(4) $i^* = (\hat{i}\hat{j})^* = \hat{j}^* \hat{i}^* = \hat{j}\hat{i} = -\hat{i}\hat{j} = -i$

and so are obtained as a simple consequence all other properties (and therefore do not need to send to mind):

(5) $j^* = -j \qquad \mathrm{T}^* = -\mathrm{T}$

$i^2 = -1 \qquad j^2 = -1 \qquad \mathrm{T}^2 = 1$

$ij = -ji \qquad i\mathrm{T} = -\mathrm{T}i \qquad j\mathrm{T} = -\mathrm{T}j$

$(\mathrm{T}ji)^* = \mathrm{T}ji \qquad (\mathrm{T}ji)^2 = -1$

The algebra is constructed by all possible products between $\hat{i}\ \ \hat{j}\ \ \hat{k}\ \ \hat{\mathrm{T}}$.

The algebra has 16 items

1, $\hat{i}\ \ \hat{j}\ \ \hat{k}\ \ \hat{\mathrm{T}}$ (4 items), $\hat{i}\hat{j}\ \ \hat{i}\hat{\mathrm{T}}$ etc. (6 items), $\hat{i}\hat{j}\hat{k}$ etc. (4 items), $\hat{i}\hat{j}\hat{k}\hat{\mathrm{T}}$

and contains a subalgebra of 8 elements ("even subalgebra of a Clifford Algebra", Hestenes)

$1,\qquad \hat{i}\hat{j}\quad \hat{i}\hat{\mathrm{T}}$ etc. (6 items), $\qquad \hat{i}\hat{j}\hat{k}\hat{\mathrm{T}}$

It can be rewritten at will as consisting of all possible products between $i \;\; j \;\; \mathrm{T}$

$$1\ ,\ i\ ,\ \ j\ ,\ \mathrm{T}\ \ ,\ ij\ \ ,\ i\,\mathrm{T}\ \ ,\ \ j\,\mathrm{T}\ \ ,\ \mathrm{T}\ ji$$

On xy plane symbols or operators

(6) $$\partial = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y}$$

$$\partial^* = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$$

are, respectively, to express the derivative and the Cauchy Riemann conditions. These are generalized in

(7) $$\partial = \frac{\partial}{\partial x} - i\frac{\partial}{\partial y} - j\frac{\partial}{\partial z} - \mathrm{T}\frac{\partial}{\partial \tau}$$

$$\partial^* = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y} + j\frac{\partial}{\partial z} + \mathrm{T}\frac{\partial}{\partial \tau}$$

and the property is

(8) $$\partial\partial^* = \partial^*\partial = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2}$$

Alternatively, the symbol or operator ∂^* used to express the analyticity can use the operator that is obtained by multiplying $\hat{i}$ left (note: if $\partial^* f = 0$ also $\hat{i}\partial^* f = 0$ and vice versa).
The operator thus obtained

(9) $$\hat{i}\partial^* = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} + \frac{\partial}{\partial \tau}\hat{\mathrm{T}} = \vec{\partial}_V$$

is formally seen as a four-vector, as $\vec{P}$.
In Clifford algebra a product naturally arises that incorporates scalar product and vector product.

It starts from the obvious equality:

(10) $$ab = \frac{1}{2}(ab + ba) + \frac{1}{2}(ab - ba)$$

This truism becomes a raison d'etre for the fact that there are elements of Clifford Algebra which anticommute, so it makes sense to speak of ba distinct from ab. They are also potentially opposite.
An analysis of this formula with some examples immediately shows that

(11) $$\frac{1}{2}(ab + ba) = a \bullet b$$

is the usual inner product between vectors and is a scalar, while what should be called *exterior* product:

(12) $$\frac{1}{2}(ab - ba) = a \wedge b$$

remember, but do not call it that, the vector product $a \times b$.
For if a and b are vectors, $a \wedge b$ is a bivector, while $a \times b$ è is a vector.
Between the two there is the formula:

(13) $$a \wedge b = (\hat{i}\hat{j}\hat{k})(a \times b)$$

you can also use reversed

(14) $$a \times b = -(\hat{i}\hat{j}\hat{k})(a \wedge b)$$

The (14) is not necessary to send her to mind because it is easily remembered by the example:

(15) $$\hat{i}\hat{j} = (\hat{i}\hat{j}\hat{k})\hat{k}$$

that relates the bivector $\hat{i}\hat{j}$ with the vector $\hat{i} \times \hat{j} = \hat{k}$.
(Note: the introduction of $a \times b$ due to Gibbs hides the true quality of the product of two orthogonal vectors, which are those of an entity bivector. However the formula (14) make things right).
We extend the (14) to the vector operator $\vec{\partial}_V$ (3D):

(16) $$\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} = \vec{\partial}_V$$

From (11)...(14) we have successively

(17) $$\vec{\partial}_V a = \vec{\partial}_V \bullet a + \vec{\partial}_V \wedge a = \vec{\partial}_V \bullet a + \left(\hat{i}\hat{j}\hat{k}\right)\left(\vec{\partial}_V \times a\right)$$

and therefore the operators div and rot are "embedded" in the 'Clifford algebra through the relation:

(18) $$\vec{\partial}_V a = diva + \left(\hat{i}\hat{j}\hat{k}\right)\left(rota\right)$$

This algebra differs from the STA for the choice of the base with the properties (1). The STA choice is for a basis of "spacelike" vectors $\gamma_k (k = 1,2,3)$ and a "timelike" vector that instead of (1) has the properties:

(19) $$\gamma_k^2 = -1, \gamma_0^2 = 1$$

So doing to obtain a unit vector basis of three axes x, y, z with modulus (+1) three spacetime bivectors should be defined (Hestenes, [4]):

(20) $$\sigma_k = \gamma_k \gamma_0$$

Hestenes note explicitly the opportunities of either choice ([4], p.25):
“If instead we had chosen $\gamma_k^2 = 1, \gamma_0^2 = -1$ we could entertain the solution $\sigma_k = \gamma_k$, which may seem more natural, because (.....)”,
......because vectors in space would also be vectors in spacetime.
I prefer to keep this option best suits to engineers ($\hat{i}\ \ \hat{j}\ \ \hat{k}$ with $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = +1$, vectors in space equal to vectors in spacetime, the symbol i for the imaginary unit on x, y plane, complex numbers in x, y plane $x + iy$, etc.).
I should also note that all the names that I used as a vector, complex number, imaginary unit vector and so on recall mnemonically concepts of the past and we can sometimes help but are materially misleading. All the things we have introduced are simply <u>numbers</u>, and if we can correctly called "Clifford numbers", the simple underlying rules, the sum product and division of Clifford algebra. The same goes for symbols such as the arrow in $\vec{P}$ etc. here have the sole function of mnemonic recall. What matters are only the properties of algebra I have briefly summarized.

APPENDIX 2

Recall briefly the description of a four vector $\vec{P} = \psi \hat{T} \psi^*$ by a spinor ψ associated with it.
Let’s start from the study of a plane motion with complex numbers, rather than through the velocity vector tangent to the trajectory. Placing

$\vec{V} = \rho e^{i\varphi}\hat{i}$

instead of the analysis in terms of velocity vector $\vec{V}$ goes to the study of complex number $z = \rho e^{i\varphi}$.
We can say (after Hestenes) that the operation that has made introduced a Clifford algebra constructed on the basis of the two unit vector $\hat{i}, \hat{j}$ of the x, y plane and having identified as "imaginary" the bivector $i = \hat{i}\hat{j}$.
The space of complex numbers z is thus identified as "even subalgebra of a Clifford Algebra" of components, so if you want to call it, "real" 1 and "imaginary" i. The essential thing is that everything is clear, all roles, including geometric, are clarified. The word "complex" or "imaginary" is essentially useless or misleading.
Let us now jump next to move from 2D to 3D space.
Everything is repeated with the added fact, that I consider irrelevant even if it is necessary, that now the complex number must be applied half right and half left. All of this is known.
The number now has 4 components and is called quaternion.
With the usual language and the clarity of clarification we owe to David Hestenes (although my symbols) we can say that this introduces an "even subalgebra of a Clifford Algebra" built on a 3-unit vector space $\hat{i}, \hat{j}, \hat{k}$.
The components of quaternions are precisely the components "even" of algebra $1, \hat{i}\hat{j}, \hat{i}\hat{k}, \hat{j}\hat{k}$.
The last and decisive step is to pass in 4D, ie the study of a vector of spacetime or four-vector with a complex number, according to the usual technique that we have seen in 2D and in 3D space.
It is necessary (and sufficient) to introduce a Clifford algebra on a basis of 3 unit spacelike vectors and one timelike:
$\hat{i}\,,\ \hat{j}\,, \hat{k}\,, \hat{T}$

and this identifies a "even subalgebra of a Clifford algebra" to 8 components

$1, \hat{i}\hat{j}, \hat{i}\hat{k}, \hat{j}\hat{k}, \hat{i}\hat{T}, \hat{j}\hat{T}, \hat{k}\hat{T}, \hat{i}\hat{j}\hat{k}\hat{T}$

Complex numbers ψ are now Dirac spinors with the exception of details and / or notations. Even now, the complex number must be applied half right and half left. For example, if $\vec{P}$ is an energy momentum vector then $\vec{P} = \psi\hat{T}\psi^*$ as with the quaternions (except here the use of $\hat{T}$) and so on.
Note that the sub-case with components $1, \hat{i}\hat{j}, \hat{i}\hat{k}, \hat{j}\hat{k}$ provides the aforementioned quaternions in 3D space while the components $1, \hat{i}\hat{j}$ give the ordinary complex numbers in the 2D x,y plane.

Among the various consequences of the rotation in spacetime there is one now eg. a four-velocity can be rotated with a bivector like $\hat{i}\hat{j}$ and then rotate on $\hat{i},\hat{j}$ plane, but also with a bivector like $\hat{i}\,\hat{T}$ (Lorentz transformation) and then rotate on $\hat{i},\hat{T}$ plane or speeds up or slows down.
So we can summarize how the energy propagates in the waveguide in the z direction is provided by an energy momentum four vector $\vec{P} = \psi\hat{T}\psi^*$ through the spinor ψ associated to $\vec{P}$.
I remember that the time axis $\hat{T}$ rotated through a Lorentz transformation becomes the four-velocity $\hat{u}$ ($\hat{T}^2 = -1, \hat{u}^2 = -1$). Indeed let $\psi = R$ unitary:

$$R = e^{\hat{k}\hat{T}\frac{\varphi}{2}}$$

and rotate $\hat{T}$ doing

$$\hat{u} = R\hat{T}R^* = e^{\hat{k}\hat{T}\frac{\varphi}{2}}\hat{T}e^{-\hat{k}\hat{T}\frac{\varphi}{2}} = \hat{T}e^{-\hat{k}\hat{T}\varphi}$$

So

$$\hat{u} = \hat{T}e^{-\hat{k}\hat{T}\varphi} = \hat{T}\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - \hat{k}\hat{T}\frac{\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}}\right)$$

where

$$\varphi = arcth\frac{V}{c}$$

The four vector $\hat{u}$ is the four-velocity of the body. Its square is (-1) for any velocity V. In the example considered the motion is the z axis having been made a Lorentz transformation (rotation) according to bivector $\hat{k}\hat{T}$ normal to the (z,τ) plane .
Summarize.
A rotation with $\psi = R$ gives the four-velocity $R\hat{T}R^* = \hat{u}$.
Multiplying by mc as in relativistic mechanics ($p_i = mcu_i$ [5]) yields instead the energy momentum vector $\vec{P} = mc\hat{u} = \psi\hat{T}\psi^*$.
While $\vec{P} = mc\hat{u} = \psi\hat{T}\psi^*$ transforms as a vector, ψ transforms as a spinor. The law of transformation "single-sidedly" of spinors is summarized effectively by Doran et. al. ([6] "States and operators in the Spacetime Algebra", Found. Phys. 23 (9), 1993).
If a vector, such $\hat{s} = \psi\hat{k}\psi^*$ is rotated through $R'(\_)R'^*$, the result of the rotation is

$$\hat{s}' = R'\hat{s}R'^*$$

then the corresponding spinor ψ must become

$\psi' = R'\psi$

“We use the term spinor to denote any object which transforms single–sidedly under a rotor R” (Doran, [6]).

APPENDIX 3

The Dirac equation is obtained by introducing an 8-components "even number" exactly structured as F , unless the different notations for the components.
Let:

(1) $\psi = \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4$

where $\psi_1\psi_2\psi_3\psi_4$ are with indexes $1, i$. The Dirac equation is:

(2) $\partial^*\psi = -\hat{i}m\psi i\hat{\mathrm{T}}$

Developing and equating the components we obtain the Dirac equation in the usual extended form, see ex. Schiff [7]:

(3)
$$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_4 + \frac{\partial}{\partial z}\psi_3 + \left(\frac{\partial}{\partial \tau} + im\right)\psi_1 = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 - \frac{\partial}{\partial z}\psi_4 + \left(\frac{\partial}{\partial \tau} + im\right)\psi_2 = 0$$

$$\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 + \frac{\partial}{\partial z}\psi_1 + \left(\frac{\partial}{\partial \tau} - im\right)\psi_3 = 0$$

$$\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1 - \frac{\partial}{\partial z}\psi_2 + \left(\frac{\partial}{\partial \tau} - im\right)\psi_4 = 0$$

From ψ may form several "squares" for example
(4) $\psi\psi^*$

or the four-velocity $\hat{u}$

(5) $\psi\hat{\mathrm{T}}\psi^* = \hat{u} \qquad \hat{u}^2 = -1$

true if ψ is unitary ie if ψ is a "rotor"

(6) $\psi = R \qquad RR^* = 1$

Conditions of relativistic invariance for (2), i.e. invariance with respect to spacetime rotations of an angle φ, make that ψ transforms with half angle $\varphi/2$. This implies (the fact is a consequence of the other and vice versa) all quantities like $\psi \hat{\Gamma} \psi^*$ transform like vectors.

APPENDIX 4

From the TE equations

$$\frac{\partial}{\partial z} i\vec{E}_{TE} - i\omega \vec{H}_{TE} = 0$$
$$\frac{\partial}{\partial z} i\vec{H}_{TE} + i\omega(1 - \frac{\omega_0^2}{\omega^2})\vec{E}_{TE} = 0$$

write

$$\vec{E}_{TE} = V(z,t)\vec{e}(x,y)$$
$$i\vec{H}_{TE} = I(z,t)\vec{h}(x,y)$$

While compliance with the values of $\vec{E}_{TE}$ and $i\vec{H}_{TE}$ you can alter at will the scale of amplitudes between $V \leftrightarrow \vec{e}$ and $I \leftrightarrow \vec{h}$. In particular, put

$$\vec{e}(x,y) = \mathrm{A}\vec{h}(x,y)$$

The equations become:

$$\frac{\partial}{\partial z} VA + i\omega I = 0$$
$$\frac{\partial}{\partial z} I + i\omega(1 - \frac{\omega_0^2}{\omega^2})VA = 0$$

Choose $\frac{\vec{e}}{\vec{h}} = 1$ means to choose for $\frac{V}{I}$ the same ratio that exists between $\vec{E}_{TE}$ and $i\vec{H}_{TE}$, ie $\frac{\vec{E}_{TE}}{i\vec{H}_{TE}} = \frac{\omega}{k_z}$. With this choice we come to the usual form of the equation of the equivalent transmission line [3]. In fact if I take A=1 I get the equations for V, I:

$$\frac{\partial}{\partial z}V = -i\omega I$$
$$\frac{\partial}{\partial z}I = -i\omega(1-\frac{\omega_0^2}{\omega^2})V$$

But one can proceed in a different way by observing that the equations are

$$\frac{\partial}{\partial z}VA + i\omega I = 0$$
$$\frac{\partial}{\partial z}I + i\frac{(\omega-\omega_0)(\omega+\omega_0)}{\omega}VA = 0$$

which it is found that choosing instead:

$$A = \frac{\omega}{\omega+\omega_0}$$

equations are obtained directly in the request "Dirac form", as I did in the text.

APPENDIX 5

From

$$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\mathrm{E}_{TE} + \mathrm{T}ji\mathrm{H}_{TE} + \mathrm{E}_{TM} + \mathrm{T}ji\mathrm{H}_{TM}\right)\hat{\imath}$$

exploiting the aforementioned

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} = \begin{bmatrix}\mathrm{E}_{TM}\\ j\mathrm{E}_{TE}\, j\\ i\mathrm{H}_{TM}\\ ij\mathrm{H}_{TE}\, j\end{bmatrix}$$

get

$$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\psi_1 + \mathrm{T}j\psi_3 + j\psi_2(-j) + \mathrm{T}\psi_4(-j)\right)\hat{\imath}$$

equivalent to

$$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \psi_+\hat{\imath} + \psi_-(-j)\hat{\imath}$$

being

$$
\begin{aligned}
\psi &= \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4 \\
\psi_+ &= (\psi_1 + \mathrm{T}j\psi_3) \\
\psi_- &= (j\psi_2 + \mathrm{T}\psi_4)
\end{aligned}
$$

This also justifies the reason for the name with the subscripts (+) and (-)

$$
\begin{bmatrix} \mathrm{E}_{TM} \\ \mathrm{E}_{TE} \\ \mathrm{H}_{TM} \\ \mathrm{H}_{TE} \end{bmatrix} = \begin{bmatrix} \mathrm{E}_+ \\ \mathrm{E}_- \\ \mathrm{H}_+ \\ \mathrm{H}_- \end{bmatrix}
$$

It is apparent

$$
\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\mathrm{E}_+ + \mathrm{T}ji\mathrm{H}_+ + \mathrm{E}_- + \mathrm{T}ji\mathrm{H}_-\right)\hat{i}
$$

fully equivalent to

$$
\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \left(\psi_1 + \mathrm{T}j\psi_3 + j\psi_2(-j) + \mathrm{T}\psi_4(-j)\right)\hat{i}
$$

For comparison:

$$
\begin{aligned}
\mathrm{E}_+ &= \psi_1 \\
\mathrm{T}ji\mathrm{H}_+ &= \mathrm{T}j\psi_3 \\
\mathrm{E}_- &= j\psi_2(-j) \\
\mathrm{T}ji\mathrm{H}_- &= \mathrm{T}\psi_4(-j)
\end{aligned}
$$

The meaning in summary is this.
The part $(\mathrm{E}_+ + \mathrm{T}ji\mathrm{H}_+)$ is contained in the spinor $\psi = \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4$ in part $\psi_+ = (\psi_1 + \mathrm{T}j\psi_3)$ commuting with i.
The part $(\mathrm{E}_- + \mathrm{T}ji\mathrm{H}_-)$ is contained in the spinor $\psi = \psi_1 + j\psi_2 + \mathrm{T}j\psi_3 + \mathrm{T}\psi_4$ in part $\psi_- = (j\psi_2 + \mathrm{T}\psi_4)$ anticommuting with i.
We can illustrate this in more detail.

The Dirac equation for plane wave at rest has the following 4 solutions

$\psi = e^{-i\omega t}$ $\quad \psi_1 \neq 0$, electron
$\psi = je^{-i\omega t}$ $\quad \psi_2 \neq 0$, electron
$\psi = \mathrm{T}jie^{+i\omega t}$ $\quad \psi_3 \neq 0$, positron
$\psi = \mathrm{T}ji(je^{+i\omega t})$ $\quad \psi_4 \neq 0$, positron

Take the two solutions "electron"

$$\psi = e^{-i\omega t} \qquad \psi_1 \neq 0, \text{ electron}$$
$$\psi = je^{-i\omega t} \qquad \psi_2 \neq 0, \text{ electron}$$

The two solutions have components $1, i, j, ij$.
The first of the two components $1, i$ is interpreted in a natural way as transverse (fictitious) electric field, just ask $\vec{\mathrm{E}}_+ = \psi\hat{\imath} = e^{-i\omega t}\hat{\imath}$.
For the second component j, ij you can not have an interpretation as a transverse field. They do not see a reason.
Moreover certainly in quantum mechanics it represents the solution “electron” with opposite spin. In order to have $1, i$ components and rotate in the opposite direction multiply $-j$ from right. The final formula is $\vec{\mathrm{E}} = \psi_+\hat{\imath} + \psi_-(-j)\hat{\imath}$.
Therefore the mapping that we have established between the even number ψ and $\vec{\mathrm{E}}$ is so done, that the positions j, ij are still related to transverse components $1, i$, but rotating in opposite directions.
The same applies to the ψ components having $\mathrm{T}ji$ in front, which have the same meaning but are magnetic components.
As saying that the mapping $\vec{\mathrm{E}} = \psi_+\hat{\imath} + \psi_-(-j)\hat{\imath}$ holds even if ψ is 8 components, and this provides not only $\vec{\mathrm{E}}$ but also $\vec{\mathrm{H}}$ in the form that we have already written

$$\vec{\mathrm{E}} + \mathrm{T}ji\vec{\mathrm{H}} = \psi_+\hat{\imath} + \psi_-(-j)\hat{\imath}$$

APPENDIX 6

We can compare this observation with that contained in Wang et. al. [11]:
“For the fields produced by an electric source, the electric field E is the large component and the magnetic field B the small component, while for the fields produced by a magnetic source, the magnetic field B is the large component and the electric field E the small component. Similarly, in the electron field, ζ is the large component and χ the small component, while in the positron field, χ is the large component and ζ the small component”.

APPENDIX 7

As an exercise we show that E, H transform like the components of a spinor ψ.
What we can do is:
1° - take the solution at rest for, example, a TE;
2° - then take the solution with velocity $V = v_g$ (group velocity in waveguide);

3° - finally verify that it passes from one to another by one side transformation $R\psi$ on $\psi = \mathrm{E} + Tji\mathrm{H}$, as it should be for a spinor.
(Note: in reality this is obvious because there was only a change of name $\psi = (\psi_1 + \mathrm{T}j\psi_3) = \mathrm{E} + Tji\mathrm{H}$
Let's start with some formulas (example: TE)
The solution in motion, speed $V = v_g$, is:

$$\begin{aligned} \mathrm{E} = \psi_1 &= e^{-i\omega t + ik_z z} \\ i\mathrm{H} = \psi_3 &= \frac{\sqrt{\omega - \omega_0}}{\sqrt{\omega + \omega_0}} e^{-i\omega t + ik_z z} \\ k_z^2 &= \omega^2 - \omega_0^2 \end{aligned}$$

(remember $V = c\sqrt{1 - \frac{\omega_0^2}{\omega^2}}$)

The solution at rest is:

$$\begin{aligned} \mathrm{E} = \psi_1 &= e^{-i\omega_0 t} \\ i\mathrm{H} = \psi_3 &= 0 \\ k_z^2 &= \omega^2 - \omega_0^2 \end{aligned}$$

Let $R = e^{\hat{k}\hat{\mathrm{T}}\frac{\varphi}{2}}$. The vectors are transformed with $R\vec{P}R^*$. Spinors are transformed with $\psi \to R\psi$.

Rotate the spinor $\psi = (\psi_1 + \mathrm{T}j\psi_3) = \mathrm{E} + Tji\mathrm{H}$ with $\psi \to R\psi = e^{\hat{k}\hat{\mathrm{T}}\frac{\varphi}{2}}\psi$.

Contrary to the expression of $e^{\hat{k}\hat{\mathrm{T}}\varphi}$, the expression of $e^{\hat{k}\hat{\mathrm{T}}\frac{\varphi}{2}}$ is less usual.
I proceed step by step:

$$e^{\hat{k}\hat{\mathrm{T}}\frac{\varphi}{2}} = ch\frac{\varphi}{2} + \hat{k}\hat{T}sh\frac{\varphi}{2}$$

Express the hyperbolic functions of $\frac{\varphi}{2}$ as a function of φ:

$$ch\frac{\varphi}{2} = \sqrt{\frac{ch\varphi + 1}{2}}$$

$$sh\frac{\varphi}{2} = \sqrt{\frac{ch\varphi - 1}{2}}$$

However

$$ch\varphi = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$sh\varphi = \frac{\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}}$$

and then replacing

$$ch\frac{\varphi}{2} = \sqrt{\frac{1}{2}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}+1)}$$

$$sh\frac{\varphi}{2} = \sqrt{\frac{1}{2}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1)}$$

Thus from the solution at rest $\mathrm{E} = \psi_1 = e^{-i\omega_0 t}$ the solution in motion $R\psi = e^{\hat{k}\hat{\mathrm{T}}\frac{\varphi}{2}}\psi$ becomes

$$\psi = \left(\sqrt{\frac{1}{2}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}+1)} + \hat{k}\hat{T}\sqrt{\frac{1}{2}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1)}\right)e^{-i\omega t+kz}$$

(note: $(-i\omega t + ikz)$ comes from the transformation of phase $(-i\omega_0 t)$ in new reference). Despite the odd appearance, this is exactly the solution in motion, differs only by a different normalization (here is normalized to $\psi\psi^* = 1$). To see more explicitly write $\hat{k}\hat{T} = Tj$ and $V = c\sqrt{1-\frac{\omega_0^2}{\omega^2}}$ ie $\sqrt{1-\frac{V^2}{c^2}} = \frac{\omega}{\omega_0}$.

Thus we have

$$\psi = \left(\sqrt{\frac{1}{2}\frac{\omega+\omega_0}{\omega_0}} + Tj\sqrt{\frac{1}{2}\frac{\omega-\omega_0}{\omega_0}}\right)e^{-i\omega t+kz}$$

which is clearly the already written solution in motion

$$\psi = \mathrm{E} + Tji\mathrm{H} = e^{-i\omega t+ik_z z} + Tj\frac{\sqrt{\omega-\omega_0}}{\sqrt{\omega+\omega_0}}e^{-i\omega t+ik_z z}$$

$$\mathrm{E} = \psi_1 = e^{-i\omega t+ik_z z}$$

$$i\mathrm{H} = \psi_3 = \frac{\sqrt{\omega-\omega_0}}{\sqrt{\omega+\omega_0}} e^{-i\omega t + ik_z z}$$
$$k_z^2 = \omega^2 - \omega_0^2$$

if normalized to $\psi\psi^* = 1$ through multiplication by $\left(\sqrt{\frac{1}{2}\frac{\omega+\omega_0}{\omega_0}}\right)$

APPENDIX 8

The $\vec{E}_{TE} \times \vec{H}_{TE} = (\vec{\mathrm{E}}\times\vec{\mathrm{H}})eh^*$ is obtained from all these steps in detail.
We recall formulas

$$\frac{1}{2}(ab - ba) = a \wedge b$$
$$a \times b = -\left(\hat{i}\hat{j}\hat{k}\right)(a \wedge b)$$
$$a \wedge b = \left(\hat{i}\hat{j}\hat{k}\right)(a \times b)$$

Let

$$a = \vec{E}_{TE} = \vec{\mathrm{E}}e^*$$
$$b = \vec{H}_{TE} = \vec{\mathrm{H}}h^*$$

We have

$$\vec{E}_{TE} \times \vec{H}_{TE} = -\left(\hat{i}\hat{j}\hat{k}\right)\frac{1}{2}\left(\vec{\mathrm{E}}e^*\vec{\mathrm{H}}h^* - \vec{\mathrm{H}}h^*\vec{\mathrm{E}}e^*\right)$$

or

$$\vec{E}_{TE} \times \vec{H}_{TE} = -\left(\hat{i}\hat{j}\hat{k}\right)\frac{1}{2}\left(\vec{\mathrm{E}}(\vec{\mathrm{H}}eh^*) - (\vec{\mathrm{H}}eh^*)\vec{\mathrm{E}}\right)$$

This is rewritten so as exterior product $\frac{1}{2}(ab - ba) = a \wedge b$ between $\left(\vec{\mathrm{E}}\right)$ e $\left(\vec{\mathrm{H}}eh^*\right)$ which finally $\vec{E}_{TE} \times \vec{H}_{TE} = (\vec{\mathrm{E}}\times\vec{\mathrm{H}})eh^*$.

APPENDIX 9

It 's interesting the physical meaning of $\psi\psi^*$ or better $\frac{1}{2}\psi\psi^*$. For this we note that the following remarkable equality holds

$$(\psi\hat{T}\psi^*)^2 = \psi\hat{T}\psi^*\psi\hat{T}\psi^* = (\psi\psi^*)^2(\hat{T})^2 = -(\psi\psi^*)^2$$

But from the energy momentum vector

$$\frac{1}{2}\psi\hat{T}\psi^* = \frac{1}{2}(|\vec{\mathrm{E}}|^2 + |\vec{\mathrm{H}}|^2)\hat{T} - (\vec{\mathrm{E}}\times\vec{\mathrm{H}})$$

we also have

$$(\frac{1}{2}\psi\hat{T}\psi^*)^2 = -(ENERGY)^2 + (MOMENTUM)^2$$

so by

$$(\frac{1}{2}\psi\psi^*)^2 = -(\frac{1}{2}\psi\hat{T}\psi^*)^2$$

the identification follows

$$(\frac{1}{2}\psi\psi^*)^2 = (MASS)^2 = (ENERGY)^2 - (MOMENTUM)^2$$

which can also be viewed

$$(\frac{1}{2}\psi\psi^*)^2 = (\omega_0)^2 = (\omega)^2 - (k_z)^2$$

So the physical meaning of $(\frac{1}{2}\psi\psi^*)^2$ is the mass-squared or $(\omega_0)^2$.

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